Chapter 2
Observer Design for Discrete-Time Switching Nonlinear Models

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Abstract Switched systems are often described by continuous and discrete dynamics as well as their interactions. Although results are available for linear switching systems, for nonlinear switching models few results exist. In this chapter, we consider observer design for discrete-time switching nonlinear systems with a Takagi–Sugeno representation. For designing the observers, a switching nonquadratic Lyapunov function is used. Such Lyapunov functions have shown a real improvement of the design conditions for discrete-time Takagi–Sugeno models. The Lypunov function can be defined for each subsystem or just for the moments when switching takes place. In the first case the results are more general, but also more conservative. The second case represents a significant improvement for periodic models. Thanks to the Lyapunov function used, it is possible to design observers for some switching systems with unobservable subsystems. The developed conditions are formulated as linear or bilinear matrix inequalities. Their advantages and shortcomings are illustrated on numerical examples.

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2.1 Introduction

2.1.1 Switching Systems

Switching systems are a class of hybrid systems that switch between a family of modes or subsystems. In the last decades, analysis and synthesis of switching systems has attracted much attention, mostly in the continuous-time case, with linear subsystems.

For instance, linear switching systems where the switching laws can be arbitrarily chosen have been considered in [1] to study the reachable set of such systems. Stabilization and tracking conditions for continuous-time linear switching systems have been developed in [3, 4], delay-dependent stabilization in [33], and observability with unknown input has been investigated in [8]. The results for switching systems have been applied in [26] for the decentralized stabilization of multiagent systems. State feedback controller design for nonlinear switching systems has been presented in [7] and optimal control in [5]. A notable result, although for continuous-time linear switching systems, is the one in [30], which concerns the design of switching sequences for stabilization and proves that it is sufficient for stabilization to employ a periodic switching law.

For the discrete-time case considerably fewer results exist. Most of these concern linear subsystems, such as [31], where stabilization in the presence of input saturation and uncertainties is considered, or [14] which considers the computation of the mode-dependent dwell time. Observability for switching discrete-time linear systems has been investigated in [13], while a linear controller with integral action has been used for the stabilization of switching systems in [7]. Other recent approaches have been reported in [11, 19, 29].

This chapter deals with nonlinear switching models. These models can be found in various domains [41, 47, 58, 60–62], such as automotive, networked control, DC converters, mobile robots, etc. In the case of automotive applications, switching approaches have been used for different parts of the vehicle: engine control, HCCI combustion [40], air path with turbocharger [42–44], clutch actuator control [38].

A very promising recent field of research for switching control concerns systems controlled via network [17, 50]. In this context, stability and stabilization are subject to communication imperfections. Switching methods allow to take into account those constraints.

In power electronics, applications concern power converter structures. Multicellular converters are components which require the control of several switches. A way to consider the different possible modes is to use a switching structure [28].

A particular class of switching models represents those models that switch periodically. Such systems can be found in numerous domains such as automotive, aeronautic, aerospace, and computer control of industrial process. For example, in [10], a periodic dynamic model is used to estimate the air/fuel ratio in each cylinder on an internal combustion engine, Gaiani et al. [25] proposes a periodic model for the rotor blades of helicopter, [56] deals with the problem of an onboard automatic station keeping of a small spacecraft on a specific orbit of reference and proposes a periodic state feedback control law. Other examples are provided in [6] related to computer control and communication systems.

The stability of linear periodic systems is characterized by the monodromy transition matrix and by its eigenvalues, called the characteristic multipliers (often referred to as the poles of the system). If all the characteristic multipliers are in the open unit disc of the complex plane, then the system is asymptotically stable [22]. Concerning the stabilization problem of those models, results are available in [21]. For models including time-varying delays, Stepan and Insperger [51] proposed methods based on Floquet's transformation, which is only applied to autonomous systems, and led to conditions for exponential stability.

Extensions exist to polytopic linear parameter varying periodic models, where the stability analysis is based on the use of quadratic [2, 21] or nonquadratic [12] Lyapunov functions. Results for stabilization has been reported in [32, 34] but observer design has not been considered.

In what follows, we consider observer design for general and periodic switching systems. We represent the switching nonlinear models by switching quasi-linear parameter varying or Takagi–Sugeno (TS) fuzzy systems [52], presented in what follows.

