Turbulent Thermal Convection

Enrico Fonda and Katepalli R. Sreenivasan

Abstract Turbulent thermal convection is a phenomenon of crucial importance in understanding the heat transport and dynamics of several natural and engineering flows. Real world systems such as the Earth's atmosphere—its oceans as well as the interior—and the interior of stars such as the Sun, are all affected to various degrees by thermal convection. The simplified physical model used to understand this ubiquitous heat transport mechanism is the Rayleigh-Bénard convection, which is a fluid flow driven by a temperature difference between the top and bottom plates of an experimental cell with adiabatic sidewalls. Despite the long history of the subject and the recent progress in theoretical, numerical and experimental domains, many questions remain unresolved. We report some recent results and discuss a few open issues.

1 Introduction

The density of fluids in general decreases with increasing temperature. A temperature difference can then drive a flow via buoyancy force. This ubiquitous phenomenon, called convection, has a history that goes back to the 18th century with the work of Hadley, Lomonosov, Thompson and others, but is still the subject of active research (e.g., Ahlers et al. 2009a; Lohse and Xia 2010; Chillà and Schumacher 2012; Xia 2013). The possibility that convection greatly enhances heat transport compared to thermal conduction makes it basic for heat transfer engineering—for example, ovens, nuclear reactors, ventilation systems, crystallization processes and casting. In natural phenomena, motion due to non-uniform heating is perhaps the most widespread kind of fluid motion in the universe.

E. Fonda (✉) · K.R. Sreenivasan
New York University, New York, USA
e-mail: enrico.fonda@gmail.com

K.R. Sreenivasan
e-mail: katepalli.sreenivasan@nyu.edu

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On the Earth's surface, convection plays a fundamental role in weather and climate, driving the oceans (Marshall and Schott 1999) as well as the atmosphere (Hartmann et al. 2001). In the Earth's interior, convection plays a role both in the outer core (Cardin and Olson 1994) and in the mantle (Schubert et al. 2001). The convective liquid metal flows in the core are thought to be responsible of the Earth's magnetic field through a dynamo mechanism (Buffett 2000). The convective mantle motion is related to the phenomenon of plate tectonics (Tackley 2000); even when planets do not show plate tectonics, as in the case of Venus or Mars, mantle convection is important because of its role in transporting heat from the planet interior to the surface. In giant planets, which have a hot interior, convection occurs when conduction and radiation are not sufficient to transport all the heat (Guillot 2005).

In the Sun, the nuclear energy generated at the core is transported by the convective motion in the outer 30 % of the radius. This motion shows cellular structures on different scales, distinguished usually as granular (2,000 km), mesoscale (5,000–10,000 km), supergranular (3×10^4 km) and giant ($1–2 \times 10^5$ km) cells (Nordlund et al. 2009). Cellular structures driven by convective motions are also evident in the atmosphere (see Fig. 1).

Many basic questions relating to these diverse systems are similar. How do we predict the heat transfer due to the convective motion? Conversely, what flow structure is needed to transport a given amount of thermal energy? How important are the macroscopic large scale structures? What is their dynamics? What are the properties of the small scale turbulence? How does the small scale motion relate to large scales? And so forth.

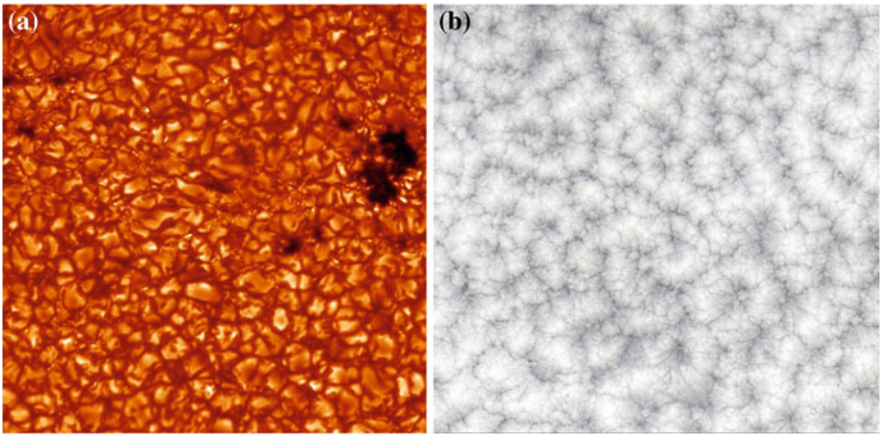

Fig. 1 Cellular convective structures in natural phenomena. **a** Sun's granular structure where each cell is on the order of 1,000 km. Image from Swedish 1-m Solar Telescope (SST)—Royal Swedish Academy of Sciences—Oddbjorn Engvold, Jun Elin Wiik, Luc Rouppe van der Voort. **b** Closed cells structures in a layer of marine stratocumulus over the southeastern Pacific Ocean. Each cell is on the order of 5 km. Image by NASA/GSFC/LaRC/JPL, MISR Team

Even though all convective flows share similarities, they are also different from each other in detail, and include additional physical processes such as rotation, stratification, salinity, pressure gradients, gravity gradients, phase changes, and magnetic fields. These effects complicate what is already one of the most challenging problems in nature, namely turbulence. It is thus useful, indeed essential, to study an idealized form of thermal convection in order to gain basic physical understanding. This idealized flow is the Rayleigh-Bénard convection (RBC).

