Interactive Four-dimensional Space Exploration Using Viewing Direction Control Based on Principal Vanishing Points Operation

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Abstract. This chapter presents an interactive 4-D visualization technique that controls a 4-D viewing direction via handling of principal vanishing points. Principal vanishing points are represented by projecting points at infinity of 4-D principal coordinate axes onto 3-D space. Our previous studies have confirmed that, because the principal vanishing points relate to a 4-D eye-point and the 4-D viewing direction, they can be landmarks to intuitively move in 4-D space. In this chapter, we propose an algorithm that utilizes principal vanishing points as an interface for the intuitive 4-D viewing direction control, and apply an algorithm to a framework of a system which enables one to fly through 4-D space. The developed system can achieve to visualize and explore an intricate 4-D scene such as a maze in 4-D space. We evaluate effectiveness of the proposed 4-D interaction technique by user experiments.

Keywords: 4-D space · 4-D viewing direction · 4-D interaction · 4-D interface · 4-D visualization · Principal vanishing points

1 Introduction

A concept of "dimension" forms a basis for human knowledge, science and culture. Although it is said that the humans live in the physical world of three dimensions, some mathematicians, thinkers and artists have realized that they can imagine 4-D space beyond 3-D space without any problems. Here, 4-D space is space defined by four axes which are orthogonal to each other. However, in general, 4-D space has been mysterious and incomprehensible for the humans, because the humans have only experience of 3-D space and cannot see 4-D space directly with their eyes. Conversely, if an interactive environment which can allow the user to look around and move in 4-D space is provided,

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it is expected that humans will be able to obtain intuitive understanding of 4-D space. In fact, there exists research reporting that mathematicians can suddenly feel 4-D space while they interact with a computer visualized 4-D object [5].

This chapter presents a novel interactive 4-D visualization technique that controls a viewing direction in 4-D space via a pick and move operation of principal vanishing points in 3-D space. The principal vanishing points are represented by projecting points at infinity of 4-D principal coordinate axes onto 3-D space. From our previous studies, we found that principal vanishing points can be landmarks to intuitively move in 4-D space, because they closely correlate with a 4-D eye-point and the 4-D viewing direction. We utilize the principal vanishing points as an interface for the intuitive 4-D viewing direction control. Our proposed algorithm can determine the 4-D viewing direction from the positions of the principal vanishing points in 3-D space. Furthermore, we apply the algorithm to a framework of a system which enables one to fly through 4-D space. The developed system consists of recent commodity components such as a personal computer, a motion sensor, a head mounted display with a built in a 6-DoF sensor and a five-button wireless mouse. Using the system, a user can freely explore 4-D space with a 4-D fly-through action, while smoothly changing the 4-D viewing direction by the simple handling of principal vanishing points. Through user experiments, we confirm usability and efficiency of the proposed 4-D interaction technique.

This chapter is organized as follows. Firstly, related work and our previous work on 4-D visualization and interaction are introduced in Sect. 2. A relationship between the 4-D viewing direction and the principal vanishing points is shown in Sect. 3. In Sect. 4, we propose the algorithm to determine the viewing direction in 4-D space from the principal vanishing points in 3-D space. Then, we outline the interactive system that enables the user to intuitively explore 4-D space in Sect. 5. In Sect. 6, we demonstrate some 4-D scenes such as insides of 4-D solids, multiple solids and a 4-D maze using the system. In Sect. 7, we discuss effectiveness of our 4-D interaction technique by user experiments. Finally, conclusions are given in Sect. 8.

This chapter is a revised and extended version of our paper entitled "Four-dimensional Viewing Direction Control by Principal Vanishing Points Operation and Its Application to Four-dimensional Fly-through Experience" [13].

2 Related Work and Our Previous Work

There are a number of approaches to visualizing multi-dimensional data via dimensionality reduction techniques. A principal components analysis, a multi-dimensional scaling and a parallel coordinate plot are typical methods. These methods are useful to analyze multi-dimensional statistical data. However, they are not suited to overview higher-dimensional geometric data without missing any information.

Starting with the pioneering work of Abbott [1] and Banchoff [3], visualization of the 4-D objects has been studied in various fields. Most of 4-D visualization techniques project the 4-D object into 3-D space, based on an analogy of that we can imagine a 3-D object from its 2-D projection drawings [6, 8, 10, 11, 14]. There are some work that slices 4-D data with a hyperplane in 4-D space [15, 22], by extending in such a way that we cut the 3-D object with a 2-D plane. As an extension of 3-D computer graphics techniques such as a lighting model and GPU computing, shading and lighting techniques in 4-D space have also been studied [4, 9, 21]. As mentioned above, various rendering methods of 4-D objects have been reported. Here, we focus our attention on research using the projection method for the 4-D object [6, 8, 10, 11, 14]. An advantage of projecting the 4-D object to 3-D space from different eye-points in 4-D space is that it keeps various original 4-D geometric features, not only a structural continuity but also parallelism and orthogonality. In approaches to 4-D visualization, this method will be effective to observe and understand shapes of the 4-D object. However, in some work [6, 8, 10], the 4-D eye-point is fixed. In the others [11, 14], changes of the 4-D eye-point and a 4-D viewing direction are limited. Therefore, it is not sufficient to move freely around 4-D space and overview the 4-D object in the same manner of 3-D space.

Some researchers have attempted to develop 4-D interaction techniques by associating common input devices such as a mouse, keyboard input, a joystick and a touchscreen with geometric operations of the 4-D object [2, 7, 23]. For example, these approaches allow a user to interactively observe rotations of the 4-D object. However, the user cannot move in 4-D space, because of the fixed 4-D eye-point. As described above, although some research into approaches to 4-D interaction has been conducted, direct manipulation interfaces to freely explore 4-D space have not been developed so far.

In our former work, we constructed a 4-D visualization algorithm and a 4-D geometric algorithm via 5-D homogeneous processing [16, 18, 19]. 5-D homogeneous processing is uniformly expressed using 5×5 matrices and 5×5 determinants. Different from the conventional visualization techniques [6, 8, 10, 11, 14] and the interaction techniques [2, 7, 23], our algorithm enables us to visualize various 3-D perspective drawings of any 4-D data onto 3-D space from an arbitrary 4-D eye-point, viewing direction and viewing field. Moreover, 5-D homogeneous processing can be applied to all points including points at infinity in 4-D space. That allows the display of principal vanishing points overlaying the 3-D perspective drawings of 4-D data, by projecting the points at infinity in directions of 4-D principal coordinate axes onto 3-D space.

By extending this 4-D visualization algorithm, we developed an intuitive and interactive 4-D space display system that made human actions in 3-D space correspond to movements and moving directions of the eye-point in 4-D space [16]. By controlling a flight-controller pad associated with human actions, the user can observe various types of 4-D data such as a 4-D solid, 3-D time-series data and 4-D mathematical data with an arbitrary 4-D viewing field, while moving around a 4-D spherical surface which surrounds the observed 4-D data [17, 18, 20]. Through our studies, we found that users utilized principal vanishing points as a landmark to understand their own location in 4-D space, when they moved around 4-D space. Inspired by this perception, we developed a novel intuitive and interactive system that employed the principal vanishing points as an interface to control the 4-D eye-point movement [12]. Using the system, we moved the 4-D eye-point along the 4-D spherical surface by simply and directly moving the principal vanishing points in 3-D space. From results of user experiments, we showed that the system using the principal vanishing points operation [12] can provide more intuitive and interactive 4-D interaction than the former system using the flight-controller pad associated with human actions [16].

Following on from our latest study [12], in this chapter, we extend the principal vanishing points operation to the 4-D viewing direction control. Furthermore, we apply this framework to a 4-D fly-through action which enables us to freely travel around 4-D space.