2.1.2 Discrete-Time TS Models

Dynamic systems are modeled in the state space framework, using a state transition model, which describes the evolution of the states over time, and a measurement model, which relates the measurement to the states. Mathematically, we describe such systems, in discrete time, as:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) \\ y_k &= h(x_k) \end{aligned} \tag{2.1}$$

where f denotes the state transition function, describing the evolution of the states over time, h is the measurement function, relating the measurements to the states, x is the vector of the state variables, u is the vector of the input or control variables, and y denotes the measurement vector.

We represent the system above by Takagi–Sugeno (TS) fuzzy models of the form

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} h_i(z_k)(A_i x_k + B_i u_k) \\ y_k &= \sum_{i=1}^{r} h_i(z_k) C_i x_k \end{aligned} \tag{2.2}$$

where r is the number of local models, A_i, B_i, C_i, are the matrices of the ith local model, z is the vector of the scheduling variables, which may depend on the states, inputs, measurements, or other exogenous variables, and $h_i(z_k)$, $i = 1, 2, \ldots, r$ are normalized membership functions, i.e., $h_i(z_k) \geq 0$ and $\sum_{i=1}^{r} h_i(z_k) = 1$, $\forall k \in \mathbb{N}$.

Such a model presents several advantages. The TS model is a universal approximator [20], and many nonlinear systems can be exactly represented in a compact set of the state space as TS systems [45]. Moreover, they are convex combination of local affine models, which facilitates stability analysis and controller and observer design for such systems. In addition, many stability and design conditions for TS systems can be formulated as linear matrix inequalities [9, 48, 53, 55], for which efficient algorithms exist.

Once a TS representation of the nonlinear system (2.1) is available, the analysis is performed using Lyapunov's direct method. Most commonly, common quadratic Lyapunov functions have been used and conditions developed independently of the membership functions. In the last years, results have been significantly improved by the use of nonquadratic Lyapunov functions, in particular for the discrete-time case.

Controller and observer design using TS models has gained an increased interest. The most well-known structure used is the so-called PDC (parallel distributed compensation) controller or observer design, where a constant gain corresponds to each rule, and the synthesis is done based on a common quadratic Lyapunov function. With the use of nonquadratic [15, 27, 35] Lyapunov functions, more complex state feedback controllers and observers were developed, and the design conditions became less conservative. These conditions are generally derived in the form of linear matrix inequalities (LMIs).

Switched TS systems have been investigated mainly in the continuous case where the stability is based on the use of a quadratic Lyapunov function [36, 37, 46, 54] or a piecewise one [23, 24]. For discrete-time switching TS models, using nonquadratic Lyapunov functions, few results exist [16, 18].

In this chapter, switching discrete-time TS fuzzy models are considered and observer design conditions are developed. For the ease of the notation, a graph representation of the switching system is employed and to develop the conditions, a nonquadratic switching Lyapunov function is used.

2.2 Switching TS Models

2.2.1 Preliminaries

To design observers for discrete-time switching TS systems, we consider subsystems of the form

$$x_{k+1} = \sum_{i=1}^{r} h_i(z_k)(A_{j,i}x_k + B_{j,i}u_k)$$
$$y_k = \sum_{i=1}^{r} h_i(z_k)C_{j,i}x_k$$

denoted in what follows as

$$\begin{aligned} x_{k+1} &= A_{j,z}x_k + B_{j,z}u_k \\ y_k &= C_{j,z}x_k \end{aligned} \tag{2.3}$$

where j is the number of the current subsystem, $j = 1, 2, \ldots, n_s$, n_s being the number of the subsystems, x denotes the state vector, r is the number of rules, z is the scheduling vector, h_i, $i = 1, 2, \ldots, r$ are normalized membership functions, and $A_{j,i}$, $B_{j,i}$, and $C_{j,i}$, $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, n_s$, are the local models. Once activated, a subsystem may be active for at least $p_i^m \in \mathbb{N}^+$ and at most $p_i^M \in \mathbb{N}^+$ samples, that are assumed known.

