Chapter 2
The Fundamental Properties of Dielectrics. The Doctrines of Faraday and Mossotti

2.1 The Theory of Magnetization by Induction, Precursor to the Theory of Dielectrics

The theory of magnetism has influenced to such a point the development of our knowledge regarding dielectric bodies that we must, first of all, say a few words about this theory.

Aepinus represented magnets as bodies on which two magnetic fluids, equal in amount, are separated such that the one fluid is at one end of the bar, the other fluid at the other end. Coulomb[1] changed this way, universally accepted in his time, of seeing things. [18] He said:

> I believe that one could reconcile the result of experiments with the calculations by making a few changes to the hypotheses; here is one that seems to explain all the magnetic phenomena of which the preceding tests give accurate measurements. It consists in assuming, in the system of Aepinus, that the magnetic fluid is withdrawn in each molecule or integral part of the magnet or steel; that fluid can be transported from one end to the other of this molecule, giving each molecule two poles, but this fluid may not move from one molecule to another. Thus, for example, if a magnetic needle were very small in diameter, or if each molecule could be regarded as a small needle whose north end would be united to the south end of the needle that precedes it, then there are only the two ends, *n* and *s*, of the needle that would give signs of magnetism; thus it would only be at both ends where one of the poles of the molecules would not be in contact with the opposite pole of another molecule.
>
> If such a needle were cut into two parts after having been magnetized, in *a* for example, the end *a* of part *na* would have the same force as the end *s* the whole needle had, and the end *s* of the part *sa* would also have the same force that the end *n* of the whole needle had before being cut.

[1] Coulomb, *Septième Mémoire sur l'Électricité et le Magnétisme*.—*Du Magnétisme* (Mémoires de l'Académie des Sciences pour 1789, p. 488.—Collection de Mémoires relatifs a la Physique, publiés par la Société française de Physique, t. I: *Mémoires de Coulomb*).

P.M.M. Duhem, *The Electric Theories of J. Clerk Maxwell*,
Boston Studies in the Philosophy and History of Science 314,
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> This fact is very accurately confirmed by experience; because if a very long, thin needle is cut into two parts after having been magnetized, each part, tested on a balance, is magnetized to saturation, and although it is magnetized again, it will not acquire a larger force.

Poisson read this passage. He said[2]:

> Before the works of Coulomb, one assumed the two transported fluids, in the process of magnetization, traveled to both ends of compass needles and accumulated at their poles; while, following this illustrious physicist, boreal and austral fluid only experience [19] infinitely small displacements and do not escape from the molecule of the magnetized body to which they belong.

The concept of a magnetic element, thus introduced into physics by Coulomb, is the basis on which the theory given by Poisson, the magnetic induction of the soft iron, rests; here, indeed, is how Poisson sets out[3] the basic hypotheses of this theory:

> Consider a body magnetized by induction, of any shape and dimensions, in which the *coercive* force is zero and which we will call A, for brevity.
>
> From the foregoing, we will look at this body as an assemblage *of magnetic elements*, separated from each other by gaps inaccessible to magnetism, and behold, with respect to these elements, the various hypotheses resulting from the discussion in which we have just entered:
>
> 1. The dimensions of the magnetic elements, and those spaces that insulate them, are unaffected and can be treated as infinitely small relatively to the body A.
> 2. The material of this body places no obstacle to the separation of the two *boreal* and *austral* fluids in the interior of the magnetic elements.
> 3. Portions of the two fluids that the magnetization separates in an any element are still very small relative to the *neutral fluid* that contains this element, and this neutral fluid is never exhausted.
> 4. These portions of fluid, so separated, travel to the surface of the magnetic element where they form a layer whose thickness, variable from one point to another, is everywhere very small and can also be considered infinitely small, even compared to the dimensions of the element.

The theory of magnetization founded by Poisson on these hypotheses is far from perfect, more than a key argument, it lacks rigor or sins against exactitude.[4] But these flaws, to which it was possible to remedy, [20] must not make us forget the results of paramount importance that the theorist definitively introduced into science. Let us recall some of these results, of which we will have to make use in what follows:

Let $d\omega$ be a volume element cut out of any magnet. If it is straight and directed in the magnetic axis of this element, carrying a length equal to the ratio of its magnetic moment by its volume, we get a directed quantity which is the *intensity of magnetization* at a point on the element $d\omega$; M is this size and A, B, C are the components.

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[2]Poisson, *Mémoire sur la théorie du Magnétisme*, lu à l'Académie des Sciences, le 2 février 1824 (MÉMOIRES DE L'ACADÉMIE DES SCIENCES pour les années 1821 et 1822, t. V. p. 250).

[3]Poisson, *loc. cit.*, p. 262.

[4]*Étude historique sur l'aimantation par influence* (ANNALES DE LA FACULTÉ DES SCIENCES DE TOULOUSE, t. II, 1888).

The components X, Y, Z of the *magnetic field*, at a point (x, y, z) outside the magnet, are given by the formulas

$$X = -\frac{\partial V}{\partial x}, \quad Y = -\frac{\partial V}{\partial y}, \quad Z = -\frac{\partial V}{\partial z},$$

V being the *magnetic potential function* of the magnet; this function is defined by the equality:

$$V = \int \left(A_1 \frac{\partial \frac{1}{r}}{\partial x_1} + B_1 \frac{\partial \frac{1}{r}}{\partial y_1} + C_1 \frac{\partial \frac{1}{r}}{\partial z_1} \right) d\omega_1, \tag{2.1}$$

(x_1, y_1, z_1) being a point of the element $d\omega_1$,
A_1, B_1, C_1, the components of magnetization at this point,
r, the distance of two points (x, y, z) and (x_1, y_1, z_1),
and the integration extending over the entire magnet.

This potential function is identical to that which comes from a *fictional distribution* of magnetic fluid, a density distribution, at each point (x, y, z) of the mass of the magnet,

$$\rho = -\left(\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} \right), \tag{2.2}$$

and, at each point of the surface of the magnet, where N_i is the normal directed to the inside of the magnet, having surface density

$$\sigma = -[A \cos(N_i, x) + B \cos(N_i, y) + C \cos(N_i, z)]. \tag{2.3}$$

[21] At each point inside the magnet, we have

$$\Delta V = -4\pi\rho = 4\pi \left(\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} \right). \tag{2.4}$$

At each point of the surface of the magnet, we have

$$\frac{\partial V}{\partial N_i} + \frac{\partial V}{\partial N_e} = -4\pi\sigma = 4\pi [A \cos(N_i, x) + B \cos(N_i, y) + C \cos(N_i, z)] \tag{2.5}$$

If a perfectly soft body is subjected to the influence of a magnet, it is magnetized so that the components of magnetization at each point (x, y, z) of the magnet are linked by the following equalities to the potential function of both the inducing and the induced magnetization:

$$A = -K \frac{\partial V}{\partial x}, \quad B = -K \frac{\partial V}{\partial y}, \quad C = -K \frac{\partial V}{\partial z}. \tag{2.6}$$

In these equalities, K is a constant amount for a given body at a given temperature; it is called *coefficient of magnetization* of the body.