3 Relationship Between 4-D Viewing Direction and Principal Vanishing Points

In this section, we describe a relationship between the viewing direction in 4-D space and the principal vanishing points in 3-D space. We obtain 3-D perspective drawings of 4-D data by converting data defined in the 4-D world-coordinate system $x_w y_w z_w w_w$ those of 3-D screen-coordinate system $x_s y_s z_s w_s$ [16, 18]. Figure 1 shows the 4-D visualization model to observe a 4-D solid from an arbitrary eye-point, viewing direction and distance in 4-D space. The 4-D viewing direction is defined as the direction from the 4-D eye-point p_f $(x_{p_f}, y_{p_f}, z_{p_f}, w_{p_f})$ to the 4-D observed point p_a $(x_{p_a}, y_{p_a}, z_{p_a}, w_{p_a})$ in the 4-D world-coordinate system, and coincides with the negative direction of the w_e-axis of the 4-D eye-coordinate system $x_e y_e z_e w_e$, the origin of which lies at the 4-D eye-point. The center of the 3-D screen and the center of the background hyperplane are located on the 4-D visual axis at distance h and f ($> h$) from the 4-D eye-point, respectively. The 3-D screen has a dimension of $2k \times 2k \times 2k$ in the $x_s y_s z_s$-space. The 4-D viewing field is defined as a truncated pyramid that is formed by the 4-D eye-point, the 3-D screen and the background hyperplane. Only 4-D data inside the 4-D viewing field is visualized as the 3-D perspective drawing on the 3-D screen. This visualization algorithm is composed of a view field transformation, a perspective transformation and a clipping operation in 4-D space. Including points at infinity in 4-D space, the transformation from the data of the 4-D world-coordinate system to that of the 3-D screen-coordinate system is represented in homogeneous coordinates V_s as the following equation:

$$
\begin{aligned}
V_s &= [X_s \quad Y_s \quad Z_s \quad W_s \quad v_s] \\
&= [X_w \quad Y_w \quad Z_w \quad W_w \quad v_w] \boldsymbol{T}_v(p_f, p_a) \boldsymbol{T}_p(k, h, f), \\
x_s &= \frac{X_s}{v_s}, \; y_s = \frac{Y_s}{v_s}, \; z_s = \frac{Z_s}{v_s}, \; w_s = \frac{W_s}{v_s},
\end{aligned}
\tag{1}
$$

where the transformation matrices $\boldsymbol{T}_v$ and $\boldsymbol{T}_p$ are the 4-D view field transformation matrix and the 4-D perspective transformation matrix, respectively. This algorithm enables one to observe various types of 4-D data from an arbitrary 4-D viewing direction at an arbitrary 4-D eye-point. Moreover, by changing the parameters k, h and f of the 4-D perspective transformation matrix $\boldsymbol{T}_p$, we can visualize 4-D data not only with various 4-D viewing fields but also with projection methods such as a perspective projection, a parallel projection and a slice operation.

Now, we consider the relationship between the viewing direction in 4-D space and the principal vanishing points in 3-D space. The points at infinity in x_w-, y_w-, z_w- and w_w-directions are represented as $V_{x_w}\overline{(1, 0, 0, 0, 0)}$, $V_{y_w}\overline{(0, 1, 0, 0, 0)}$, $V_{z_w}\overline{(0, 0, 1, 0, 0)}$,

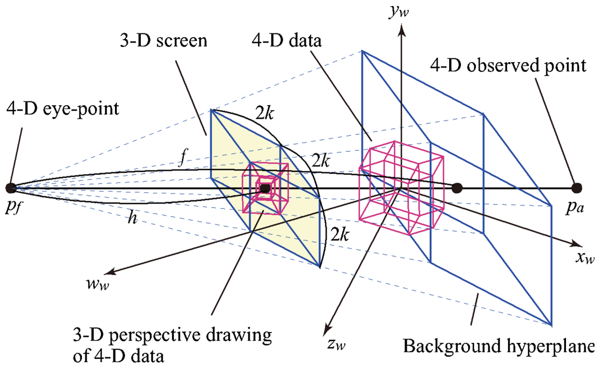

Fig. 1. Visualization model of 4-D space.

and $V_{z_w}\overline{(0,0,0,1,0)}$. Substituting these points to Eq. (1), the principal vanishing points vp_x, vp_y, vp_z and vp_w on the 3-D screen are obtained as follows:

$$\begin{aligned}
vp_x &= \left(x_{vp_x},\, y_{vp_x},\, z_{vp_x}\right) \\
&= \left(\frac{h}{k}\frac{1}{\tan\beta\tan\gamma},\, -\frac{h}{k}\tan\gamma,\, 0\right), \\
vp_y &= \left(x_{vp_y},\, y_{vp_y},\, z_{vp_y}\right) \\
&= \left(0,\, \frac{h}{k}\frac{1}{\tan\gamma},\, 0\right), \\
vp_z &= \left(x_{vp_z},\, y_{vp_z},\, z_{vp_z}\right) \\
&= \left(-\frac{h}{k}\frac{\tan\beta}{\cos\gamma},\, -\frac{h}{k}\tan\gamma,\, -\frac{h}{k}\frac{1}{\tan\alpha\cos\beta\cos\gamma}\right), \\
vp_w &= \left(x_{vp_w},\, y_{vp_w},\, z_{vp_w}\right) \\
&= \left(-\frac{h}{k}\frac{\tan\beta}{\cos\gamma},\, -\frac{h}{k}\tan\gamma,\, \frac{h}{k}\frac{\tan\alpha}{\cos\beta\cos\gamma}\right),
\end{aligned} \tag{2}$$

where α, β and γ are the parameters regarding the 4-D viewing direction for the 4-D view field transformation, which are derived from the 4-D eye-point p_f and the 4-D observed point p_a [16, 18]. It is assumed from the converse relation of Eq. (2) that we can calculate the parameters regarding the 4-D viewing direction using some given principal vanishing points. Thus, it is expected that the principal vanishing points will be useful for a geometric reference to detect the viewing direction and the position during moving in 4-D space.

Figure 2 shows the relationship between the principal vanishing points and the 3-D perspective drawings of a hypercube. In each picture shown in Fig. 2, the 3-D screen corresponds to an inside of a cubic region closed by a white dashed line. Corresponding to changes of the 4-D eye-point and the 4-D viewing direction, various 3-D perspective drawings of the hypercube inside the 4-D viewing field appear on the 3-D screen. On the other hand, the points at infinity in x_w-, y_w-, z_w- and w_w-directions are displayed as four principal vanishing points vp_x, vp_y, vp_z and vp_w in the entire 3-D space without defining any 4-D viewing field. According to the geometric relationship of the 4-D world-coordinate system and the 3-D screen-coordinate system, positions and the number of the principal vanishing points change, correspondingly. The principal vanishing points are represented by small spheres with different color. For example, as the top-left picture shown in Fig. 2, although one principal vanishing point vp_w is displayed on the 3-D screen, the other three principal vanishing points vp_x, vp_y and vp_z also appear in 3-D space by changing the 4-D viewing direction as the bottom-right picture shown in Fig. 2. The 3-D perspective drawings are classified by the number of the principal vanishing points; one-, two-, tree- and four-point 3-D perspective drawings.

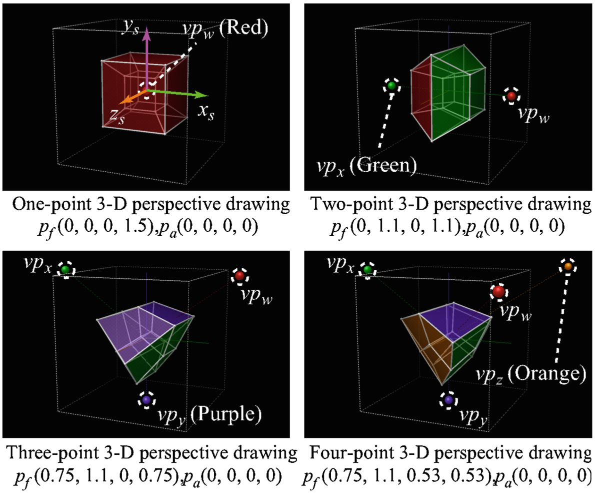

Fig. 2. Relationship between the principal vanishing points and the 3-D perspective drawings of a hypercube. The parameters k, h and f regarding the 4-D viewing field are given as 0.5, 0.5 and 100, respectively.

4 Algorithm of 4-D Viewing Direction Control via Principal Vanishing Points Operation

In this section, we explain an algorithm to determine the viewing direction in 4-D space from principal vanishing points in 3-D space. This algorithm is mainly composed of two processing steps. The first one estimates positions of principal vanishing points. The second one estimates parameters regarding the 4-D viewing direction. In order to apply to the 4-D visualization algorithm described in Sect. 3, in the second processing step, the position of the 4-D observed point is calculated from the parameters regarding the 4-D viewing direction. Different from our previous 4-D visualization method [12], we can freely look in all 4-D viewing directions from an arbitrary 4-D eye-point via simple handling of the principal vanishing points.