In this chapter, we will make use of the following results:

Lemma 2.1 [49] *Consider a vector $x \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{m \times n_x}$ such that rank$(R) < n_x$. The two following expressions are equivalent:*

1. $x^T Qx < 0$, $x \in \{x \in \mathbb{R}^{n_x}, x \neq 0, Rx = 0\}$
2. $\exists M \in \mathbb{R}^{n_x \times m}$ *such that* $Q + MR + R^T M^T < 0$

Analysis and design for TS models often lead to double-sum negativity problems of the form

$$x^T \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z_k)h_j(z_k)\Gamma_{ij}x < 0 \tag{2.4}$$

where Γ_{ij}, $i, j = 1, 2, \ldots, r$ are matrices of appropriate dimensions.

Lemma 2.2 [59] *The double-sum* (2.4) *is negative, if*

$$\begin{aligned} &\Gamma_{ii} < 0 \\ &\Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r, \ i < j \end{aligned}$$

Lemma 2.3 [57] *The double-sum* (2.4) *is negative, if*

$$\begin{aligned} &\Gamma_{ii} < 0 \\ &\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r, \ i \neq j \end{aligned}$$

0 and I denote the zero and identity matrices of appropriate dimensions, and a $(*)$ denotes the term induced by symmetry. The subscript $z + m$ (as in $A_{1,z+m}$) stands for the scheduling vector being evaluated at the current sample plus mth instant, i.e., $z(k + m)$.

2.2.2 Graph Representation

For the easier notation, we use a directed graph representation of the switching system (2.3). The associated graph is $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, with $\mathscr{V}$ being the set of vertices representing the subsystems and $\mathscr{E}$ the set of admissible transitions or switches. As such, $(v_i, v_j) \in \mathscr{E}$ if a switch from subsystem i to subsystem j is possible. Note that we assume that self-transitions are also possible: these correspond to the subsystem being active for more than one sample.

A path $\mathscr{P}(v_i, v_j)$ between two vertices v_i and v_j in the graph $\mathscr{G}$ is a sequence of vertices $\mathscr{P}(v_i, v_j) = [v_{p_1}, v_{p_2}, \ldots, v_{p_{n_p}}]$ so that $v_i = v_{p_1}$, $v_j = v_{p_{n_p}}$, and $(v_{p_k}, v_{p_{k+1}}) \in \mathscr{E}$, $k = 1, 2, \ldots, n_p - 1$. A path between two vertices is in general not unique.

A cycle $\mathscr{C} = [c_1, c_2, \ldots, c_{n_c}, c_1]$ is a path having the same initial and final vertex. Two cycles are equivalent if the vertices in one are a cyclic permutation of the vertices in the other. In this chapter, when referring to cycles, we mean elementary cycles. A graph is strongly connected if there is a path between any two vertices in $\mathscr{V}$.

In a weighted graph $\mathscr{G} = \{\mathscr{V}, \mathscr{E}, \mathscr{W}\}$, the weight (adjacency) matrix is defined as $\mathscr{W}(i, j) = w_{i,j}$, with $w_{i,j} \in \mathbb{R}\backslash\{0\}$, if there exists an edge (v_i, v_j), or $\mathscr{W}(i, j) = \infty$, if a switch from subsystem i to subsystem j is not possible. The elements on the diagonal $\mathscr{W}(i, i) = w_{i,i}$ are the weights associated to the vertices.

We define the weight of a path $W(\mathscr{P}(v_i, v_j))$ as the product of all weights of the vertices and edges that appear in the path, i.e.,

$$W([v_{p_1}, v_{p_2}, \ldots, v_{p_{n_p}}]) = \prod_{k=1}^{n_p} w_{p_k,p_k} \cdot \prod_{k=1}^{n_p-1} w_{p_k,p_{k+1}}$$

The weight of a cycle is similarly defined. A cycle is subunitary, if its weight is less than 1.

A path in a graph associated to a switching system induces a switching law. The length of a path is given by the number of edges it contains. A cycle in a graph associated to a switching system corresponds to a periodic switching law.

The notations above are illustrated on the following example.

Example 2.4 Consider a switching system composed of four subsystems:

$$x_{k+1} = A_{i,z} x_k$$

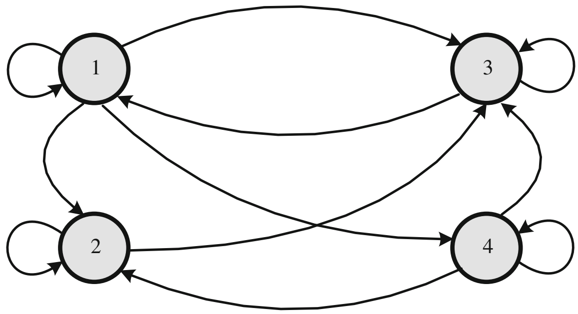

Fig. 2.1 Graph representation of the switching system in Example 2.4

for $i = 1, 2, 3, 4$, and with admissible switches (1, 2), (1, 3), (1, 4), (2, 3), (3, 1), (4, 2), (4, 3). Next to this, each subsystem can be active for more than one sample. The corresponding graph representation is illustrated in Fig. 2.1.