This starting point is sufficient to put the problem of magnetization by induction on bodies devoid of a coercive force completely into equations.

These various results, we said, remained committed to science; only equalities (2.6) have been changed. To account for various phenomena presented by highly magnetic bodies, such as soft iron, and, in particular, the phenomenon of *saturation*, G. Kirchhoff proposed[5] replacing the coefficient of magnetization K by a *magnetizing function* $f(M)$ which varies not only with nature and the body temperature, but [22] also with intensity M of the magnetization. Equalities (2.6) are then replaced by the equalities

$$A = -f(M)\frac{\partial V}{\partial x}, \quad B = -f(M)\frac{\partial V}{\partial y}, \quad C = -f(M)\frac{\partial V}{\partial z}. \tag{2.7}$$

For weakly magnetic bodies, this magnetizing function is reduced, as Poisson wanted, to a coefficient of magnetization.

One can, as indicated by Émile Mathieu[6] and later, by H. Poincaré,[7] remove the inaccuracies in reasoning which mar the theory of Poisson and avoid the experimental difficulties which militate against it. However, the same hypotheses on which this theory is based have something naive which shocks the habits of contemporary physicists. W. Thomson said[8]:

> [I]n the present state of science, no theory founded on Poisson's hypothesis of "two magnetic fluids" moveable in the "magnetic elements" could be satisfactory, as it is generally admitted that the truth of any such hypothesis is extremely improbable. Hence it is at present desirable that a complete theory of magnetic induction in crystalline or non-crystalline matter should be established independently of any hypothesis of magnetic fluids, and, if possible, upon a purely experimental foundation. With this object, I have endeavoured to detach the hypothesis of magnetic fluids from Poisson's theory, and to substitute elementary principles deducible from it as the foundation of a mathematical theory identical with Poisson's in all substantial [23] conclusions.

[5] G. Kirchhoff, *Ueber den inducirten Magnetismus eines unbegrenzten Cylinders von weichem Eisen* (CRELLE'S JOURNAL FÜR REINE UND ANGEWANDTE MATHEMATIK, Bd. XLVIII, p. 348, 1853.—G. KIRCHHOFF'S ABHANDLUNGEN, p. 103, Berlin, 1882).

[6] É. Mathieu, *Théorie du Potentiel et ses applications à l'Électrostatique et Magnétisme*; 2e partie: *Applications* (Paris, 1886).

[7] H. Poincaré, *Électricité et Optique*, I.—*Les théories de Maxwell et la théorie électromagnétique de la lumière*, leçons professées à la Sorbonne pendant le second semestre 1888–1889, p. 44 (Paris, 1890).

[8] W. Thomson, *On the Theory of Magnetic Induction in Crystalline and Non-Crystalline Substances* (PHILOSOPHICAL MAGAZINE, 4th series, vol. I, pp. 177–186, 1851.—PAPERS ON ELECTROSTATICS AND MAGNETISM, art. XXX, Sect. 604; London, 1872).

Instead of imagining a magnet as a cluster of magnetic particles equally charged by austral and boreal fluid, and embedded in a medium impermeable to magnetic fluids, Sir W. Thomson treats this magnet as a continuous body whose properties depend on the value taken at each point, by a certain directed quantity, the intensity of magnetization. The fundamental hypotheses that characterize this quantity in magnets in general and in bodies devoid of coercive force in particular are equivalent to the diverse equations that are generally admitted today; it makes the developments of the theory of magnetism easier and more elegant, and at the same time satisfying more our desire to make physical hypotheses independent of any supposition about the existence or properties of molecules.

It is, in the study of magnetism, a special point that has certainly influenced the theory of dielectrics and, in particular, has contributed to introducing the idea of Faraday that the ether, empty of all ponderable matter, is endowed with dielectric properties. This point is the study of *diamagnetic* bodies.

Faraday acknowledged that a bar of bismuth took on, at each point, a magnetization directed not as the magnetic field, but in the direction opposite of this field; bismuth is *diamagnetic*.

At first, diamagnetism seems scarcely compatible with the theory of magnetism by Poisson; magnetic particles can be magnetized only in the direction of the field. The contradiction disappears assuming a hypothesis by Edmond Becquerel.[9]

According to this hypothesis, all bodies, even bismuth, would be magnetic; but ether, deprived of any other material, would also be magnetic. Under these conditions, the bodies we call magnetic would be more magnetic than [24] ether; the bodies less magnetic than ether would seem diamagnetic.

The impossibility of properly diamagnetic bodies, manifest in the hypothesis of Poisson, is no longer so when it exposes the foundations of the theory of magnetism as suggested by W. Thomson; nothing, it seems, prevents one from assigning a negative value to the magnetizing function in Eq. (2.7), which become mere hypotheses. Also, in many places in his writings on magnetism, W. Thomson does not bother to treat actual diamagnetic bodies.

The contradictions that would lead to the existence of such bodies appear again when comparing the laws of magnetism to the principles of thermodynamics.

These contradictions were seen for the first time by W. Thomson, in the testimony of Tait[10]:

> The commonly received opinion, that a diamagnetic body in a field of magnetic force takes the *opposite* polarity to that produced in a paramagnetic body similarly circumstanced, is thus attacked by Thomson by an application of the principle of energy. Since all paramagnetic bodies require time for the full development of their magnetism, and do not instantly lose it when the magnetising force is removed, we may of course suppose the same to be true for diamagnetic bodies; and it is easy to see that in such a case a homogeneous non-crystalline diamagnetic sphere rotating in a field of magnetic force would, if it always tended to take the opposite distribution of magnetism to that acquired by iron under the same circumstances,

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[9]Edmond Becquerel, *De l'action du Magnétisme sur tous les corps* (Comptes Rendus, t. XXXI, p. 198; 1850.—Annales de Chimie et de Physique, 3^{e} série t. XXVIII, p. 283, 1850).

[10]Tait, *Sketch of Thermodynamics* [p. 88].

> be acted upon by a couple constantly tending to turn it in the same direction round its centre, and would therefore be a source of the perpetual motion.

John Parker,[11] by similar reasoning, has shown that the existence of the diamagnetic body would be inconsistent with the principle of Carnot. [25]

Finally, E. Beltrami[12] and ourselves[13] arrived at the conclusion that if we can find, on a diamagnetic body placed in a given field, a magnetic distribution that satisfies Eq. (2.7), this distribution corresponds to a state of unstable equilibrium. It is therefore impossible to admit the existence of a diamagnetic body properly so-called and necessary for the hypothesis of Edmond Becquerel: the ether is susceptible to being magnetized.