Let us discuss the first processing step. When one principal vanishing point is picked and moved in 3-D space, the other three principal vanishing points will be automatically allocated at the corresponding correct positions so that they satisfy their geometric positional relationship in 3-D space. Suppose, for instance, the 3-D perspective drawing of 4-D data and principal vanishing points vp_{x_b}, vp_{y_b}, vp_{z_b} and vp_{w_b} are displayed in 3-D space to steer a visual axis in the direction from the 4-D eye-point p_f to the 4-D observed point p_{a_b} in the 4-D world-coordinate system $x_w y_w z_w w_w$. As the principal vanishing point vp_{w_b} moves to a new principal vanishing point vp_w, the other three vanishing points vp_x, vp_y and vp_z can be estimated using the position of the operated principal vanishing point vp_w as follows:

$$\begin{aligned} vp_x &= \left(x_{vp_x},\, y_{vp_x},\, z_{vp_x}\right) \\ &= \left(-\frac{1}{x_{vp_w}}\left\{\left(\frac{h}{k}\right)^2 + {y_{vp_w}}^2\right\},\, y_{vp_w},\, 0\right), \\ vp_y &= \left(x_{vp_y},\, y_{vp_y},\, z_{vp_y}\right) \\ &= \left(0,\, -\frac{1}{y_{vp_w}}\left(\frac{h}{k}\right)^2,\, 0\right), \\ vp_z &= \left(x_{vp_z},\, y_{vp_z},\, z_{vp_z}\right) \\ &= \left(x_{vp_w},\, y_{vp_w},\, -\frac{1}{z_{vp_w}}\left\{\left(\frac{h}{k}\right)^2 + {x_{vp_w}}^2 + {y_{vp_w}}^2\right\}\right). \end{aligned} \tag{3}$$

Even if another principal vanishing point is moved, or if all the principal vanishing points are not displayed in 3-D space, the positions of the principal vanishing points can be estimated in the same manner as mentioned above. A movable region of the principal vanishing points vp_x and vp_y is restricted on the $x_s y_s$-plane and on the y_s-axis, respectively, while the principal vanishing points vp_z and vp_w can freely move in 3-D space. These restrictions mean that a 4-D upper direction, which corresponds to the y_e-direction, is kept in an upward vertical direction in the 4-D world-coordinate system.

Next, let us discuss the second processing step. We achieve the 4-D viewing direction control. That is, we consider movement of the 4-D observed point p_a along a 4-D spherical surface centered on the 4-D eye-point p_f (see Fig. 3). Parameters α, β and γ regarding the 4-D viewing direction are derived from Eq. (2) as follows:

$$\begin{aligned} \alpha &= \tan^{-1}\frac{z_{vp_w}}{\sqrt{-z_{vp_z} z_{vp_w}}}, \\ \beta &= \tan^{-1}\frac{-x_{vp_w}}{\sqrt{-x_{vp_z} x_{vp_w}}}, \\ \gamma &= \tan^{-1}\frac{-y_{vp_w}}{\sqrt{-y_{vp_z} y_{vp_w}}}. \end{aligned} \tag{4}$$

Substituting the coordinate values of the principal vanishing points vp_x, vp_y, vp_z and vp_w estimated from the above-mentioned first processing step, the coordinate values of the Eq. (3) as an example, to Eq. (4), the corresponding parameters α, β and γ regarding the 4-D viewing direction are determined. The 4-D observed point p_{a_b} is represented as the transformed coordinates (0, 0, 0, $-r$) in the 4-D eye-coordinate system $x_e y_e z_e w_e$, which is defined with the origin at the 4-D eye-point p_f and the w_e-axis in the direction from the 4-D observed point p_a to the 4-D eye-point p_f as described in Sect. 3. Therefore, the 4-D observed point p_a in the 4-D world-coordinate system $x_w y_w z_w w_w$ can be computed as the following equation:

$$\begin{aligned} p_a &= \begin{bmatrix} x_{p_a} & y_{p_a} & z_{p_a} & w_{p_a} & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 & -r & 1 \end{bmatrix} \boldsymbol{T}_{xz}^{-1}(\gamma)\boldsymbol{T}_{yz}^{-1}(\beta)\boldsymbol{T}_{xy}^{-1}(\alpha)\boldsymbol{T}_{t}^{-1}(-p_f), \end{aligned} \tag{5}$$

where the transformation matrix $\boldsymbol{T}_t$ represents the 4-D translation matrix, and the transformation matrices $\boldsymbol{T}_{xz}$, $\boldsymbol{T}_{yz}$ and $\boldsymbol{T}_{xy}$ represent the 4-D rotation matrices around *xz*-, *yz*- and *xy*-planes, respectively. When the 4-D viewing direction is beyond the range of $-\pi/2 \leq \alpha, \beta, \gamma \leq +\pi/2$, the appropriate 4-D viewing direction is estimated by comparing the before and the after its change.

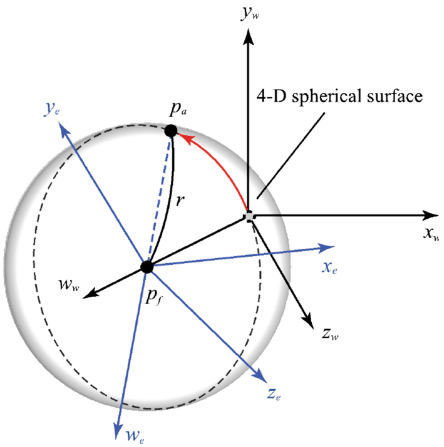

Fig. 3. Movement of the 4-D observed point along a 4-D spherical surface.

With the 4-D interaction method mentioned above, we can smoothly change the 4-D viewing direction at an arbitrary 4-D eye-point. The final 4-D visualization is achieved by introducing the 4-D observed point p_a of Eq. (5) into Eq. (1) shown in Sect. 3.

5 Construction of Interactive System for 4-D Space Exploration

In this section, we describe our interactive system that enables a user to intuitively explore 4-D space. As shown in Fig. 4, the developed system consists of commercially available products such as a personal computer (HP, Intel Core i7 3.90 GHz, 8 GB

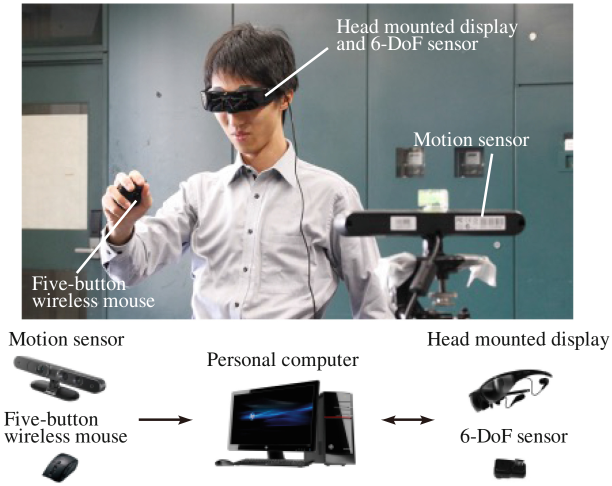

Fig. 4. Configuration of an interactive system.

RAM, NVIDIA GeForce GTX 680) installed Windows 8 (Microsoft), a motion sensor (ASUS Xtion Pro Live), a head mounted display with a built in a 6-DoF sensor (Vuzix Wrap 1200VR) and a five-button wireless mouse (ELECOM). Our proposed algorithm was implemented in C#, OpenTK, OpenNI and OpenCV under Visual Studio (Microsoft). The system guarantees real-time performance (60 frames per second) with interactive visualization of the 4-D scene including 45 or less hypercubes.