- The graph is $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, with the set of vertices $\mathscr{V} = \{1, 2, 3, 4\}$ and the set of admissible switches:

$$\mathscr{E} = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (3, 1), (3, 3), (4, 2), (4, 3), (4, 4)\}$$

- Possible paths between vertices 1 and 3 are $\mathscr{P}(1, 3) = [1, 3]$, $\mathscr{P}(1, 3) = [1, 4, 3]$ and $\mathscr{P}(1, 3) = [1, 2, 3]$.
- The sequence $[1, 3, 1]$ is a cycle and is equivalent to $[3, 1, 3]$.
- The cycle $C_{1,3} = [1, 3, 1]$ can induce the switching law $[1, 1, 3, 1, \ldots]$ or $[1, 3, 3, 1, \ldots]$ or any switching law of the form: $[\underbrace{1, 1, \ldots, 1}_{p_1}, \underbrace{3, 3, \ldots, 3}_{p_2}, \underbrace{1, 1, \ldots, 1}_{p_1}, \ldots]$.

The associated weight matrix will be constructed based on the possible switches and the number of samples a subsystem is being active. However, to illustrate the definitions, at this point let the associated weight matrix be given by:

$$\mathscr{W} = \begin{pmatrix} 2 & 2 & 0.5 & 2 \\ \infty & 1 & 3 & \infty \\ 0.5 & \infty & 1 & \infty \\ \infty & 1 & 1 & 2 \end{pmatrix}$$

where ∞ corresponds to an inadmissible switch. The graph with the weights given in $\mathscr{W}$ is illustrated in Fig. 2.2.

The weight of the path $\mathscr{P}(1, 3) = [1, 2, 3]$ is $W(\mathscr{P}(1, 3)) = w_{11}w_{12}w_{22}w_{23}w_{33} = 12$. The weight of the cycle $\mathscr{C} = [1, 3, 1]$ is $W(\mathscr{C}) = w_{11}w_{13}w_{33}w_{31} = 0.5 < 1$, so this cycle is subunitary. Since in the graph above there exists a path between any two vertices, the graph is strongly connected.

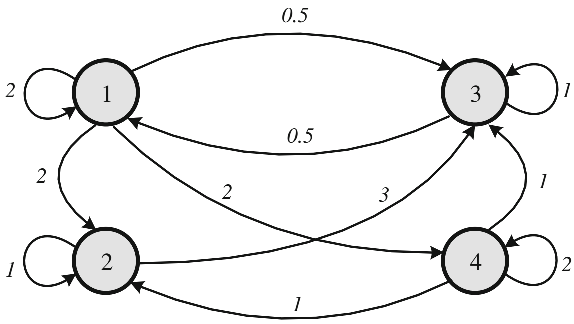

Fig. 2.2 Graph representation of the switching system with weights in Example 2.4

Our goal is to design an observer such that the estimation error dynamics converge to zero. In order to obtain relaxed conditions, we will take into account the switches that are admissible in the system. To see why this is important, consider the following example.

Example 2.5 Consider a TS system with three subsystems, each having two local models, as follows:

$$A_{1,1} = \begin{pmatrix} 0.20 & 0.06 \\ 0.56 & -0.3 \end{pmatrix} \qquad A_{1,2} = \begin{pmatrix} -1.37 & 0.52 \\ 0.96 & -0.24 \end{pmatrix}$$

$$h_{1,1} = \frac{1 - \sin(x_1)}{2} \qquad h_{1,2} = 1 - h_{1,1}$$

$$A_{2,1} = \begin{pmatrix} 0.23 & -0.55 \\ -0.80 & -1.66 \end{pmatrix} \qquad A_{2,2} = \begin{pmatrix} 1.12 & 1.20 \\ 0.49 & -0.29 \end{pmatrix}$$