2.2 The Polarization of Dielectrics

If the hypotheses of Coulomb and Poisson on the constitution of magnetic bodies extremely deviate from the principles in favor with physicists today, their sharpness, their simplicity, the ease with which the imagination could grasp them, should be, for theorists of the beginning of the century, one of the most alluring hypotheses of physics. All properties that we represent today by *directed quantities* were then attributed to *polarized molecules*, i.e. with molecules, at both ends, of opposite qualities; one sought for analogues of *magnetic polarization*.

The idea of comparing to iron, under the influence of the magnet, the insulating substances, such as glass, sulfur or shellac, subject to the action of electrified bodies, has no doubt offered itself to the minds of physicists. Already Coulomb, in the passage following what we already cited, the following[14] this: [26]

> The hypothesis that we just made seems very similar to this well-known electrical experience: when one charges a pane of glass covered with two metal planes; however thin the planes are, if one is away from the glass pane, they give very considerable signs of electricity; the surfaces of the glass, after one discharges the electricity of the linings, are themselves steeped in two contrary currents and form a very good electrophorus; this phenomenon is related somewhat to the thickness that one gives to the glass plane; thus the electric fluid, albeit of a different nature on both sides of the glass, penetrates the surface to an infinitely

[11] John Parker, *On Diamagnetism and Concentration of Energy* (PHILOSOPHICAL MAGAZINE, 5th, vol. XXVII, p. 403, 1889).

[12] E. Beltrami, *Note fisico-matematiche, lettera al prof. Ernesto Cesàro* (RENDICONTI DEL CIRCOLO MATEMATICO DI PALERMO, t. III, meeting of 10 March 1889).

[13] *Sur l'aimantation par influence* (COMPTES RENDUS, t. CV, p. 798, 1887)—*Sur l'aimantation des corps diamagnétiques* (COMPTES RENDUS, t. CVI, p. 736, 1888).—*Théorie nouvelle de l'aimantation par influence fondée sur la thermodynamique* (ANNALES DE LA FACULTÉ DES SCIENCES DE TOULOUSE, t. II, 1888).—*Sur l'impossibilité des corps diamagnétiques* (TRAVAUX ET MÉMOIRES DES FACULTÉS DE LILLE, mémoire n° 2, 1889).—*Leçons sur l'Électricité et le Magnétisme*, t. II, p. 221, 1892.

[14] Coulomb, *Septième Mémoire sur l'Électricité et le Magnétisme* (MÉMOIRES DE L'ACADÉMIE DES SCIENCES DE PARIS pour 1789, p. 489. COLLECTION DE MÉMOIRES RELATIFS A LA PHYSIQUE, publiés par la Société française de Physique; t. I: *Mémoires de Coulomb*).

> small distance, and this pane looks exactly like a magnetised molecule of our needle. And if now one placed on the other a series of panes in such a way that, in the meeting of the panes, the positive side forms the surface of the first pane located several inches away from the negative surface of the last pane, each surface of the extremities, as experience also proves, will produce, at fairly considerable distances, effects as sensitive as our magnetic needles; although the fluid of each surface of the panes on the extremities penetrates these tiles to an infinitesimally small depth and electrical fluids from all surfaces in contact balance each other, since one of the faces is positive, the other negative.

A few years later, Avogadro[15] also admitted that the molecules of a non-conductive body of electricity are polarized under the influence of a charged conductor. In the terms of Mossotti,[16] "Professor Orioli used induction exercised by one molecule on another, or one thin disk of glass on another, to explain the mode of action of the electrical machine." [27]

But it is to Faraday that we owe the first extensive developments on the electrification of insulating bodies.

Faraday was careful to specify the following about the thoughts that led him to imagine his hypotheses about the constitution of the *dielectric bodies*[17]:

> In the long-continued course of experimental inquiry in which I have been engaged, this general result has pressed upon me constantly, namely, the necessity of admitting two forces, or two forms or directions of a force…, combined with the impossibility of separating these two forces (or electricities) from each other, either in the phenomena of statical electricity or those of the current. In association with this, the impossibility under any circumstances, as yet, of absolutely charging matter of any kind with one or the other electricity only, dwelt on my mind, and made me wish and search for a clearer view than any that I was acquainted with, of the way in which electrical powers and the particles of matter are related; especially in inductive actions, upon which almost all others appeared to rest.

Two theories have, by way of analogy, guided Faraday in his hypotheses affecting the polarization of the dielectric body: the theory of magnetism and the theory of electrolytic actions.

Everyone knows about the representation, imagined by Grotthuss, of the state in which a current traversing an electrolyte is situated; each molecule is oriented in the direction of the current, the electrically positive atom on the side of the negative electrode and the electrically negative atom on the side of the positive electrode. But Faraday is struck[18] by the resemblance a voltmeter has with a capacitor. Put a plate of ice between two sheets of platinum; charge one of the leaves of positive electricity and other with negative electricity; you will have a dielectric plate capacitor; [28]

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[15] Avogadro, *Considérations sur l'état dans lequel doit se trouver une couche d'un corps non conducteur de l'électricité lorsqu'elle est interposée entre deux surfaces douées d'électricité de différente espèce* (JOURNAL DE PHYSIQUE, t. LXIII, p. 450, 1806).—*Second Mémoire sur l'Électricité* (JOURNAL DE PHYSIQUE, t. LXV, p. 130, 1807).

[16] Mossotti, *Recherches théoriques sur l'induction électrostatique envisagée d'après les idées de Faraday* (BIBLIOTHÈQUE UNIVERSELLE, Archives, t. VI, p. 193, 1847).

[17] Faraday, *On Induction*, read at the Royal Society of London, 21 December 1837 (PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY OF LONDON, 1838, p. 1.—Faraday's EXPERIMENTAL RESEARCHES IN ELECTRICITY, series I, vol. I, n° 1163, p. 361).

[18] Faraday, *loc. cit.* (EXPERIMENTAL RESEARCHES, 1. 1, p. 361).

now melt the ice; the water will be electrolyzed; you will have a voltameter. From where does this difference come? Simply, from the liquid state of water allowing ions to travel on the two electrodes; as to the electric polarization of particles, one must assume it pre-exists their mobility and that it already occurred in the ice.

> …as the whole effect in the electrolyte appeared to be an action of the particles thrown into a peculiar or polarized state, I was led to suspect that common induction itself was in all cases an *action of contiguous particles*, and that electrical action at a distance (i.e. ordinary inductive action) never occurred except through the influence of the intervening matter.

How will these contiguous particles influence each other? Faraday repeatedly describes this action.