3-D virtual space seen from the head mounted display as a side by side stereoscopic image coincides with the $x_s y_s z_s$-space in the 3-D screen-coordinate system $x_s y_s z_s w_s$. Moreover, 3-D virtual space and real space have the same scale. In this work, we set the 3-D screen which has a dimension of 300 mm × 300 mm × 300 mm in 3-D virtual space. The 3-D perspective drawings of 4-D data and the principal vanishing points are displayed onto the 3-D screen and in 3-D virtual space on the head mounted display, respectively. In order to observe these to freely walk in 3-D virtual space, we associated a location and a viewing direction in 3-D virtual space with the user's head position and orientation in real space, which are measured using the motion sensor and the 6-DoF sensor built into the head mounted display, respectively. The user's head tracking data of the motion sensor is defined in the right-hand IR camera-coordinate system $x_c y_c z_c$, the origin of which lies at the IR camera of the motion sensor, and where the z_c-axis is in the direction from the IR camera to the user. We convert them to data of 3-D virtual space. By this transformation, the user's head position in real space corresponds to the location in 3-D virtual space. Therefore, the user can observe the 3-D perspective drawing of 4-D data and the principal vanishing points from various positions and directions in 3-D virtual space.

A 3-D cursor displayed in 3-D virtual space on the head mounted display has a function to indicate an arbitrary principal vanishing point in 3-D virtual space. We associated the 3-D cursor motion in 3-D virtual space with the user's hand motion in real space, which is measured using the motion sensor. The user's hand tracking data of the motion sensor in real space is converted to data in 3-D virtual space as well as the user's head tracking mentioned above. Thus, in 3-D virtual space, the user moves the 3-D cursor to the desired position, and can indicate the target principal vanishing point. Moreover, in real space, the user is allowed to take a two-step left button operation with the five-button wireless mouse holding in his/her hand. The first step is a click operation. The second step is a drag operation. These operations in real space are used for selection and movement of one principal vanishing point in 3-D virtual space, respectively. If the moving 3-D cursor and the target principal vanishing point overlap with each other, using the above-mentioned mouse operation, the user can pick and move the principal vanishing point in 3-D virtual space, interactively. As a result of this, the viewing direction in 4-D space defined in the 4-D world-coordinate system $x_w y_w z_w w_w$ is changed, correspondingly. Therefore, onto the 3-D screen on the head mounted display, the user can observe various 3-D perspective drawings of 4-D data from different viewing directions in 4-D space. Figure 5 shows the basic correspondence relationship between the principal vanishing points operation in 3-D virtual space and the viewing direction change in 4-D space. We put a hypercube at the position (0, 0, 0, −1.5) on w_w-axis of the 4-D world-coordinate system and visualized the hypercube from the 4-D eye-point which lies at the origin of the 4-D world-coordinate system. We gave eight different colors to each cell of the hypercube. In order to clearly visualize edges inside the 3-D perspective drawing, we rendered semi-transparent faces with a reticular stipple pattern. Firstly, when the 4-D observed point is at the position (0, 0, 0, −1.5) on w_w-axis, the principal vanishing point vp_w and the 3-D perspective drawing of the hypercube are displayed at the origin of the $x_s y_s z_s$-space corresponding to 3-D virtual space and on the 3-D screen, respectively. When the user moves the principal vanishing point vp_w in the directions of principal coordinate axes of 3-D virtual space which coincide with x_s-, y_s- and z_s-directions of the 3-D screen-coordinate system, the 4-D viewing direction changes toward to x_w-, y_w- and z_w-directions of the 4-D world-coordinate system, respectively. In accordance with each operation, the hypercube gradually moves outside of the 4-D viewing field. For example, the viewing direction in 3-D space is roughly composed of a combination of horizontal and vertical directions, while the viewing direction in 4-D space includes another direction. So, especially, the operation of the principal vanishing points vp_z and vp_w in the z_s-direction will be a key to master the 4-D viewing direction control. Therefore, from the correspondence between the 4-D viewing direction and the principal vanishing points operation, it is expected that the user will be able to enhance spatial understandings of 4-D space.

In addition to the 4-D viewing direction control, the system is allowed to simultaneously move both the eye-point and the observed point back and forth along the viewing direction in 4-D space. The forward and backward movements in 4-D space are associated with each click operation of two side buttons with the five-button wireless mouse in real space, respectively. Therefore, the user can freely fly through 4-D space, while changing the moving direction in 4-D space via the principal vanishing points operation in 3-D virtual space.

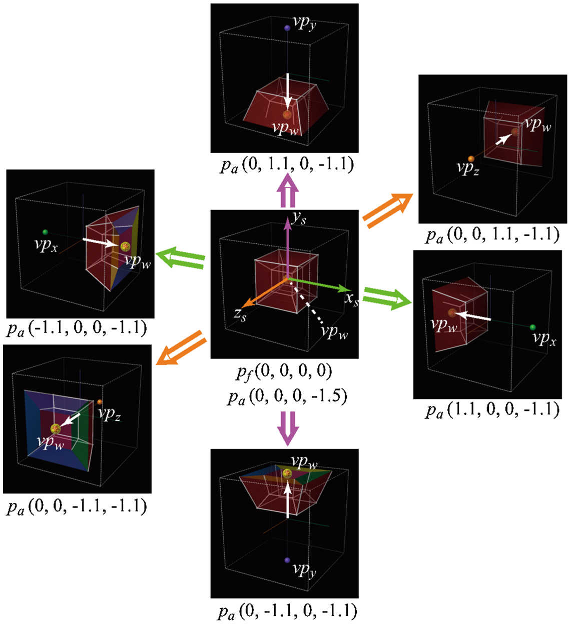

Fig. 5. Correspondence relationship between the principal vanishing points operation in 3-D virtual space and the viewing direction change in 4-D space. The parameters k, h and f regarding the 4-D viewing field are given as 0.5, 0.5, and 100, respectively.

6 Visualization of 4-D Solid Scene Using 4-D Space Exploration System

In this section, we demonstrate some 4-D scenes such as insides of 4-D solids, multiple 4-D solids and a 4-D maze using the developed system, and show that the system can provide novel experiences and understandings of 4-D space different from our previous system [12].

6.1 Inside of 4-D Solids

By putting the 4-D eye-point inside a 4-D solid and changing the 4-D viewing direction in the 4-D solid, we can visualize various insides of the 4-D solid clipped by the 3-D screen. Figures 6 and 7 show the insides of a 24-cell and a 120-cell visualized using this manner. We can understand that the 24-cell and the 120-cell is constructed by octahedrons and dodecahedrons, respectively, and that each surface of the 4-D solid is shared by two different cells, generally. In our system, eight different colors are given to each cell of the 4-D solid. This visualization will help us to perceive geometric characteristics and structure of the 4-D solid.

In addition, although the 4-D viewing direction is different in 4-D space, the obtained principal vanishing points are the same in pictures 2 and 3 of Figs. 6(a) and 7(a),

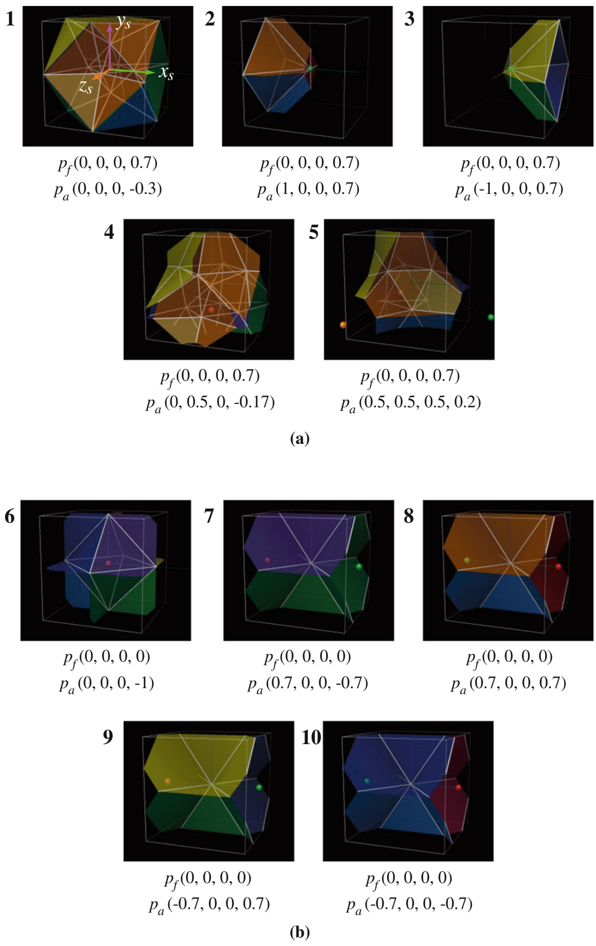

Fig. 6. Visualization of a 24-cell by 4-D viewing direction changes from 4-D eye-points inside the 24-cell. The 24-cell is placed at the origin of the 4-D world-coordinate system. The parameters k, h and f regarding the 4-D viewing field are given as 0.5, 0.5 and 100, respectively. (a) The 4-D eye-point is at the position (0, 0, 0, 0.7) in the 4-D world-coordinate system. (b) The 4-D eye-point is at the origin of the 4-D world-coordinate system.

and pictures 7 and 9, and 8 and 10 of Figs. 6(b) and 7(b). This happens when the user turns the 4-D viewing direction 180 degrees in 4-D space. Understanding of these relationships will lead to intuitively handle turning about a motion in 4-D space.