$$h_{2,1} = \frac{1 - \cos(x_1)}{2} \qquad h_{2,2} = 1 - h_{2,1}$$

$$A_{3,1} = \begin{pmatrix} 3 & 0.5 \\ 0.5 & 1.5 \end{pmatrix} \qquad A_{3,2} = \begin{pmatrix} 2 & 0.1 \\ 0.5 & 2 \end{pmatrix}$$

$$h_{3,1} = \frac{1 - \exp(-x_1^2)}{2} \qquad h_{3,2} = 1 - h_{3,1}$$

One can switch from each subsystem to any other one and any subsystem can be active for any number of samples. However, the local models $A_{1,2}$, $A_{2,1}$, and $A_{2,2}$ are unstable and both local models $A_{3,1}$ and $A_{3,2}$ of the third subsystem are unstable. The associated graph is presented in Fig. 2.3.

Let us see whether this system can be stable based on the switching law that is applied. Due to the instability of the local models, no existing result in the literature can prove the stability of the switching system. Moreover, just switching to one subsystem and keeping it continuously active does not result in a stable system. However, by switching continuously between the first and second subsystem, the states converge to zero. This can be proven by using a periodic Lyapunov function, such as the one proposed in [39]. From the third subsystem, one can switch to the

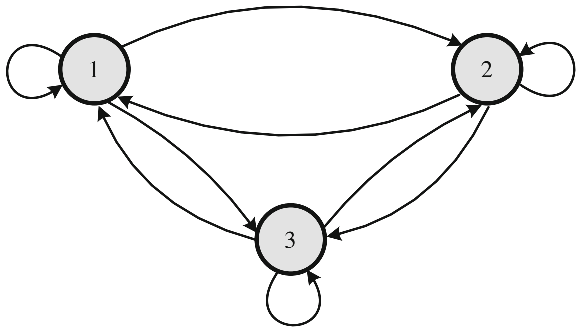

Fig. 2.3 Graph representation of the switching system in Example 2.5

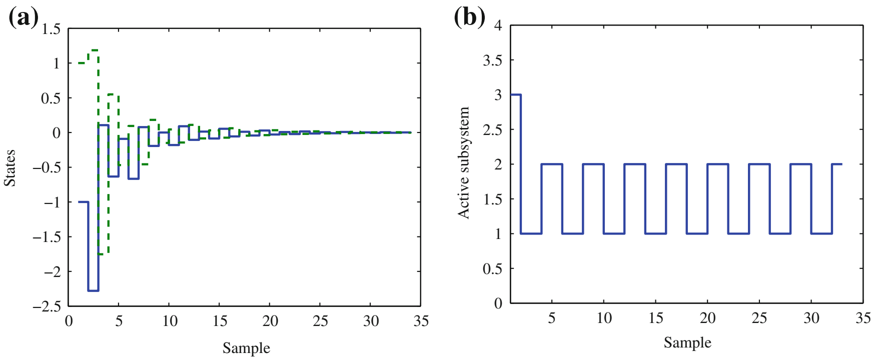

Fig. 2.4 Simulation results for Example 2.5. **a** A trajectory of the switching system. **b** The stabilizing switching law

first or second one and then switch between these two and obtain a stable system. A trajectory that confirms stability of the switching system, starting from $x_0 = [-1,\ 1]^T$ and the corresponding switching law are illustrated in Fig. 2.4a and b, respectively.

As an extension of the stability analysis shown by Example 2.5, by taking into account the switching sequence, it is possible to design more relaxed conditions. Therefore, in what follows, we develop observer design conditions for switching systems.

2.3 Observer Design

We consider observer design for the switching TS system (2.3) of the form (repeated here for convenience):

$$\begin{aligned} x_{k+1} &= A_{j,z} x_k + B_{j,z} u_k \\ y_k &= C_{j,z} x_k \end{aligned}$$

To develop the conditions, we use the switching observer

$$\begin{aligned}\widehat{x}_{k+1} &= A_{j,z}\widehat{x}_k + B_{j,z}u_k + H_{j,z}^{-1}L_{j,z}(y_k - \widehat{y}_k)\\ \widehat{y}_k &= C_{j,z}\widehat{x}_k\end{aligned} \tag{2.5}$$

for the j-th subsystem, the observer switching together with the observed subsystem. The matrices $H_{j,i}$ and $L_{j,i}$, $j = 1, 2 \ldots, n_s$, $i = 1, 2, \ldots, r$ are to be determined.