2 Rayleigh-Bénard Convection

In Rayleigh-Bénard convection the flow is confined to a cylindrical container of height H and diameter D; $\Gamma = D/L$ is called the aspect ratio. The sidewalls are adiabatic and the bottom and top walls are conducting. The fluid motion is driven by the temperature difference between the bottom hot plate and the cold top plate (see Fig. 2).

The dual-name assigned to the flow acknowledges the work of Henri Bénard and Lord Rayleigh. Bénard's contribution started with his Ph.D. thesis from 1901, in which he observed the famous hexagonal cells (see Fig. 3) on a thin layer of spermaceti (whale oil) placed on a heated metallic plate. For lack of theoretical insight, these seminal observations were received with some reserve by the thesis committee presided by the (future) 1908 Nobel laureate, Gabriel Lippman, who stated "...though Bénard's main thesis was very peculiar, it did not bring significant elements to our knowledge. ... the thesis should not to be considered as the best of what Bénard could produce" (Wesfreid 2006).

One who did not miss the theoretical insight was Lord Rayleigh, who first solved the linearized stability problem in 1916. Under the Boussinesq approximation, which assumes that the fluid properties are constant except in the buoyancy term, the equations that describe the system are relatively simple. For these equations the only

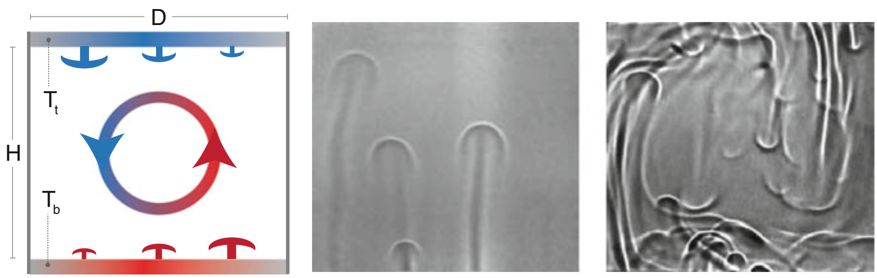

Fig. 2 The image on the *left* shows a diagram of a Rayleigh-Bénard convection cell. The shadowgraph images from Xi et al. (2004) show the structure of the thermal plumes and the mean wind into which they organize

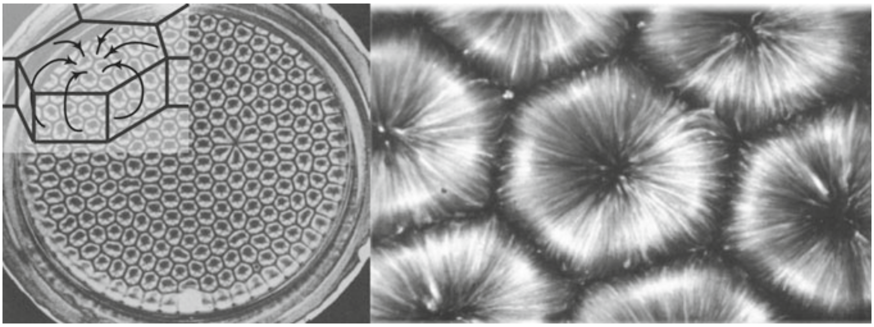

Fig. 3 The images show the hexagonal convective cells similar to those observed by Bénard in his experiments. Bénard was informed that the cells, now known to be due to the temperature dependence of surface tension, resembled the pattern of the solar granulation photographed by Janssen. Images from Van Dyke (1982): *left* photograph by Koschmieder (1974) and *right* photograph by Velarde et al. (1982). The sketch of the hexagonal flow structure in a Rayleigh-Bénard cell is inspired by Getling (1991)

dimensionless numbers necessary for dynamic similarity, given the geometry, are the Rayleigh number Ra, and the Prandtl number Pr. Here,

$$Ra = \frac{\alpha g \Delta T H^3}{\nu\kappa}, \quad Pr = \frac{\nu}{\kappa},$$

where α is the isobaric thermal expansion coefficient, g is the acceleration of gravity, ν is the kinematic viscosity, κ is the thermal diffusivity, and ΔT is the temperature difference between the top and bottom plate. Rayleigh found that the fluid starts to move when the dimensionless temperature difference, expressed by Ra, exceeds the critical value. For a detailed treatment of the linearized stability problem, see the classic text by Chandrasekhar (1961); for a general introduction, see Tritton (1988).

The RBC flow just beyond the onset of convection has been used as a paradigm for studying flow instabilities, chaotic systems and pattern formation (e.g., Busse 1978; Cross and Hohenberg 1993; Bodenschatz et al. 2000).