6.2 Multiple 4-D Solids

While most previous research has mainly handled only one type of 4-D solid with a fixed eye-point and viewing direction in 4-D space, our system can provide smooth interaction with the 4-D scene including multiple 4-D solids where users can freely

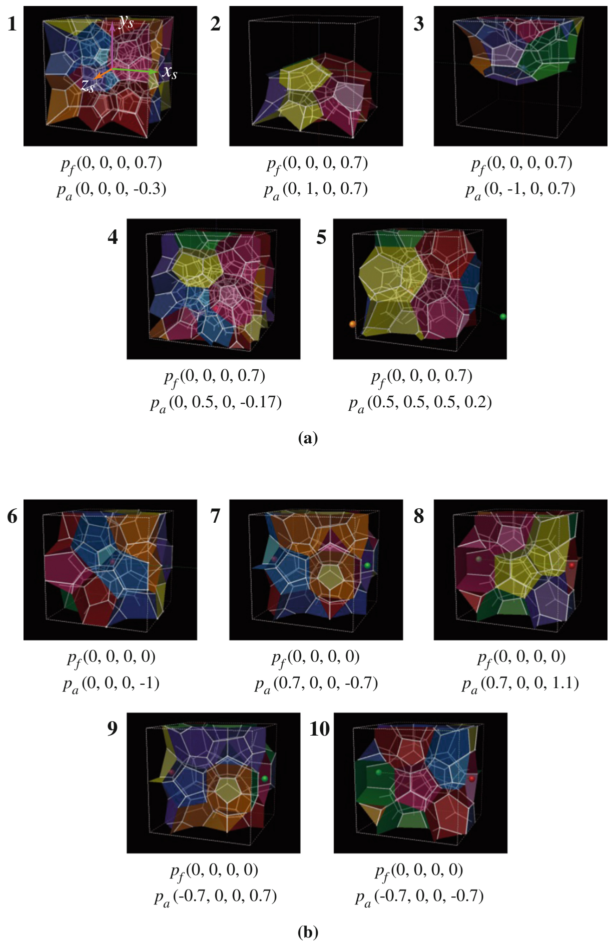

Fig. 7. Visualization of a 120-cell by 4-D viewing direction changes from 4-D eye-points inside the 120-cell. The 120-cell is placed at the origin of the 4-D world-coordinate system. The parameters k, h and f regarding the 4-D viewing field are given as 0.5, 0.5 and 100, respectively. (a) The 4-D eye-point is at the position (0, 0, 0, 0.7) in the 4-D world-coordinate system. (b) The 4-D eye-point is at the origin of the 4-D world-coordinate system.

change the eye-point and viewing direction. For example, as shown in Fig. 8(a), there are a hypercube, a 5-cell and a 16-cell at the positions (0, 0, 0, 1.5), (1.5, 0, 0, 0) and (0, 1.5, 0, 0) in the 4-D world-coordinate system, respectively, so that they surround the 4-D eye-point at the origin of the 4-D world-coordinate system. Figure 8(b) depicts an image sequence of 3-D perspective drawings obtained by turning the 4-D viewing direction to each 4-D solid. Initially, the 3-D perspective drawing of the hypercube is shown at the center of the 3-D screen. By moving principal vanishing points vp_w, vp_x and vp_y, respectively, the 4-D viewing direction changes (see Fig. 8(a)) and the 5-cell and the 16-cell come into the 3-D screen in sequence (see Fig. 8(b)).

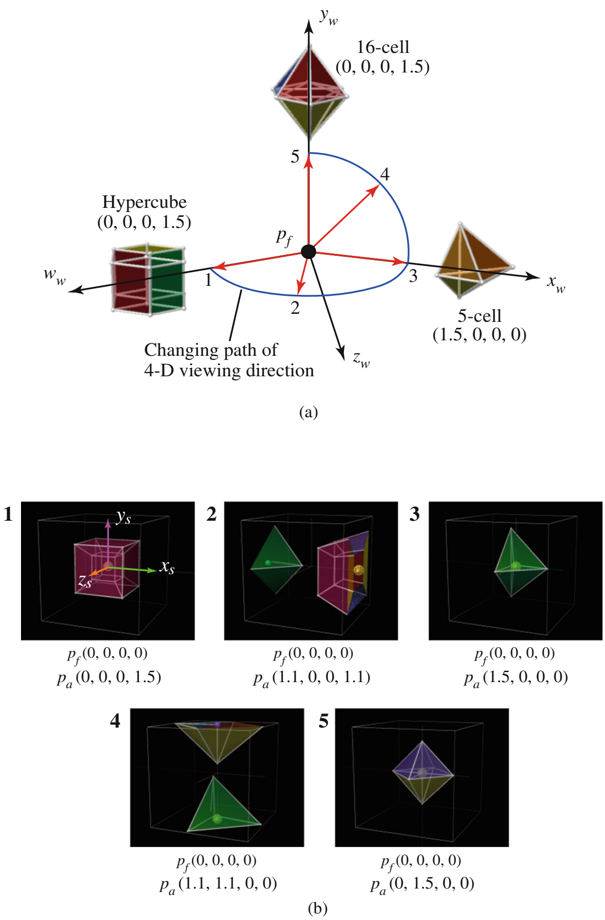

Fig. 8. Visualization of the multiple 4-D solids by 4-D viewing direction changes. (a) Layout of the 4-D solids and the changing path of the 4-D viewing direction. (b) Image sequence obtained by 4-D viewing direction changes. The parameters k, h and f regarding the 4-D viewing field are given as 0.5, 0.5 and 100, respectively. The numbers of the pictures correspond to the numbers of the 4-D viewing direction in (a).

Moreover, we demonstrate the visualization of the multiple 4-D solids with 4-D fly-through actions (see Fig. 9). As shown in Fig. 9(a), we use the 4-D scene which includes a hypercube at the origin of the 4-D world-coordinate system and a 16-cell at the position (0, 0, 0, 3.5) in the 4-D world-coordinate system (see Fig. 9(a)). Figure 9(b) depicts an image sequence of 3-D perspective drawings of 4-D solids obtained by traveling around 4-D space. In the initial projection obtained from the 4-D eye-point p_f (0, 0, 0, 1.5) on the w_w-axis, the 3-D perspective drawings of the hypercube and the 16-cell have an overlap with each other in the 3-D screen as shown in picture 1 of Fig. 9(b). This means that the 16-cell is occluded by the hypercube in 4-D space. Then, by changing the 4-D viewing direction and moving the 4-D eye-point from on the w_w-axis toward the 16-cell

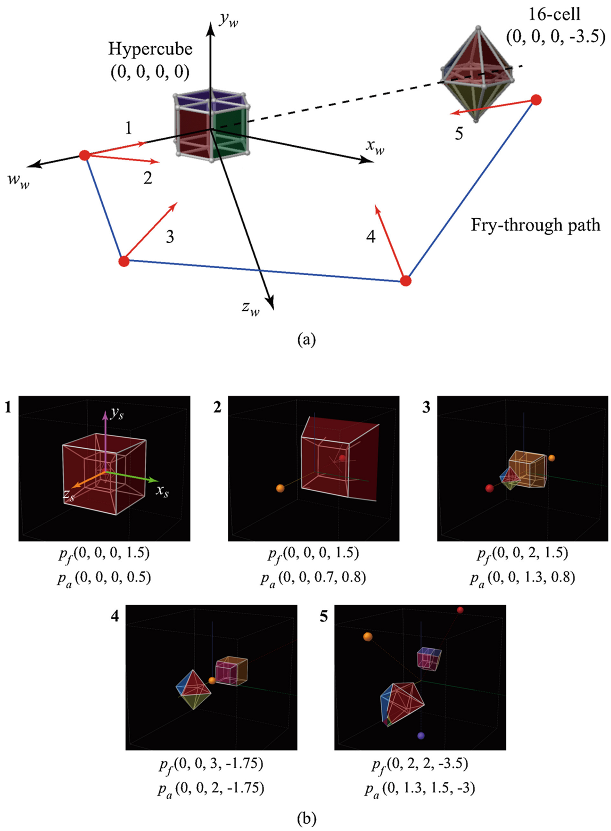

Fig. 9. Visualization of the multiple 4-D solids by 4-D fly-through actions. (a) Layout of the 4-D solids and the fly-through path. (b) Image sequence obtained by 4-D fly-through actions. The parameters *k*, *h* and *f* regarding the 4-D viewing field are given as 0.5, 0.5 and 100, respectively. The numbers of the pictures correspond to the numbers of the 4-D viewing direction in (a).