> Induction appears[19] to consist in a certain polarized state of the particles, into which they are thrown by the electrified body sustaining the action, the particles assuming positive and negative points or parts, which are symmetrically arranged with respect to each other and the inducting surfaces or particles.
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> The theory[20] assumes that all the *particles*, whether of insulating or conducting matter, are as wholes conductors. That not being polar in their normal state, they can become so by the influence of neighbouring charged particles, the polar state being developed at the instant, exactly as in an insulated conducting mass consisting of many particles.
>
> …The particles of an insulating dielectric whilst under induction may be compared to a series of small magnetic needles, or more correctly still to a series of small insulated conductors. If the space round a charged globe were filled with a mixture of an insulating dielectric, as oil of turpentine or [29] air, and small globular conductors, as shot, the latter being at a little distance from each other so as to be insulated, then these would in their condition and action exactly resemble what I consider to be the condition and action of the particles of the insulating dielectric itself. If the globe were charged, these little conductors would all be polar; if the globe were discharged, they would all return to their normal state, to be polarized again upon the recharging of the globe.

It is clear that Faraday imagines the constitution of dielectric bodies in the exact likeness of what Coulomb and Poisson assigned to magnetic bodies; it does not, however, appear that Faraday thought about bringing to his ideas on electric polarization the consequences to which the theory of magnetization by induction led Poisson.

This reconciliation is shown for the first time, in a succinct but clear manner, in one of the early writings of W. Thomson.[21] He said:

> It is therefore necessary that there be a very special action in the interior of solid *dielectric* bodies to produce this effect. It is likely that this phenomenon would be explained by giving the body an action similar to that which would occur if there were no action in the insulating dielectric medium and if there were a very large number of small conducting spheres uniformly distributed in the body. Poisson showed that the electric action, in this case, would be

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[19] Faraday, *loc. cit.* (EXPERIMENTAL RESEARCHES, vol. I, p. 409).

[20] Faraday, *Nature of the Electric Force or Forces*, read at the Royal Society of London, on 21 June 1838 (PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY OF LONDON, 1838, pp. 265 à 282.—EXPERIMENTAL RESEARCHES, série XIV, vol. I,. p. 534).

[21] W. Thomson, *Note sur les lois élémentaires de l'électricité statique* (JOURNAL DE LIOUVILLE, t. X, p. 220, 1845.—Reproduced, with some developments, under the title: *On the Elementary Laws of Statical Electricity*, in CAMBRIDGE AND DUBLIN MATHEMATICAL JOURNAL, nov. 1845, and in PAPERS ON ELECTROSTATICS AND MAGNETISM, art. II, Sect. 25).

> quite similar to the action of soft iron magnet under the influence of the magnetized bodies. Based on the theorems he gave with respect to this action, it is easily able to show that if the space between A and B is filled with a mixture thus constituted, the surfaces of equilibrium are the same as when there is only an insulating dielectric medium without dielectric power, but the potential in the interior of A will be smaller than in the latter case, in a ratio that it is easy to determine from the data [30] related to the state of the insulating medium. This conclusion seems to be sufficient to explain the facts that Faraday has observed with respect to dielectric media…[22]

Around the same time, the Italian Society of Sciences, in Modena, began to contest the following question:

> Taking as a starting point the ideas of Faraday on electrostatic induction, give a physico-mathematical theory of the distribution of electricity on conductors of various shapes.

It suffices for Mossotti[23] to resolve the problem, to make a kind of transposition of the formulas that Poisson had obtained in the study of magnetism; this transposition was then completed by Clausius.[24]

To accept the ideas of Faraday, Mossotti, and Clausius on the constitution of the dielectric body seems as difficult today as to admit the hypotheses of Coulomb and Poisson about the magnetic body; but it is easy to subject to the polarization theory a theory analogous to what W. Thomson did for the theory of magnetization; it is a theory thus stripped of any consideration of the polarized molecules of which H. von Helmholtz made use.[25]

We note the foundations of this theory.

At the beginning of the study of electrostatics, two types of undirected [31] quantities are enough to define the distribution of electricity on a body; these two quantities were the *solid electric density* σ at each point inside the body and the *surface electric density* Σ at each point on the surface of the body. Even the founders of electrostatics took this notion for that one; they regarded the surface of bodies as having a very thin, but not infinitely thin, electrical layer.

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[22] [Translated from the French].

[23] Mossotti, *Discussione analitica sull'influenza che l'azzione di un mezzo dielettrico ha sulla distribuzione dell'eleitricità alla superfizie dei piu corpi elettrici disseminati in esso* (MÉMOIRES DE LA SOCIÉTÉ ITALIENNE DE MODÈNE, t. XXIV, p. 49, 1850).—Extraits du même (BIBLIOTHÈQUE UNIVERSELLE, ARCHIVES, t. VI, p. 357, 1847).—*Recherches théoriques sur l'induction électrostatique envisagée d'après les idées de Faraday* (BIBLIOTHÈQUE UNIVERSELLE, ARCHIVES, t. VI, p. 193; 1847).

[24] R. Clausius, *Sur le changement détat intérieur qui a lieu, pendant la charge, dans la couche isolante d'un carreau de Franklin ou d'une bouteille de Leyde, et sur l'influence de ce changement sur le phénomène de la décharge* (ABHANDLUNGENSAMMLUNG ÜBER DIE MECHANISCHE THÉORIE DER WARME, Bd. II, ZUSATZ ZU ABHANDL. X, 1867.—THÉORIE MÉCANIQUE DE LA CHALEUR, traduite en français par F. Folie, t. II, ADDITION AU MÉMOIRE, X, 1869).

[25] H. Helmholtz, *Ueber die Bewegungsgleichungen der Elektrieitat für ruhende leitende Körper*, §8 (BORCHARDT'S JOURNAL FÜR REINE UND ANGEWANDTE MATHEMATIK, Bd. LXXII, p. 114, 1870.—WISSENSCHAFTLICHE ABHANDLUNGEN, Bd. I, p. 611).

Later, the study of abrupt drops of the potential level in contact with two different conductors led to the introduction of a third directed quantity, irreducible to previous ones: the *moment of a double layer* at each point of the surface of contact of the two conductors.

These three species of quantities no longer suffice to represent fully the distribution of electricity on a system when this system contains poorly conductive bodies; to complete the representation of a similar system, it is necessary to make use of a new quantity, a directed magnitude that is assigned to each point of a dielectric body and called the *intensity of polarization* at this point.

A dielectric body is thus a body in which there is an intensity of polarization at each point, defined in magnitude and direction, as a magnetic body is a body in which there is an intensity of magnetization, defined in magnitude and direction at each point. The hypotheses to which the intensity of polarization are subjected, moreover, are modeled after the basic hypotheses that characterize the intensity of magnetization. A single hypothesis—essential, it is true—is proper to the intensity of polarization. This hypothesis, to which one is necessarily led by how Faraday and his successors have represented the constitution of dielectrics, is as follows:

A dielectric element, with volume $d\omega$, whose intensity of polarization has components A, B, C, exerts on an electric charge, placed at a finite distance, the same action as two equal electric charges, the one having μ, the other having $-\mu$, placed first at a point M of the element $d\omega$, the second at a point M' of the same element, so that the direction $M'M$ is that of the polarization; and so we have the equality

$$M.\overline{MM'} = \left(A^2 + B^2 + C^2\right)^{\frac{1}{2}} d\omega.$$

[32] On the contrary, it is recognized that *a magnetic element is not on an electric charge*.