When the convective flow is strong enough, and the Rayleigh number is on the order of 10^5, the flow is characterized by thermal plumes, mushroom-shaped features of hot (cold) fluid detaching from the bottom (top) thermal boundary layer (e.g. Zocchi et al. 1990). At higher Rayleigh numbers, the plumes organize into a large scale motion linked synergistically with the turbulent flow (Qiu and Tong 2001; Xi et al. 2004). At very large Rayleigh numbers, the mean wind may itself be destroyed (Niemela et al. 2001; Sreenivasan et al. 2002).

The Rayleigh number, which can be interpreted as the ratio of buoyancy to viscous and thermal dissipation, is of the order $Ra \sim 10^{18} - 10^{22}$ in the atmosphere ($Pr \sim 0.7$), $Ra \sim 10^{20}$ in the ocean ($Pr \sim 7$), $Ra \sim 10^{20} - 10^{24}$ in the Sun ($Pr \sim 10^{-7} - 10^{-3}$), and $Ra \gtrsim 10^{20}$ for most astrophysical phenomena (see Sreenivasan and Donnelly 2001). These values indicate the upper end of the Rayleigh number in which we are interested.

3 High *Ra* and Ultimate Regime

Studying flows at Ra comparable to those of natural phenomena is challenging: numerical simulations and experiments cannot yet reach the parameter limits, and extrapolating the behavior from the dynamics at lower Ra is not always an option, in particular because different theories suggest different scenarios. Hence, answering even the apparently simple question of how much thermal energy is carried by the flow for a stipulated temperature difference is very hard when the temperature difference is large. This question can be formalized by looking for the dependence of the Nusselt number, the dimensionless heat transfer coefficient, on the Rayleigh number. The Nusselt number is defined as $Nu = hH/k$, where h is the measured heat transfer coefficient and k is the thermal conductivity of the fluid.

Several scaling theories have been proposed over the years. The oldest is the marginal stability theory by Malkus (1954), in which $Nu \sim Ra^{1/3}$. Written out in detail, this means that the heat transport is independent of the container height. The experiment with cryogenic helium in Chicago from the late 80s (Castaing et al. 1989) led to theories where the scaling exponent was 2/7. Additional and more precise experiments and simulations have made it clear that none of these theories could explain all the observations (though there have been suggestions in favor of the 1/3 power, see Urban et al. (2014)). A phenomenological theory that tries to account for the observed results is that of Grossmann and Lohse (2000). In this theory, there are two coupled equations for the Nusselt number and the flow Reynolds number, with six free parameters fitted to experimental and numerical data (for the updated prefactors based on the latest experiments and simulations, see Stevens et al. (2013)). The theory divides the $Pr - Ra$ parameters space into several regions in which either the bulk or the boundary layer dominates the dissipation rates (see Fig. 4).

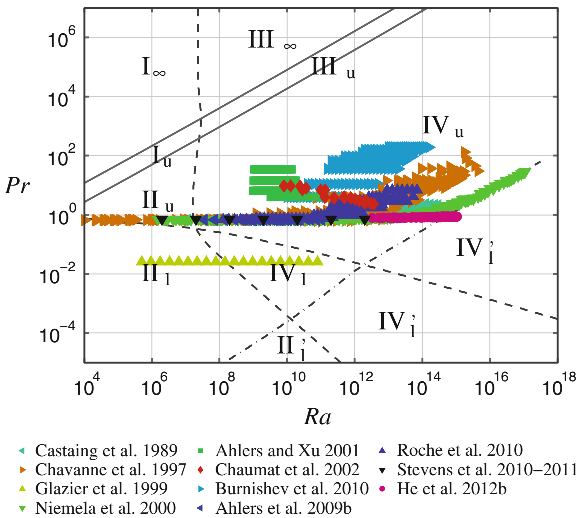

Fig. 4 Rayleigh-Bénard convection phase diagram in the $Ra - Pr$ space for $\Gamma = 1/2$ where the different regimes are from the Grossmann-Lohse theory. There is a dearth of data from experiments and simulation for the so-called ultimate regime (IV'_l), in which the kinetic boundary layer is assumed to be "completely" turbulent. The figure is adapted from Stevens et al. (2013)

The regime that is still very puzzling is the asymptotic regime for high Ra, denoted in Fig. 4 by IV_l', corresponding to "completely" turbulent boundary layers, dubbed also as the ultimate or Kraichnan regime. Indeed, Kraichnan (1962) predicted that, when the Rayleigh number exceeds some large value (for which he provided preliminary estimates), the boundary layer becomes "completely" turbulent, leading to an enhancement of heat transport; in this regime, he predicted that the Nusselt number scales as $Nu \sim Ra^{1/2}/(\log Ra)^{3/2}$.