(see Fig. 9(a)), these 3-D perspective drawings are separated off on the 3-D screen, and align in the direction of the principal vanishing point vp_w (see Fig. 9(a)). Therefore, we can find that the hypercube and the 16-cell locate on different positions on the w_w-axis of the 4-D world-coordinate system. In this way, the 4-D fly-through action will be useful to understand the layout of the 4-D solids in the 4-D scene, even if it is intricate with occlusions.

6.3 4-D Maze

We show a 4-D maze exploration using 4-D fly-through actions. As shown in Fig. 10 (a), a 4-D maze is formed so that three straight-line paths on x_w-, y_w- and z_w-axes in the 4-D world-coordinate system intersect at the origin of the 4-D world-coordinate system. The first path is constructed by connecting five hypercubes, which align in the negative and positive directions on x_w-axis. The second and third paths are constructed by connecting three hypercubes, which align in the positive directions on z_w- and w_w-axes from the origin, respectively. Two neighboring hypercubes on the straight-line path share one cell. The intersection of three straight-line paths coincides with the four-way intersection with three corners, and includes the middle hypercube in the first straight-line path and the one end hypercubes in the second and third straight-line paths. Each end hypercube of the 4-D maze is colored by different color.

Figure 10 (b) is an image sequence obtained from the exploration of the 4-D maze by 4-D fly-through actions. We explore this maze so that the 4-D eye-point passes through the hypercubes on the straight-line paths. Starting from the green hypercube on the w_w-axis, we first moved ahead the straight-line path and then turned to the positive direction of x_w-axis at the four-way intersection. After that, we moved ahead again and stopped at the orange hypercube on x_w-axis. Edges and small cubes which concentrate to the principal vanishing points indicate an existence of the next way. Knowledge regarding orthogonality and parallelism in 4-D space obtained from characteristics of the principal vanishing points will help us to understand the spatial structure of such 4-D maze.

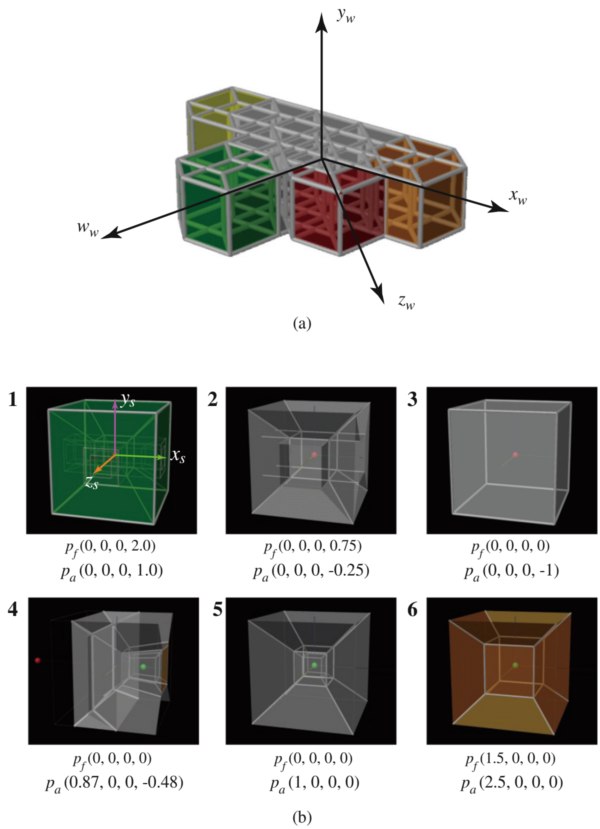

Fig. 10. Exploration of the 4-D maze by 4-D fly-through actions. (a) Appearance of the 4-D maze visualized using the parallel projection. (b) Image sequence obtained from the exploration of the 4-D maze by 4-D fly-through actions. The parameters k, h and f regarding the 4-D viewing field are given as 0.5, 0.5 and 2.5, respectively.

7 User Experiments with Interactive 4-D Space Exploration System

In this section, we describe a user experiment in order to examine whether the proposed system can provide sufficient usability and learnability to handle the 4-D viewing direction in 4-D space. In the experiment, subjects spot the target 4-D object in 4-D space, while changing the 4-D viewing direction via the principal vanishing points operation. If the subjects can reduce response time to complete the tasks and show a positive impression for the interaction, it will be proof of the usability and learnability of the proposed system.

7.1 Method

Participants. Eleven male subjects and one female subject participated in the experiment without monetary compensation. The subjects were students or graduate students who majored in science and engineering at Waseda University. Their mean age was 21.4 years. Seven male subjects had some previous knowledge of 4-D space and objects, since they had observed the hypercube with our previous interactive system [12]. However, they had never experienced the 4-D viewing direction control and had no knowledge of the current experiment. The other four male and one female subjects had never had any 4-D interaction. They did not have any knowledge of 4-D space and the proposed interactive system. In addition, they had no experience of an immersive virtual reality system using the head-mounted display, either.

We divided the twelve subjects into two groups based on their previous knowledge. Thus, group A had seven male experienced subjects of mean age 21.6 years. Group B had five beginner subjects of mean age 21.0 years. Both groups were given the same tasks.

Apparatus. We ran the experiment inside a meeting space of our university building (see Fig. 4). Subjects used the head-mounted display and the five-button wireless mouse to negotiate the experimental tasks. The 3-D screen was placed 1.8 meters away from the motion sensor. This configuration provided the enough spatial resolution to track the subjects' head and hand positions. The 4-D scene was rendered in 60 frames per second, consistently.

Stimuli and Tasks. The experiment was performed in two stages. In each stage, firstly, subjects engaged in 20 trials, then following a three-minute break repeated the same 20 trials. Thus, the subjects performed 80 trials in total.

Stimuli were simple 4-D scenes. We constructed 20 scenes for each stage. We used the scenes twice in each stage. Each 4-D scene included one hypercube as the target object in 4-D space. The hypercube had a dimension of $1 \times 1 \times 1 \times 1$ and we gave eight different colors to each cell as well as Fig. 5. The position of the target hypercube was different in each scene. In the first stage, the target was placed at distance 2 from the origin of the 4-D world-coordinate system so that at least one vertex of the target hypercube was included in the 4-D viewing field when the subjects started the trial.

In order to make 20 scenes, we randomly chose 20 positions in 4-D space. The mean angle between the initial 4-D viewing direction which coincided with the negative direction of the w_w-axis and the direction to the target hypercube was 42.4 degrees. In the second stage, the target hypercube was placed at distance 2 from the origin of the 4-D world-coordinate system as well as the first stage. However, the position of the target hypercube was behind the 4-D eye-point when the subjects started the trials. We chose 20 positions in 4-D space to make 20 scenes, again. In the second stage, the mean angle between the initial 4-D viewing direction and the direction to the target hypercube was 116.8 degrees.

Tasks of the experiment were to spot the target hypercube in 4-D space. In each trial, the subjects were asked to change the 4-D viewing direction from the negative direction of the w_w-axis to the center of the target hypercube in 4-D space. The 4-D eye-point was fixed at the origin of the 4-D world-coordinate system. In other words, the goal of the trial was to capture the target hypercube at the center of the 3-D screen. In each trial, the subjects could check whether or not they completed the task at any time they desired. When the subjects checked their answer, the system calculated an anguler error which corresponds to the angle between the current 4-D viewing direction and the direction to the target. If the angular error was under a threshold, the subjects proceeded to the next trial. Otherwise, the subjects had to continue the current trial until they reduced the angular error sufficiently. In the present experiment, we determined the threshold of 15 degrees based on results of our preliminary experiment.