Before summing up the consequences that can be drawn from these hypotheses, let us insist a moment still on the transformation that the hypotheses made by the founders of electrostatics have undergone.

Four species of quantities—the solid electric density, the surface electric density, the moment of a double layer, the intensity of polarization—are used today to represent the electrical distribution on a system. The founders of electrostatics—Coulomb, Laplace, and Poisson—made use of only one of these quantities, solid electric density; they admitted it willingly in their theories because they succeeded without difficulty to imagine the density as of a certain fluid; they reduced the other three quantities to this one. Instead of regarding the electrical layer that covers a body as lacking thickness and assigning it a surface density, they imagined it as a finite, though very small, thickness in which electricity has a finite, though very large, solid density; two such layers, identical in sign near the electricity which they are formed, placed a small distance from the each other other, replaced our present double layer, without thickness. Finally, instead of conceiving, at each point of a dielectric, an intensity of polarization of set magnitude and direction, they placed a conductive particle covered with an electrical layer which contained as much positive as negative fluid.

Today, we no longer require of physical theories a simple and easy-to-imagine mechanism which explains the phenomena. We look at them as rational and abstract constructs that are intended to symbolize a set of experimental laws; therefore, to *represent* the *qualities* that we are studying, we accept without difficulty in our theories quantities of any nature, provided only that these quantities are clearly defined, regardless of whether or not the imagination seizes the properties served by these quantities. For example, the concepts of intensity of magnetization or intensity of polarization remain inaccessible to the imagination, which captures very well, on the contrary, the magnetic particles of Poisson, the [33] electric corpuscles of Faraday, covered at both ends, by fluid layers of opposite signs. But the concept of intensity of polarization involves a much smaller number of arbitrary hypotheses than the notion of a polarized particle; it is more completely cleared of any hypothesis on the constitution of matter. Substituting continuity for discontinuity, it lends to simpler and more rigorous calculations; we owe it preference.

2.3 Key Propositions of the Theory of Dielectrics

The principles we have analyzed allow the development of a complete theory of the electrical distribution on systems of conductive bodies and dielectric bodies. We briefly indicate, and without any demonstration,[26] the key proposals which we will have to use later.

We imagine two small bodies, placed at the distance r the one from the other and carrying quantities q and q' of electricity; we conceive these two small bodies placed not in *ether*, i.e. in what would contain a container where one would have made the physical vacuum, but in the *absolute vacuum*, i.e. in a medium identical to the space of the geometers, having length, width and depth, but devoid of any physical property, in particular the power to magnetize or polarize. The distinction is important; indeed, we have seen that the existence of diamagnetic bodies would be contradictory if the faculty of magnetizing were not attributed to ether, according to the hypothesis admitted by Edmond Becquerel; and, since Faraday, all physicists agree to assign dielectric polarization to the ether.

By an extension of Coulomb's laws (experience verifies these laws for a body placed in the air, but it is not conceivable for a body placed in the absolute vacuum), we assume that these two small bodies repel with a force

$$F = \varepsilon \frac{qq'}{r^2}, \tag{2.8}$$

ε being some positive constant.

[26]The reader may find these demonstrations in our *Leçons sur l'Électricité et le Magnétisme*, t. II, 1892.

[34] Suppose that an ensemble of electrified bodies is placed in space and let

$$V = \sum \frac{q}{r} \tag{2.9}$$

be their *potential function*. At any one point (x, y, z) outside the charged conductor, or *inside* one of them, an electric charge μ undergoes an action whose components are μX, μY, μZ, and we have

$$X = -\varepsilon \frac{\partial V}{\partial x}, \quad Y = -\varepsilon \frac{\partial V}{\partial y}, \quad Z = -\varepsilon \frac{\partial V}{\partial z}. \tag{2.10}$$

Now imagine a set of polarized dielectric bodies. Let $d\omega_1$ be a dielectric element, (x_1, y_1, z_1) a point of this element, and A_1, B_1, C_1 the components of polarization at the point (x_1, y_1, z_1).

$$\overline{V}(x, y, z) = \int \left(A_1 \frac{\partial \frac{1}{r}}{\partial x_1} + B_1 \frac{\partial \frac{1}{r}}{\partial y_1} + C_1 \frac{\partial \frac{1}{r}}{\partial z_1} \right) d\omega_1, \tag{2.11}$$

where the integration extends over the ensemble of polarized dielectrics. This formula defines, at the point (x, y, z), the *potential function* of this set. In formula (2.11), which recalls exactly the expression (2.1) of the magnetic potential function, r is the mutual distance of two points (x, y, z), (x_1, y_1, z_1).

The electrostatic field created by the dielectric at the point (x, y, z) has for components

$$\overline{X} = -\varepsilon \frac{\partial \overline{V}}{\partial x}, \quad \overline{Y} = -\varepsilon \frac{\partial \overline{V}}{\partial y}, \quad \overline{Z} = -\varepsilon \frac{\partial \overline{V}}{\partial z}. \tag{2.12}$$

The potential function V, defined by equality (2.11), is identical to the electrostatic potential function that formula (2.9), applied to a certain *fictitious electrical distribution*, defines; in this [35] fictitious distribution, each point (x, y, z) inside the polarized dielectric is assigned a solid density

$$e = -\left(\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} \right), \tag{2.13}$$

and every point on the surface of two different polarized bodies, designated by indices 1 and 2, corresponds to a surface density

$$\begin{aligned} E = -[&A_1 \cos(N_1, x) + B_1 \cos(N_1, y) + C_1 \cos(N_1, z) \\ &+ A_2 \cos(N_2, x) + B_2 \cos(N_2, y) + C_2 \cos(N_2, z)]. \end{aligned} \tag{2.14}$$

If one of the two bodies, body 2 for example, is incapable of dielectric polarization, it is sufficient, in the previous formula, to suppress the terms in A_2, B_2, C_2.

We see that at any point inside a continuous dielectric, we have

$$\Delta \overline{V} = -4\pi e = 4\pi \left(\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} \right), \tag{2.15}$$

while at any point on the surface of two dielectrics, we have

$$\frac{\partial \overline{V}}{\partial N_1} + \frac{\partial \overline{V}}{\partial N_2} = -4\pi E \tag{2.16}$$

$$\begin{aligned} = 4\pi \, [\ & A_1 \cos(N_1, x) + B_1 \cos(N_1, y) + C_1 \cos(N_1, z) \\ & + A_2 \cos(N_2, x) + B_2 \cos(N_2, y) + C_2 \cos(N_2, z)] \,. \end{aligned}$$

Consider a system where all bodies likely to be charged are good conductive bodies, homogeneous and non-decomposable by electrolysis, and where all the bodies likely to be polarized are perfectly soft dielectrics; on such a system, electrical equilibrium will be ensured by the following conditions:

1. In each of the conductive bodies, we have

$$V + \overline{V} = \text{const.} \tag{2.17}$$

2. [36] At each point of a dielectric, we have

$$\begin{cases} A = -\varepsilon F(M) \dfrac{\partial}{\partial x}(V + \overline{V}), \\ B = -\varepsilon F(M) \dfrac{\partial}{\partial y}(V + \overline{V}), \\ C = -\varepsilon F(M) \dfrac{\partial}{\partial z}(V + \overline{V}). \end{cases} \tag{2.18}$$

In these formulas,

$$M = \left(A^2 + B^2 + C^2 \right)^{\frac{1}{2}}$$

is the intensity of polarization at the point (x, y, z) and $F(M)$ is an essentially positive function of M; this function depends on the nature of the dielectric at the point (x, y, z); from one point to the other, it varies continuously or intermittently depending on whether the nature and the state of the bodies vary in a continuous or discontinuous manner.