A regime in which $Nu \sim Ra^{1/2}$ has been attained in configurations with no top and bottom boundaries, in simulations by Lohse and Toschi (2003) and in experiments in a vertical open-ended channel by Cholemari and Arakeri (2009) and Gibert et al. (2006). However, the experimental confirmation of the ultimate regime in the presence of boundary layers on solid boundaries, as one has in the classical RBC, is still awaited. For contradictory points of view on the ultimate regime, one may consult Chavanne et al. (1997), Niemela and Sreenivasan (2003, 2010), Roche et al. (2010), He et al. (2012a), Ahlers et al. (2012) and Urban et al. (2014), and references therein. One hopes that this issue will soon be resolved satisfactorily.

4 Large Aspect Ratio and Large Scales

Most experiments at high Ra are performed so far for $\Gamma = 0(1)$. However, most natural phenomena are not constrained laterally in this way, with Γ on the order $10 - 10^2$. Unfortunately, little is known for high-Ra turbulent convection in containers of large aspect ratio, and, in general, also for different geometries. Theories do not account for the aspect ratio explicitly; and while geometry effects were discussed briefly in Grossmann and Lohse (2003), only qualitative predictions are known for Γ larger than unity. Historically, convection theories have been guided by empirical results, hence the lack of data prevented a proper theoretical formulation of geometrical effects. Just recently data for Γ significantly different from order unity at high Ra are starting to appear both at moderately large aspect ratios ($\Gamma \sim 10$)—see Hogg and Ahlers (2013), du Puits et al. (2013)—and small aspect ratios, $\Gamma \sim 1/10$—see Huang et al. (2013). We should call attention to experiments by Niemela and Sreenivasan (2006) for $\Gamma = 4$, covering Rayleigh numbers up to 10^{15}, the highest to-date. Several experiments studied the convection in large aspect ratio, $\Gamma \sim 100$, using pressurized gas, and using a shadowgraph visualization technique. However, almost all the studies were done near the onset of convection (see, for example, de Bruyn et al. (1996), Bodenschatz et al. (2000)). The reason that the high Ra-high Γ parameter space is largely unexplored is that it is very challenging: while keeping all other quantities constant, we find that $Ra \propto \Gamma^{-3}$.

From available data, it would seem that the dependence of the Nusselt number Nu on the aspect ratio Γ is rather weak. Indeed, there is a strong indication that the mean wind may become weaker with increasing Rayleigh numbers (Sreenivasan et al. 2002). No major change in the Nusselt number scaling was observed in the process. This points to the likelihood that the mean wind may not play a particularly fundamental role in determining the heat transport. Further, the flow structure and the mean wind are affected by the geometry of the system. Yet, the mean wind is interesting in its own right. This large-scale mean flow was first observed experimentally by Krishnamurti and Howard (1981) and, in experiments with helium, by Sano et al. (1989) up to 10^{12} and by Niemela et al. (2001) up to 10^{13}. The importance of the aspect ratio for the mean wind was evident already in the experiments of Niemela and Sreenivasan Niemela and Sreenivasan (2003, 2006, 2010) for aspect ratios 1/2, 1 and 4, all other aspects remaining the same. Other experiments have examined different aspects of the mean wind. Several studies characterized the dynamics of these large scale motions (Ciliberto et al. 1996; Qiu and Tong 2001; Xi et al. 2004; Brown and Ahlers 2007; Funfschilling et al. 2008) in cells of aspect ratio $\Gamma \simeq 1$ and moderate Ra. One important observation is that, once in a while, the mean wind reverses direction. Such dramatic phenomenon has fascinating analogies. Statistics of the wind reversal in convection experiments show the same statistical signature as solar flare activity, which is driven by Sun's outer layer convective motion (Sreenivasan et al. 2002), and the abrupt changes in mean flow direction in large-scale atmospheric winds (van Doorn et al. 2000). Another important analogy is the reversal of Earth's magnetic field which, despite the data from past reversals obtained from geological footprints, lacks full understanding with predictive power (even for a short time).

As for Nusselt number scaling, meager amount of data is available for the case of high aspect ratio and high Ra. For $\Gamma \simeq 10$ and moderate Rayleigh numbers, simulations show the presence of cellular coherent structures (Cattaneo et al. 2001; Hartlep et al. 2003; Parodi et al. 2004; Shishkina and Wagner 2005; von Hardenberg et al. 2008). In particular, recent direct numerical simulations up to $Ra = 10^8$ (Bailon-Cuba et al. 2010) detected polygonal structures that resemble those observed right above the onset of convection. Experimentally it would be a challenging task to visualize these structures at high Ra. See Fig. 5.