Procedure. Before the subjects started the trials, we gave each subject a brief explanation on 4-D space and objects, the visualization model and the interactive system. This explanation took approximately 10 to 15 minutes. Firstly, we defined 4-D space and the structure of the hypercube; 4-D space was defined by four principal coordinate axes which were orthogonal to each other; the hypercube of the 4-D solid was constructed by eight cells (3-D cubes) and each surface is shared by two different cells. Then, we explained the 4-D visualization model based on the analogy with 3-D space; the hypercube in 4-D space is displayed as the 3-D perspective drawing into the 3-D screen by the projection method; the principal coordinate axes in 4-D space were displayed as the principal vanishing points in 3-D space by the projection method. After that, we showed how to use the interactive system by displaying the 3-D perspective drawing, principal vanishing points and 3-D cursor in 3-D virtual space on the head-mounted display. The 3-D cursor was used to pick and move the specific principal vanishing point. The subjects could move around 3-D virtual space by walking around real space.

After the explanation was finished, we gave the subjects five minutes to practice using the 4-D viewing direction control using the interactive system. We used the 4-D scene where the target hypercube was placed at the position (0, 0, 0, −2) in the 4-D world-coordinate system as an example. The 4-D eye-point and the initial 4-D viewing direction were as same as the condition of the task. The subjects freely handled the 4-D viewing direction by the principal vanishing points operation and saw the hypercube being in and out of the 4-D viewing field. Note, we adjusted the stereoscopic parallax to display the 3-D image on the head-mounted display in this practice phase.

The subjects received a three minute break after the practice run. They, then, proceeded to the first stage of the trials, followed three-minute break, after which second stage of the trials was conducted. These two stages took approximately 45 to 60 minutes. Although every subject challenged the same trials, the order of the trials was shuffled to reduce an order effect. We recorded their response time, accuracy (the final angular error) and action histories for the 4-D viewing direction control in each trial. These results were used to evaluate the usability and the learnability of the proposed system.

After the subjects finished tasks, they answered a questionnaire to elucidate their impression of the proposed system. The questionnaire included the following seven questions and a free comment column.

1. How was overall impression of the system, positive or negative?
2. How easy to learn operations of the system?
3. How easy to use the system?
4. How easy to turn the 4-D viewing direction to the target hypercube according to your plans?
5. Did you understand the relationship between operations of the principal vanishing points and the resulting changes on the 3-D screen?
6. Did you enhance your sense of direction in 4-D space?
7. Did you reduce mistakes of the 4-D viewing direction control during the trials?

The rating used a seven-point scale. The subjects answered a score between “minus three” and “plus three” for each question, a score of “plus three” being at the positive end of the spectrum while the score of “zero” indicates neutral impression. A score of “minus three” is the most negative response. These subjective reports were used to assess the usability of the proposed system.

7.2 Results

960 trials were run in total by twelve subjects. We regarded 945 trials as valid results, because, in 15 trials, the motion sensor suddenly failed to properly track the subjects’ head and hand positions.

Response Time. Firstly, we describe the results of 40 trials of the first stage. Figure 11 shows the mean response time of the trials. In order to remove noise of time data, we plotted moving average of each four trials with standard errors displayed as error bars. Initially, the mean response time for group A was 64.1 seconds. The subjects reduced their response time in the first five trials. Then, their performance stabilized at the mean response time of 12.3 seconds. Group B yielded similar results. Their initial mean response time was 115.8 seconds. Their response time rapidly dropped in the first five trials. This drop gradually continued in the sixth to the 15th trials. Then, their performance stabilized at the mean response time of 15.1 seconds.

Improvement in subjects of group B was also confirmed in their action histories. Figure 12 shows time courses of the angular error between the 4-D viewing direction and the direction to the target hypercube. Because subjects of group B followed similar

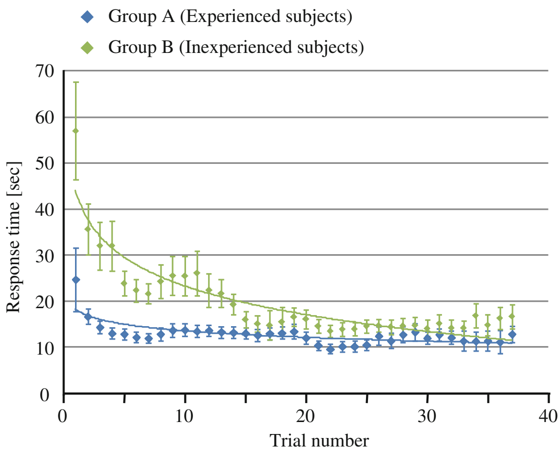

Fig. 11. Relationship between the mean response time and the trial number in the first stage.

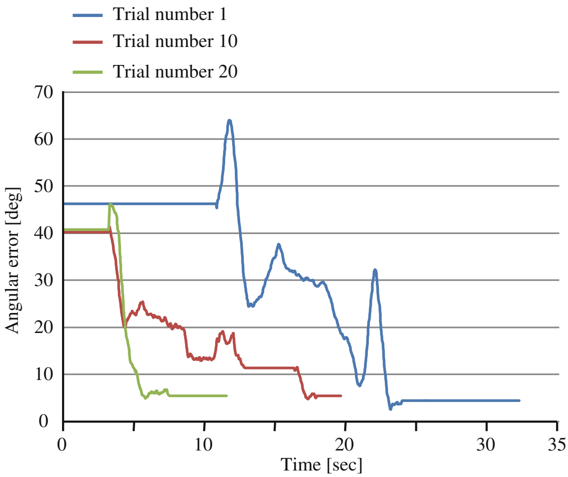

Fig. 12. Relationship between angular error decreasing and time.

trends, we plotted the first, tenth and 20th trials of one subject as representative results. Increases in angular error represent subjects turning the 4-D viewing direction to the wrong direction in 4-D space. The results showed that as the subject practiced, they reduced the number of incorrect operations and time spent to correct the misguiding.

Next, we describe the results of 40 trials of the second stage. Figure 13 shows the mean response time of the trials. Subjects gave similar results to the first stage. Both groups reduced their response time in the first ten trials, after which their performance stabilized. The mean response times of the 11th to the 40th trials were 23.2 seconds for group A and 33.7 seconds for group B. The subjects succeeded in searching for and spotting the target hypercube behind the 4-D eye-point in 4-D space.

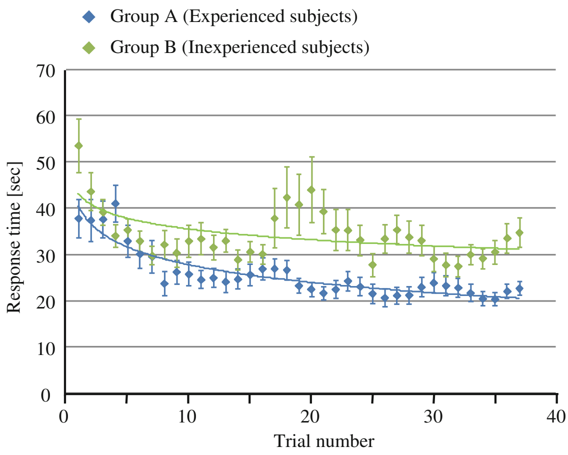

Fig. 13. Relationship between the mean response time and the trial number in the second stage.

Accuracy. We also describe the subjects' accuracy of the 4-D viewing direction control by calculating the mean angular error of 80 trials for each group. The mean angular error of group A was 5.37 degrees. The mean angular error of group B was 6.54 degrees. These angular errors correspond the target hypercube displacement from the center of the 3-D screen.