In general, as a first approximation, we are content to replace $F(M)$ by a *coefficient of polarization* F, independent of the intensity M of the polarization; with this approximation, equalities (2.18) become

$$\begin{cases} A = -\varepsilon F \dfrac{\partial}{\partial x}(V + \overline{V}), \\ B = -\varepsilon F \dfrac{\partial}{\partial y}(V + \overline{V}), \\ C = -\varepsilon F \dfrac{\partial}{\partial z}(V + \overline{V}). \end{cases} \tag{2.19}$$

This immediately leads to two relationships that will have, in this study, a great importance.

In the first place, compared to equality (2.13), equalities (2.19) show that we have, at any point of a continuous dielectric medium, the equality

$$\varepsilon \frac{\partial}{\partial x}\left[\frac{\partial(V + \overline{V})}{\partial x}\right] + \varepsilon \frac{\partial}{\partial y}\left[\frac{\partial(V + \overline{V})}{\partial y}\right] + \varepsilon \frac{\partial}{\partial z}\left[\frac{\partial(V + \overline{V})}{\partial z}\right] = e. \tag{2.20}$$

[37] In the second place, compared to equality (2.14), equalities (2.19) show that at any point on the surface of two different media, we have

$$\varepsilon F_1 \frac{\partial(V + \overline{V})}{\partial N_1} + \varepsilon F_2 \frac{\partial(V + \overline{V})}{\partial N_2} = E. \tag{2.21}$$

From these equalities we draw some important consequences. In the case where it is applied to a homogeneous dielectric, the formula (2.20) becomes

$$\varepsilon F \varDelta(V + \overline{V}) = e.$$

This equality, combined with equalities (2.15) and

$$\varDelta V = 0,$$

satisfied at any point where there is no real electricity, gives the equality

$$(1 + 4\pi\varepsilon F)\varDelta(V + \overline{V}) = 0,$$

and since F is essentially positive, this equality is, in turn,

$$\varDelta(V + \overline{V}) = 0, \tag{2.22}$$

and

$$e = 0. \tag{2.23}$$

Hence the following proposition, demonstrated by Poisson in the case of the magnetic induction and transposed by W. Thomson and Mossotti to the case of dielectrics:

When a dielectric, homogeneous, and perfectly soft body is polarized by induction, the fictitious electric distribution that would equal the polarization of this body is a purely superficial distribution.

Imagine now that dielectric 1 is in contact along an area with a charged body 2, but incapable of any polarization. To each point on this surface, [38] two electric surface densities correspond: a *real* density Σ and a *fictitious* density E; with equalities (2.16) et (2.21), we can attain the well known equality

$$\frac{\partial V}{\partial N_1} + \frac{\partial V}{\partial N_2} = -4\pi\Sigma$$

and also the equality

$$\frac{\partial V}{\partial N_2} + \frac{\partial \overline{V}}{\partial N_2} = -4\pi\Sigma,$$

which derives from the condition (2.17). We thus obtain equality

$$4\pi\varepsilon F_1 \Sigma + (1 + 4\pi\varepsilon F_1)E = 0. \tag{2.24}$$

On the surface of contact of a conductor and a dielectric, the density of the actual electrical layer Σ is to the density of the fictitious electrical layer E in a negative ratio $\left(-\frac{1+4\pi\varepsilon F}{4\pi\varepsilon F}\right)$, *larger than* 1 *in absolute value and only dependent on the nature of the dielectric.*

The formulas and theorems we have just quickly reviewed pertain to placing into equations the issues raised by the study of dielectrics. Two of these issues will play a major role in the discussions that will follow; it is important to recall the solution in a few words.

The first of these problems concerns capacitors.

Imagine an enclosed capacitor. At any point of the internal armature, the sum $(V + \overline{V})$ has the same value U_1, while at any point of the external armature, it has the value U_0. The gap between the two armatures is occupied by a homogeneous dielectric D where F is the coefficient of polarization. It is shown without difficulty that, in these circumstances, the internal armature becomes covered with a real electric charge Q given by the formula

$$Q = \frac{1 + 4\pi\varepsilon F}{4\pi} A(U_1 - U_0),$$

A being a quantity that depends only on the geometric shape [39] of the space between the two armatures. The *capacitance of the capacitor*, i.e. the ratio

$$C = \frac{Q}{\varepsilon(U_1 - U_0)},$$

has the value

$$C = \frac{1 + 4\pi\varepsilon F}{4\pi\varepsilon} A. \tag{2.25}$$

Take a capacitor of identical shape to the previous one and place between the armatures of this capacitor a new dielectric D', having a coefficient of polarization F'; the capacitance of this second capacitor will have the value

$$C' = \frac{1 + 4\pi\varepsilon F'}{4\pi\varepsilon} A.$$

As Cavendish did it, in 1771, in some researches[27] that remained unpublished for one hundred years, so Faraday[28] did it again as early as 1837, experimentally determining the ratio of the capacitance of the second capacitor to the capacitance of the first; the result of this measurement will be the number

$$\frac{C'}{C} = \frac{1 + 4\pi\varepsilon F'}{1 + 4\pi\varepsilon F}. \tag{2.26}$$

This number will only depend on the nature of two dielectrics D and D'; this number is given the name of *specific inductive capacity of the dielectric D', relative to the dielectric D*.

By definition, the *absolute specific inductive capacitance* of a dielectric D is the number $(1 + 4\pi\varepsilon F)$; for a non-polarizable medium, it is equal to 1. [40]

The consideration of the second problem is more strictly needed when one considers ether as susceptible to dielectric polarization.

Electrostatics as a whole is built assuming that conductive or dielectric bodies are isolated in the absolute vacuum. If one accepts the hypothesis that we have just discussed, such electrostatics is a pure abstraction, unable to give a picture of reality; but, by a fortunate circumstance, one can easily transform this electrostatics into another where unlimited space, which was empty in the first, is filled by a homogeneous, incompressible, and polarizable ether.