The work of Niemela and Sreenivasan (2006) for $\Gamma = 4$ suggests the presence of a single coherent mean wind over the whole container, consistent with the observations of Krishnamurti and Howard (1981) at much lower Rayleigh numbers. However, at very high Ra the former authors reported the absence of the wind, a result confirmed

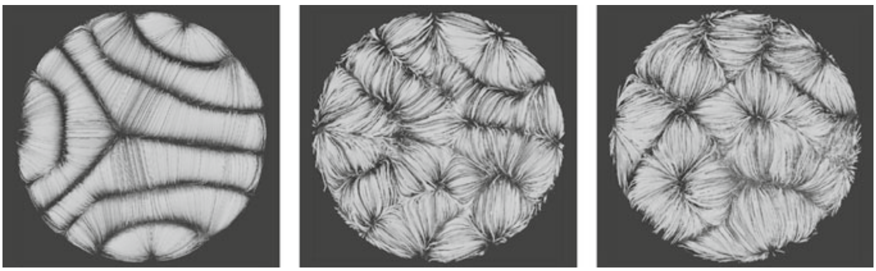

Fig. 5 Images from Bailon-Cuba et al. (2010) showing the polygonal structures of the time-averaged streamlines of the direct numerical simulations in a Rayleigh-Bénard cell with $\Gamma = 8$ for $Ra =$ 6,000, 10^7, and 10^8, respectively

subsequently by du Puits et al. (2007). The absence of wind could be either a consequence of a completely random motion, or due to the formation of more complex structures. Resolving this kind of questions experimentally requires the visualization of the global flow structure. To our knowledge, the highest Rayleigh number in a large aspect ratio experiment is $\sim 10^9$ for $\Gamma = 9$ in the air experiments by du Puits et al. (2013). If we consider only the experiments that visualized the global flow structures, the maximum Rayleigh number is $Ra = 2 \times 10^7$ in the $\Gamma = 11$ cell by Hogg and Ahlers (2013). More experiments at large aspect ratios and large Ra are therefore needed, in particular with capabilities for visualizing the flow structures (see Fig. 6).

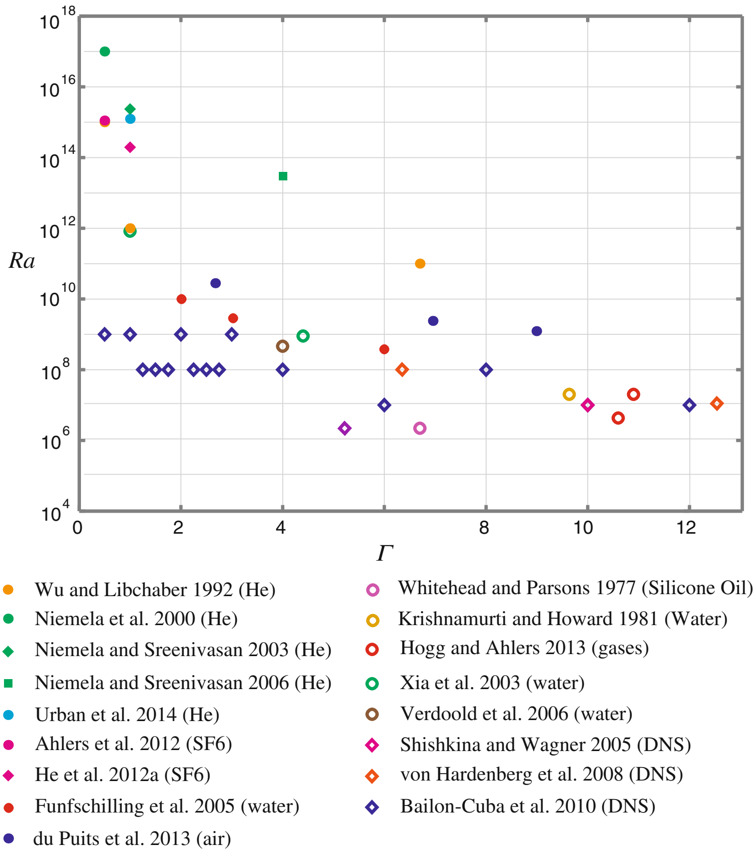

Fig. 6 The plot shows the maximum Rayleigh numbers achieved by some selected experiments and simulations for different values of the aspect ratio. Open symbols denote conditions for possible visualization

Understanding the dynamics of these flow structures, besides being of interest to the RBC community, is also of interest for the Solar Physics community. Indeed, with the exception of solar granulation, the cellular structures of the Sun are still not completely understood (Rieutord and Rincon 2010). It is worth noting that despite the traditional view that separates the scales into discrete features with different names, there is some evidence for a continuous spectrum of motion (Nordlund et al. 2009).

The appearance of discrete wavelengths or the presence of a continuous spectrum in RBC at high Ra and high Γ is still unexplained. As already mentioned, simulations at moderately high Ra show the presence of coherent structures but there are no data available at very high Ra. The appearance of regular structures from the random turbulence motion is a fascinating phenomenon that seems to go against the classical notion of turbulence, and can be seen more broadly in the context of self-organization of a system far from equilibrium (Nicolis and Prigogine 1977). In hydrodynamic turbulence such regular structures have been observed in the Taylor-Couette flow (Lathrop et al. 1992), the von Kármán flow (La Porta et al. 2001) and the pipe flow (Hof et al. 2004). In general, there is a renewed interest in the role of coherent structures in turbulent flows (Ouellette 2012), also thanks to the new tools for detecting Lagrangian coherent structures (Haller 2015). It will be interesting to see the role that RBC will play in this context in the years to come.