Subjective Impression. Finally, we show the subjects' impression of the 4-D interaction. Tables 1 and 2 summarize full data of the results of the questionnaire for groups A and B. The boxplot for full data shown in Tables 1 and 2 appears in Fig. 14. Question number 1 asked the overall impression. The median responses were a score of "plus two" for group A and a score of "plus three" for group B, respectively. Every subject had positive impressions of the proposed 4-D interaction. Question numbers 2 and 3 enquired as to the learnability and the usability of the interaction. The median response of group A were "plus two" for each question, respectively. The median response of group B were "plus one" and "plus three", respectively. No one gave negative responses. Question numbers 4 to 7 asked the impression of the 4-D viewing direction control. Question number 4 asked how easy the subjects could change the 4-D viewing direction to the hypercube. The mean responses were the score of "plus one" for both groups. Only subject B3 responded negatively. Question number 5 asked whether subjects can grasp the relationship between their operation and the resulting 4-D viewing direction changes. The median responses were "plus one" for group A and "plus two" for group B, respectively. Question number 6 asked whether they improved their perception of 4-D space. The number of the neutral and negative responses were high compared to the other six questions in this question. We discuss this point in the next section. Question number 7 asked whether they felt that they could improve their performance by themselves. Both groups showed the median score of the "plus two" and no one responded negatively. Summarily, these results suggest that subjects found the interactive system easy to learn and use.

Table 1. Results of the questionnaire phase on the group A.

Question No.	Participant						
	A1	A2	A3	A4	A5	A6	A7
1	+2	+2	+1	0	0	+2	+1
2	+2	+2	+2	+2	+2	+2	0
3	+2	+2	+1	+2	+1	+1	+2
4	+3	+2	+1	+1	+3	+1	+1
5	+2	+2	0	+1	+2	0	−1
6	+1	+2	0	−1	+2	−1	+1
7	+2	+3	+2	+2	0	+2	+1

Table 2. Results of the questionnaire phase on the group B.

Question No.	Participant				
	B1	B2	B3	B4	B5
1	+3	+3	+3	+3	+3
2	+2	+3	0	0	+1
3	+1	+3	+2	+3	+3
4	0	+1	−3	+1	+1
5	+2	+2	+1	−2	+2
6	0	−1	0	0	+2
7	+2	+3	+3	+1	+2

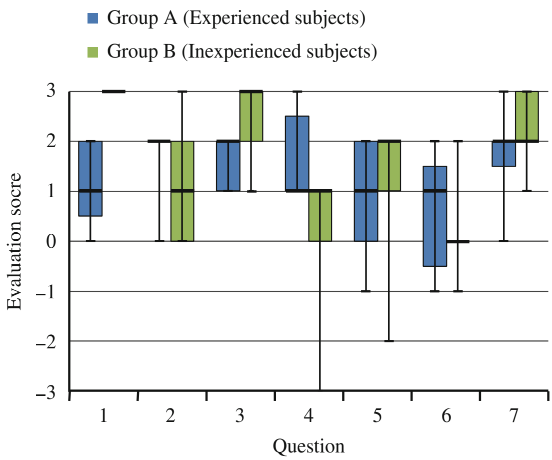

Fig. 14. Results of the questionnaire about usability and learnability of the developed system.

7.3 Discussions

Usability and Learnability. Both groups showed a similar pattern of results in the first stage. Although the learning speeds were different for each of the subjects, they reduced their response time in early stage of the first 20 trials and consistently performed smooth 4-D viewing direction control in the latter 20 trials. The interpretation of this pattern of performance is that subjects rapidly learn the correspondences between the principal vanishing points operation and the 4-D viewing direction changes in the first five to ten trials. Then, they use that obtained understanding over the trials. This interpretation is confirmed by their positive impression shown in the questionnaire scores and comments. Subjects found the task difficult at the beginning of the experiment. However, once they understand the correspondence between the principal vanishing point operation and its result visualized on the 3-D screen, trials become easier to complete.

In the second stage, the subjects initially searched the target by looking around 4-D space. Once they found the target hypercube in the 4-D viewing field, they captured it into the center of the 3-D screen by handling the 4-D viewing direction as well as the first stage. We find that subjects' response times are roughly twice as long as the first stage. Because the target hypercube is located behind the 4-D eye-point, the subjects have to turn back the 4-D viewing direction in 4-D space to find the hypercube. In order to achieve this action, the subjects need to operate at least two principal vanishing points. Moreover, their amount of interaction is approximately twice as much as the first stage. Considering these conditions, the increase in response time is not unexpected.

In summary, we conclude that the proposed system makes it easy to learn and handle the 4-D viewing direction control, even if the user does not have previous knowledge of 4-D space and objects. The system is suitable for looking around and finding 4-D objects in 4-D space.

Evaluation of Accuracy. Based on the results, there were approximately six degrees of the angular error, when subjects completed the task. This error can be explained by a combination of the following interpretations. Firstly, we consider the primary cause to be that it is difficult for the users to completely fix their hand in the air. This may be particularly pronounced when subjects slightly move the principal vanishing points to fine-adjust the 4-D viewing direction. In fact, many subjects commented on this point.

Secondly, there is approximately 150 milliseconds latency in the motion sensor due to the smoothing filter reducing the positional noise of the head and hand tracking. Although there has been no feedback regarding the latency, it is possible that this affects the interaction.

Thirdly, the error is due to the subjects confusing the depth of the center of the 3-D screen on the stereoscopic visualization, for example, when the opaque part of the 3-D perspective drawings of the target hypercube occludes the center of the 3-D screen which is indicated as a cross-point of the x_s-, y_s- and z_s-axes displayed in 3-D virtual space.

Effect of Previous Knowledge. In the both stages, the response time of group A was significantly shorter than group B ($p = 3.28 \times 10^{-3} < 0.05$ for the first stage, and $p = 1.37 \times 10^{-5} < 0.05$ for the second stage). We also confirm that the performance of group A was more stable among the trials than group B. In addition, the scores of Question number 4 of the questionnaire for group A were significantly higher than those of group B ($p = 0.0254 < 0.05$). These results suggest that it is easy for subjects of group A to solve the task, compared to group B. We speculate that this difference in performance occurs due to differences in subjects' previous knowledge of 4-D space and objects. Because subjects of group A have observed the hypercube by controlling the 4-D eye-point along the 4-D spherical surface via the principal vanishing points operation [12], they may naturally grasp the 4-D viewing direction control in the present experiment. This is proof, at least in part, that humans were able to improve their 4-D spatial perception with the various experiences in the 4-D interactive environment.

Limitation of Present Study. We confirm that the proposed system makes it easy to learn and provide the sufficient usability to handle the 4-D viewing direction, however, some feedback comments highlight limitations of the system. For example, one subject felt that the viewing field on the head-mounted display was narrow. Some subjects found the view bobbing in 3-D virtual space, which occurred especially when their hand occluded their head from the motion sensor, annoying. Combined with a discussion about the accuracy, the improvement of the tracking system and the stereoscopic display system will be the subject of further work, with the aim to provide more comfortable interaction.

As shown by Question number 6 of the questionnaire, in the present experiment, we cannot confirm whether the subjects obtain the general spatial perception of 4-D space. This is because we designed the experiment to primarily assess the usability of the proposed system. In order to evaluate the human 4-D spatial perception, it will be necessary to plan the cognitive test to examine whether subjects grasp the spatial structure of the 4-D objects and the 4-D scenes such as 4-D maze. The proposed system can be used as a test bed for such cognitive tests in the future work.

8 Conclusions

In this chapter, we described the novel 4-D interaction technique that enables the user to intuitively control the viewing direction in 4-D space via the principal vanishing points operation in 3-D space. Using an interactive interface, our system can visualise the 4-D scenes such as the inside of the 4-D solids, the multiple 4-D solids and the 4-D maze. In such 4-D scenes, the principal vanishing points are useful in understanding the orthogonality and the parallelism in 4-D space. In order to assess the usability of the proposed interactive system, the user experiment was conducted with simple 4-D scenes. We examined whether the subjects correctly handled the 4-D viewing direction to the predetermined direction. The results showed that the principal vanishing points operation provided the sufficient usability for the 4-D viewing direction control. Subjects easily learned the interaction and intuitively looked around 4-D space, even if they had no previous knowledge of 4-D space and objects.

General human spatial perception in 4-D space was outside the scope of this research, due to experiment design. Future work may include spatial cognitive tests in

4-D space such as mental rotation, spatial orientation and maze exploration. The proposed system will be used as the test bed of the cognitive tests. In addition, we will try to apply the system to modeling and editing of 4-D solids and data.

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