Let F_0 be the coefficient of polarization of the medium in which the studied bodies are immersed. These bodies are of homogeneous conductors of electricity and perfectly soft dielectric. What will the distribution of electricity on such a system in equilibrium be? What forces will the various bodies of which it consists produce?

The following rule reduces the solution of these questions to classical electrostatics:

Replace the polarizable vacuum for the ether; for each conductive body, leave the total electrical charge it bears in reality; to each dielectric, attribute a coefficient φ of fictitious polarization, equal to the excess of its real coefficient of polarization F over the coefficient of polarization F_0 of the ether:

$$\varphi = F - F_0; \tag{2.27}$$

[27] *The electrical Researches of the honourable* Henry Cavendish, F. R. S., written between 1771 and 1781; edited by J. Clerk Maxwell (Cambridge).

[28] Faraday, Experimental Researches in Electricity, series XI, *On Induction*; §5. *On Specific Induction*, On Specific Inductive Capacity. Read at the Royal Society of London, 21 December 1837.

finally, replace the constant e by a fictitious constant

$$\varepsilon' = \frac{\varepsilon}{1 + 4\pi\varepsilon F_0}. \tag{2.28}$$

You will get a fictitious system corresponding to the actual given system.

The electrical distribution on the conductive bodies will be the same in the fictional system as in the actual system.

The ponderomotive actions will be the same in the fictional system as in the actual system.

As for the polarization at each point of one of the dielectric bodies [41] *other than the ether, it has the same direction in the fictional system and in the actual system; but, to obtain its value in the second system, the value that it has in the former must be multiplied by* $\frac{F}{F-F_0}$.

2.4 The Particular Idea of Faraday

From the ideas of Faraday on the polarization we have extracted so far what is more general, what gave birth to the theory of dielectrics. These general ideas are far from representing, in their fullness, the thought of Faraday. Faraday professed, in addition, a very particular opinion on the relationship that exists between the electric charge comprising a conductor and the polarization of the dielectric medium in which the conductor is immersed. This opinion of Faraday did not escape Mossotti, which he adopted; on the other hand, it seems to have struck no contemporary physicist. Heinrich Hertz[29] has exhibited this opinion, observing that it is a limiting case of the theory of Helmholtz, already reported by the great physicist; but neither Helmholtz, nor Hertz, attributed it to Faraday and Mossotti.

For him who reads Faraday with careful attention, it is clear that he admitted the following law:

When a dielectric medium is polarized under the action of charged conductors, at each point on the surface of contact of a conductor and dielectric, the density of the fictitious surface layer that covers the dielectric is EQUAL AND OPPOSITE IN SIGN *to the density of the actual electrical layer that covers the conductor:*

$$E + \Sigma = 0. \tag{2.29}$$

Faraday wrote to D^r^ Hare[30]:

[29] Heinrich Hertz, *Untersuchungen über die Aushreitung der elektrischen Kraft: Einleitende Uebersicht*; Leipzig, 1892. [English translation: Hertz(1893)]—Traduit en français par M. Raveau (LA LUMIÈRE ÉLECTRIQUE, t. XLIV, pp. 285, 335 et 387; 1892).

[30] Faraday, *An Answer to D^r^ Hare's Letter on Certain Theoretical Opinions* (SILLIMANN'S JOURNAL, vol. XXXIX, p. 108; 1840.—EXPERIMENTAL RESEARCHES IN ELECTRICITY, vol. II, p. 268; London, 1844).

> Using the word charge in its simplest meaning, I think that a body *can* be [42] charged with one electric force without the other, that body being considered in relation to itself only. But I think that such charge cannot exist without induction, or independently of what is called the development of an equal amount of the other electric force, not in itself, but in the neighbouring consecutive particles of the surrounding dielectric, and through them of the facing particles of the uninsulated surrounding conducting bodies, which, under the circumstances, terminate as it were the particular case of induction.

It is the existence, in the immediate vicinity of each other, of these two layers, equal in density and opposite in sign, that the possibility is due, for Faraday, of maintaining an electrical layer at the surface of a conductor.

Since the theory assumed the medium which surrounds conductive bodies to be perfectly insulating, it does not seek what force keeps the electrical layer adhering to the surface of the conductor; what maintains it is the property attributed to the medium for not allowing the passage of electricity. If we can talk about the *pressure* that the medium exerts on the electricity for maintaining it, it is in the sense where we talk about mechanical strength of binding; this pressure is the electromotive action that *should be* applied to the electrical layer so that it remains on the surface of the conductor, *if the medium ceased to be insulated*. This idea seems to have been clearly perceived by Poisson[31]; he said:

> The pressure that the fluid exerts against the air that contains it is partly composed of the repulsive force and the thickness of the layer; and since one of these elements is proportional to the other, it follows that pressure changes on the surface of an electrified body and is proportional to the square of the thickness or the amount of electricity accumulated at each point on this surface. The air impermeable to electricity must be regarded as a vessel whose shape is determined by that of the electrified body; the fluid contained in this vessel exerts against the walls different pressures [43] at different points, so the pressure that occurs at certain points is sometimes very big and infinite compared to what others experience. In places where the pressure of the fluid overcomes the resistance of the air that opposes it, the air yields, or, if desired, the tank bursts, and fluid flows through such an opening. It is what happens at the end points and sharp edges of angular bodies.

Faraday does not understand the thought of Poisson; he confuses the resistance that the air opposes to the escape of electricity, in virtue of its non-conductibility, with the *atmospheric pressure*, i.e. with the resistance that this same air opposes to the movement of the material masses, under gravity and inertia; and, easily interpreted as the explanation, he draws advantage for his theory which attributes to the action of the layer spread on the dielectric the equilibrium of the layer covering the conductor. He said[32]:

> Here I think my view of induction has a decided advantage over others, especially over that which refers the retention of electricity on the surface of conductors in air to the pressure of the atmosphere. The latter is the view which, being adopted by Poisson and Biot is also, I believe, that generally received; and it associates two such dissimilar things, as the ponderous

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[31] S. D. Poisson, *Mémoire sur la distribution de l'électricité à la surface des corps conducteurs*, lu à l'Académie des sciences le 9 mai et le 3 août 1812 (Mémoires de la classe des sciences mathématiques et physiques in the year 1811, Mémoires des savants étrangers, p. 6).

[32] Faraday, Experimental Researches in Electricity, series XII, *On Induction*, vol. I, p. 438.

> air and the subtile and even hypothetical fluid. …Hence a new argument arises[33] proving that it cannot be mere pressure of the atmosphere which prevents or governs discharge, but a specific electric quality or relation of the gaseous medium. It is, hence, a new argument for the theory of molecular inductive action.

Moreover, an attentive reader of *The Experimental Researches in Electricity* easily recognizes, in the hypothesis that we develop [44] at this time, what Faraday intends to articulate when he says that electric action is not exercized at a distance, but only between contiguous particles; he certainly wants to say that no amount of electricity can develop on the surface of a material molecule without a charge of equal and opposite sign developing on the surface facing another extremely close molecule.