5 Conclusions

Turbulent convection has been studied more extensively in recent years, and much more sophisticated data have become available in both experiment and simulation. Unprecedented ranges of parameters have been explored. However, many of the old questions are still open—for example, the basic heat transport law for asymptotically large Ra. We discussed some of the open problems for the simplest case of Rayleigh-Bénard convection. Variations that include rotation, phase change and magnetic fields have also gained a lot of attention recently but are not included in this review for brevity.

One feature of experiments in the last two decades is the effort in achieving higher and higher Rayleigh numbers by building larger and larger facilities, and using different test fluids. The experiments that used cryogenic helium and pressurized SF_6 are currently the record holders for the highest Rayleigh number achieved in controlled laboratory experiments. An alternative to helium and its increasing price, or to large storage and compressing facilities of SF_6, could be the use of cryogenic nitrogen (Fonda et al. 2012), which we believe could be the fluid of choice for a landmark facility with the goal of reaching $Ra \sim 10^{20}$.

References

Ahlers G, Grossmann S, Lohse D (2009a) Heat transfer and large scale dynamics in turbulent Rayleigh-Bénard convection. Rev Mod Phys 81(2):503

Ahlers G, Funfschilling D, Bodenschatz E (2009b) Transitions in heat transport by turbulent convection at Rayleigh numbers up to 10^{15}. New J Phys 11(12):123001

Ahlers G, He X, Funfschilling D, Bodenschatz E (2012) Heat transport by turbulent Rayleigh-Bénard convection for $Pr \simeq 0.8$ and $3 \times 10^{12} \leq Ra \leq 10^{15}$: aspect ratio $\Gamma = 0.50$. New J Phys 14(10):103012

Ahlers G, Xu X (2001) Prandtl-number dependence of heat transport in turbulent Rayleigh-Bénard convection. Phys Rev Lett 86(15):3320

Bailon-Cuba J, Emran MS, Schumacher J (2010) Aspect ratio dependence of heat transfer and large-scale flow in turbulent convection. J Fluid Mech 655:152

Bodenschatz E, Pesch W, Ahlers G (2000) Recent developments in Rayleigh-Bénard convection. Annu Rev Fluid Mech 32(1):709

Brown E, Ahlers G (2007) Large-scale circulation model for turbulent Rayleigh-Bénard convection. Phys Rev Lett 98(13):134501

Buffett BA (2000) Earth's core and the geodynamo. Science 288(5473):2007

Burnishev Y, Segre E, Steinberg V (2010) Strong symmetrical non-Oberbeck-Boussinesq turbulent convection and the role of compressibility. Phys Fluids 22(3):035108

Busse FH (1978) Non-linear properties of thermal convection. Rep Prog Phys 41(12):1929

Cardin P, Olson P (1994) Chaotic thermal convection in a rapidly rotating spherical shell: consequences for flow in the outer core. Phys Earth Planet In 82(3–4):235

Castaing B, Gunaratne G, Heslot F, Kadanoff L, Libchaber A, Thomae S, Wu X-Z, Zaleski S, Zanetti G (1989) Scaling of hard thermal turbulence in Rayleigh-Bénard convection. J Fluid Mech 204:1

Cattaneo F, Lenz D, Weiss N (2001) On the origin of the solar mesogranulation. Astrophys J 563(1):L91

Chandrasekhar S (1961) Hydrodynamic and hydromagnetic stability. Dover Publications, New York

Chaumat S, Castaing B, Chilla F (2002) Rayleigh-Bénard cells: influence of the plates' properties. Advances in turbulence IX (ed. IP Castro, PE Hancock, and TG Thomas) CIMNE, Barcelona

Chavanne X, Chilla F, Castaing B, Hebral B, Chabaud B, Chaussy J (1997) Observation of the ultimate regime in Rayleigh-Bénard convection. Phys Rev Lett 79(19):3648

Chillà F, Schumacher J (2012) New perspectives in turbulent Rayleigh-Bénard convection. Eur Phys J E 35(7):58

Cholemari MR, Arakeri JH (2009) Axially homogeneous, zero mean flow buoyancy-driven turbulence in a vertical pipe. J Fluid Mech 621:69

Ciliberto S, Cioni S, Laroche C (1996) Large-scale flow properties of turbulent thermal convection. Phys Rev E 54(6):R5901

Cross M, Hohenberg P (1993) Pattern formation outside of equilibrium. Rev Mod Phys 65(3):851

de Bruyn JR, Bodenschatz E, Morris SW, Trainoff SP, Hu Y, Cannell DS, Ahlers G (1996) Apparatus for the study of Rayleigh-Bénard convection in gases under pressure. Rev Sci Instrum 67(6):2043

du Puits R, Resagk C, Thess A (2007) Breakdown of wind in turbulent thermal convection. Phys Rev E 75(1):016302

du Puits R, Resagk C, Thess A (2013) Thermal boundary layers in turbulent Rayleigh-Bénard convection at aspect ratios between 1 and 9. New J Phys 15(1):013040