Mossotti has also understood the thought of Faraday well. He said[34]:

> This physicist, considering the state of molecular electric polarization, thinks that there must be two systems of opposing forces which alternate rapidly and hide alternately in the interior of the dielectric, but that they must manifest two special effects opposed to the ends of the same body. On one side, with the simultaneous action of the two systems of forces that develop in the dielectric body, a force equal and opposite to that with which the same layer tends to expel its atoms is born at each point of the electrical layer that covers the excited body; and the opposition of these two forces makes the fluid that makes up the layer to stay on the surface of the electric body. On the opposite side, where the dielectric body touches or envelopes the surfaces of other surrounding electrical bodies, it exerts a force of a species analogous to that of the electrified body and by means of which these surfaces are brought to the contrary electric state.

Mossotti, having demonstrated the existence of surface layers which are equivalent to a dielectric polarized by induction, adds[35]:

> These layers that represent, for the limits of the dielectric body, effects not neutralized by two reciprocal systems of internal forces, exercise, on the surface surrounding the conductive body, actions equivalent to those that these same electrical layers of these same bodies exercise directly between them without the intervention of the dielectric body. This theorem gives us the main conclusion of the question that we proposed. [45] The dielectric body, by means of the polarization of the atmospheres of its molecules, only transmits from one body to the other the action between the conductive bodies, neutralizing the electrical action on one and conveying to the other an action equal to that which the first would have exercised directly.

If it is observed that for Faraday and Mossotti the words *electric action*, *electric force* are at every moment taken as synonyms of *electric charge* or *electric density*, one cannot recognize, in the passages that we have just quoted, the hypothesis that reflects equality (2.29). So, we can say that this equality expresses the *particular Faraday and Mossotti hypothesis.*

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[33] Faraday, *ibid.*, p. 445.

[34] Mossotti, *Recherches théoriques sur l'induction électrostatique envisagée d'après les idées de Faraday* (Bibliothèque universelle, Archives, t. VI, p. 194; 1847).

[35] Mossotti, *Ibid.*, p. 196.

Taken strictly, this hypothesis is not consistent with the principles on which the theory of dielectric polarization is based. We have seen, in effect, as a result of Eq. (2.24), that the density of the actual electrical layer spread on the surface of a conductive body still had a higher absolute value than the density, at the same point, of the fictitious electrical layer which would be equivalent to the polarization of the adjacent dielectric.

But this same equality (2.24) teaches us that the hypothesis of Faraday and Mossotti, unacceptable if taken strictly, can be approximately true; it is what happens if εF_1 is very large compared to $\frac{1}{4\pi}$.

So, we can say that the *hypothesis of Faraday and Mossotti will represent an approximate law if the abstract number* εF *has, for all dielectrics, an extremely large numeric value*.

Let us examine the consequences to which this hypothesis leads.

The capacitance of a variable capacitor varies little when, in this capacitor, a vacuum is made as perfect as possible; one can therefore admit that the specific inductive capacity of air compared to the ether hardly surpasses unity or that the number $(1 + 4F\pi\varepsilon F)$ relative to the air can be substituted for the number $(1 + 4F\pi\varepsilon F)$ relative to the ether.

Take two electrical charges Q and Q' placed in the ether [46] (practically in the air) and let r be the distance between them; these charges repel with a force which has the value

$$R = \frac{\varepsilon}{1 + 4\pi\varepsilon F_0} \frac{QQ'}{r^2}. \tag{2.30}$$

If one accepts the hypothesis of Faraday and Mossotti, this value differs little from

$$R = \frac{1}{4\pi F_0} \frac{QQ'}{r^2}. \tag{2.31}$$

Suppose that one uses the C. G. S. system of electromagnetic units; that the numbers Q, Q', r—which measure, in this system, the charges and their distances—be numbers of moderate magnitude; and that, for example, they be, all three, equal to 1. Experience shows us that the repulsive force is not measured by a very small number, but, on the contrary, by a large number; the coefficient of polarization F_0 of the ether cannot therefore be regarded as having a very high value in the C. G. S. electromagnetic system. The hypothesis of Faraday then entails the following proposition:

In the C. G. S. *electromagnetic system, the constant* ε *has an extremely large value; each formula can be replaced by the limiting form that one gets when* ε *is made to grow and surpass any limit.*

The experience which we have just discussed tells us, moreover, about the value of F_0. The repulsion of two charges represented by the number 1 in the C. G. S. electromagnetic system, placed at one centimeter of distance the one from the other, is measured approximately by the same number as the square of the speed of light, i.e. the number 9×10^{22}; so, if one accepts the hypothesis of Faraday, we roughly have

$$\frac{1}{4\pi F_0} = 9 \times 10^{22}$$

or

$$F_0 = \frac{1}{36\pi \times 10^{22}}.$$

[47] εF_0 being extremely large compared to $\frac{1}{4\pi}$, we see that, *in the* C. G. S. *electromagnetic system, ε must be measured by a very large number compared to* 10^{22}.

The specific inductive capacity relative to the ether (practically to the air) of a dielectric is the ratio $\frac{1+4\pi\varepsilon F}{1+4\pi\varepsilon F_0}$; for all dielectrics known, it has a finite value; it varies between 1 (ether) and 64 (distilled water).

Now, in the theory of Faraday, *the specific inductive capacity of a dielectric* D'*compared to another dielectric* D *is approximately equal to the ratio between coefficient of polarization F' of the first dielectric and the coefficient of polarization F of the second*:

$$\frac{1+4\pi\varepsilon F'}{1+4\pi\varepsilon F} = \frac{F'}{F}. \tag{2.32}$$

So, for all dielectrics, the ratio $\frac{F}{F_0}$ is understood to be between 1 and 64; in other words, for all dielectrics, the coefficient of polarization F, measured in C. G. S. electromagnetic units, is at most on the order of 10^{-22}.

Helmholtz, having developed a very general electrodynamics, suggested,[36] to find various consequences of Maxwell's theory, an operation that amounts to taking the limit of the equations obtained when εF grows beyond any limit. This supposition, it is seen, immediately reduces to the hypothesis of Faraday and Mossotti. [48]

[36] H. Helmholtz, *Ueber die Gesetze der inconstanten elektrischen Ströme in körperlich ausgedehnten Leitern* (Verhandlungen des naturhistorisch-medicinischen Vereins zu Heidelberg, 21 January 1870; p. 89.—Wissenschaftliche Abhandlungen, Bd. I, p. 513).—Ueber die Bewegungsgleichungen der Elektricität für ruhende leitende Körper (Borchardt's Journal für reine und angewandte Mathematik, Bd. LXXII, p. 127 et p. 129.—Wissenschaftliche Abhandlungen, Bd. I, p. 625 et p. 628).—See also: H. Poincaré. *Électricité et Optique*; II. *Les théories de Helmholtz et les expériences de Hertz*, p. vi et p. 103; Paris, 1891.

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