Fonda E, Sreenivasan KR, Lathrop DP (2012) Liquid nitrogen in fluid dynamics: visualization and velocimetry using frozen particles. Rev Sci Instrum 83(8):085101

Funfschilling D, Brown E, Ahlers G (2008) Torsional oscillations of the large-scale circulation in turbulent Rayleigh-Bénard convection. J Fluid Mech 607:119

Funfschilling D, Brown E, Nikolaenko A, Ahlers G (2005) Heat transport by turbulent Rayleigh-Bénard convection in cylindrical samples with aspect ratio one and larger. J Fluid Mech 536:145

Getling AV (1991) Formation of spatial structures in Rayleigh-Bénard convection. Sov Phys Usp 34(9):737

Gibert M, Pabiou H, Chillà F, Castaing B (2006) High-Rayleigh-number convection in a vertical channel. Phys Rev Lett 96(8):084501

Glazier JA, Segawa T, Naert A, Sano M (1999) Evidence against 'ultrahard' thermal turbulence at very high Rayleigh numbers. Nature 398(6725):307

Grossmann S, Lohse D (2000) Scaling in thermal convection: a unifying theory. J Fluid Mech 407:27

Grossmann S, Lohse D (2003) On geometry effects in Rayleigh-Bénard convection. J Fluid Mech 486:105

Guillot T (2005) The interior of giants planets: models and outstanding questions. Annu Rev Earth Planet Sci 33(1):493

Haller G (2015) Lagrangian coherent structures. Annu Rev Fluid Mech (August 2014):137

Hartlep T, Tilgner A, Busse F (2003) Large scale structures in Rayleigh-Bénard convection at high Rayleigh numbers. Phys Rev Lett 91(6):064501

Hartmann DL, Moy LA, Fu Q (2001) Tropical convection and the energy balance at the top of the atmosphere. J Clim 14(24):4495

He X, Funfschilling D, Bodenschatz E, Ahlers G (2012a) Heat transport by turbulent Rayleigh-Bénard convection for $Pr \simeq 0.8$ and $4 \times 10^{11} \leq Ra \leq 2 \times 10^{14}$: ultimate-state transition for aspect ratio $\Gamma = 1.00$. New J Phys 14(6):063030

He X, Funfschilling D, Nobach H, Bodenschatz E, Ahlers G (2012b) Transition to the ultimate state of turbulent Rayleigh-Bénard convection. Phys Rev Lett 108(2):024502

Hof B, van Doorne CWH, Westerweel J, Nieuwstadt FTM, Faisst H, Eckhardt B, Wedin H, Kerswell RR, Waleffe F (2004) Experimental observation of nonlinear traveling waves in turbulent pipe flow. Science 305(5690):1594

Hogg J, Ahlers G (2013) Reynolds-number measurements for low-Prandtl-number turbulent convection of large-aspect-ratio samples. J Fluid Mech 725:664

Huang S-D, Kaczorowski M, Ni R, Xia K-Q (2013) Confinement-induced heat-transport enhancement in turbulent thermal convection. Phys Rev Lett 111(10):104501

Koschmieder EL (1974) Bénard convection. Adv Chem Phys 26. Prigogine I, Rice SA (eds). Wiley, Hoboken

Kraichnan RH (1962) Turbulent thermal convection at arbitrary Prandtl number. Phys Fluids 5(11):1374

Krishnamurti R, Howard LN (1981) Large-scale flow generation in turbulent convection. Proc Natl Acad Sci USA 78(4):1981

La Porta A, Voth GA, Crawford AM, Alexander J, Bodenschatz E (2001) Fluid particle accelerations in fully developed turbulence. Nature 409(6823):1017

Lathrop DP, Fineberg J, Swinney H (1992) Transition to shear-driven turbulence in Couette-Taylor flow. Phys Rev A 46(10):6390

Lohse D, Toschi F (2003) Ultimate state of thermal convection. Phys Rev Lett 90(3):034502

Lohse D, Xia K-Q (2010) Small-scale properties of turbulent Rayleigh-Bénard convection. Annu Rev Fluid Mech 42(1):335

Malkus WVR (1954) The heat transport and spectrum of thermal turbulence. Proc R Soc Lond A 225(1161):196

Marshall J, Schott F (1999) Open-ocean convection: observations, theory, and models. Rev Geophys 37(1):1

Nicolis G, Prigogine I (1977) Self-organization in nonequilibrium systems. Wiley, New York

Niemela JJ, Skrbek L, Sreenivasan KR, Donnelly RJ (2000) Turbulent convection at very high Rayleigh numbers. Nature 404(1):837

Niemela JJ, Skrbek L, Sreenivasan KR, Donnelly RJ (2001) The wind in confined thermal convection. J Fluid Mech 449:169

Niemela JJ, Sreenivasan KR (2003) Confined turbulent convection. J Fluid Mech 481:355

Niemela JJ, Sreenivasan KR (2006) Turbulent convection at high Rayleigh numbers and aspect ratio 4. J Fluid Mech 557:411

Niemela JJ, Sreenivasan KR (2010) Does confined turbulent convection ever attain the ‘asymptotic scaling’ with 1/2 power? New J Phys 12(11):115002
Nordlund AK, Stein RF, Asplund M (2009) Solar surface convection. Living Rev Sol Phys 6
Ouellette NT (2012) On the dynamical role of coherent structures in turbulence. C R Phys 13(9–10):866
Parodi A, von Hardenberg J, Passoni G, Provenzale A, Spiegel E (2004) Clustering of plumes in turbulent convection. Phys Rev Lett 92(19):194503
Qiu X-L, Tong P (2001) Large-scale velocity structures in turbulent thermal convection. Phys Rev E 64(3):036304
Rieutord M, Rincon F (2010) The suns supergranulation. Living Rev Sol Phys 7
Roche P-E, Gauthier F, Kaiser R, Salort J (2010) On the triggering of the ultimate regime of convection. New J Phys 12(8):085014
Sano M, Wu X, Libchaber A (1989) Turbulence in helium-gas free convection. Phys Rev A 40(11):6421
Schubert G, Turcotte DL, Olson P (2001) Mantle convection in the earth and planets. Cambridge University Press, Cambridge
Shishkina O, Wagner C (2005) Analysis of thermal dissipation rates in turbulent Rayleigh-Bénard convection. J Fluid Mech 546:51
Sreenivasan K, Bershadskii A, Niemela J (2002) Mean wind and its reversal in thermal convection. Phys Rev E 65(5):056306
Sreenivasan K, Donnelly R (2001) Role of cryogenic helium in classical fluid dynamics basic research and model testing. Adv Appl Mech 37:239
Stevens RJAM, Lohse D, Verzicco R (2011) Prandtl and Rayleigh number dependence of heat transport in high Rayleigh number thermal convection. J Fluid Mech 688:31
Stevens RJAM, van der Poel EP, Grossmann S, Lohse D (2013) The unifying theory of scaling in thermal convection: the updated prefactors. J Fluid Mech 730:295
Stevens RJAM, Verzicco R, Lohse D (2010) Radial boundary layer structure and Nusselt number in Rayleigh-Bénard convection. J Fluid Mech 643:495
Tackley PJ (2000) Mantle convection and plate tectonics: toward an integrated physical and chemical theory. Science 288(5473):2002
Tritton DJ (1988) Physical fluid dynamics. Clarendon Press, Oxford
Urban P, Hanzelka P, Musilová V, Králík T, Mantia ML, Srnka A, Skrbek L (2014) Heat transfer in cryogenic helium gas by turbulent Rayleigh-Bénard convection in a cylindrical cell of aspect ratio 1. New J Phys 16(5):053042
van Doorn E, Dhruva B, Sreenivasan KR, Cassella V (2000) Statistics of wind direction and its increments. Phys Fluids 12(6):1529
Van Dyke M (1982) An album of fluid motion. Parabolic Press, Stanford
Velarde MG et al (1982) An album of fluid motion, assembled by Van Dyke M. Parabolic Press, Stanford
Verdoold J, Tummers M, Hanjalić K (2006) Oscillating large-scale circulation in turbulent Rayleigh-Bénard convection. Phys Rev E 73(5):056304
von Hardenberg J, Parodi A, Passoni G, Provenzale A, Spiegel E (2008) Large-scale patterns in Rayleigh-Bénard convection. Phys Lett A 372(13):2223–2229
Wesfreid JE (2006) Scientific biography of Henri Bénard (1874–1939). In: Mutabazi I, Wesfreid JE, Guyon E (eds) Dynamics of spatio-temporal cellular structures. Springer tracts in modern physics, vol 207. Springer, New York, pp 9–37
Whitehead JA, Parsons B (1977) Observations of convection at Rayleigh numbers up to 760,000 in a fluid with large Prandtl number. Geophys Astrophys Fluid Dyn 9(1):201
Wu X-Z, Libchaber A (1992) Scaling relations in thermal turbulence: the aspect-ratio dependence. Phys Rev A 45(2):842
Xi H-D, Lam S, Xia K-Q (2004) From laminar plumes to organized flows: the onset of large-scale circulation in turbulent thermal convection. J Fluid Mech 503:47

Xia K-Q (2013) Current trends and future directions in turbulent thermal convection. Theor Appl Mech Lett 3(5):052001

Xia K-Q, Sun C, Zhou S-Q (2003) Particle image velocimetry measurement of the velocity field in turbulent thermal convection. Phys Rev E 68(6):066303

Zocchi G, Moses E, Libchaber A (1990) Coherent structures in turbulent convection, an experimental study. Phys A 166(3):387

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