The error dynamics $e_k = x_k - \widehat{x}_k$ using this observer, under the assumption that the scheduling variables are available online at sample k, can be written as

$$\begin{aligned}e_{k+1} &= A_{j,z}e_k - H_{j,z}^{-1}L_{j,z}(y_k - \widehat{y}_k)\\ &= (A_{j,z} - H_{j,z}^{-1}L_{j,z}C_{j,z})e_k\end{aligned}$$

which in itself is a switching system.

In what follows, we will consider different possibilities of switching systems and derive observer design conditions that ensure that the estimation error dynamics converges to zero. The conditions will be relaxed depending on the possible switches and the possibility of choosing the switches.

2.3.1 Switching TS Systems

Let us first consider a general switching system, with the possible switches given by the edges in the graph associated to the system. Recall that using the observer (2.5), the error dynamics are given by

$$e_{k+1} = (A_{j,z} - H_{j,z}^{-1}L_{j,z}C_{j,z})e_k \tag{2.6}$$

This is in itself a switching system, with the same switching sequence as the original system and thus with the same graph. Naturally, the simplest way to develop conditions that ensure that the error dynamics is asymptotically stable is to use the common—for all the local models and all the subsystems—Lyapunov function

$$V(e_k) = e_k^T P e_k$$

The difference in the Lyapunov function for two consecutive samples is

$$\begin{aligned}\Delta V &= e_{k+1}^T P e_{k+1} - e_k^T P e_k\\ &= \begin{pmatrix} e_k \\ e_{k+1}\end{pmatrix}^T \begin{pmatrix} -P & 0 \\ 0 & P\end{pmatrix}\begin{pmatrix} e_k \\ e_{k+1}\end{pmatrix}\end{aligned}$$

During the transition from subsystem j to subsystem l, the dynamics of the error system are described by

$$\begin{pmatrix} A_{j,z} - H_{j,z}^{-1} L_{j,z} C_{j,z} & -I \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} = 0$$

Using Lemma 2.1, the difference in the Lyapunov function is negative, if there exists M such that

$$\begin{pmatrix} -P & 0 \\ 0 & P \end{pmatrix} + M \begin{pmatrix} A_{j,z} - H_{j,z}^{-1} L_{j,z} C_{j,z} & I \end{pmatrix} + (*) < 0$$

By choosing

$$M = \begin{pmatrix} 0 \\ H_{j,z} \end{pmatrix}$$

we have the conditions

$$\begin{pmatrix} -P & (*) \\ H_{j,z} A_{j,z} - L_{j,z} C_{j,z} & -H_{j,z} - H_{j,z}^T + P \end{pmatrix} < 0 \tag{2.7}$$

and we have

Theorem 2.6 *The error dynamics* (2.6) *is asymptotically stable, if there exist* $P = P^T > 0$, $H_{j,k}$, $j \in \mathscr{V}$, $k = 1, 2, \ldots, r$, *such that* (2.7) *holds for all vertices* $j \in \mathscr{V}$.

Note that the conditions (2.7) are nonlinear, but sufficient LMI conditions can easily be formulated using, e.g., Lemma 2.3, as follows:

Corollary 2.7 *The error dynamics* (2.6) *is asymptotically stable, if there exist* $P = P^T > 0$, $H_{j,k}$, $j \in \mathscr{V}$, $k, l = 1, 2, \ldots, r$, *such that*

$$\begin{aligned} &\Gamma_{i,k}^j < 0 \\ &\frac{2}{r-1} \Gamma_{ii}^j + \Gamma_{ik}^j + \Gamma_{ki}^j < 0 \\ &i, \ k = 1, \ 2, \ \ldots, \ r \end{aligned}$$

with

$$\Gamma_{ik}^j = \begin{pmatrix} -P & (*) \\ H_{j,k} A_{j,i} - L_{j,k} C_{j,i} & -H_{j,k} + (*) + P \end{pmatrix}$$

Due to the common Lyapunov function for all the local models and all the subsystems, the conditions above are very restrictive. One possibility to reduce the conservativeness is to use a nonquadratic Lyapunov function common for all the subsystems. Thus, one can consider the Lyapunov function

$$V(e_k) = e_k^T P_z e_k \tag{2.8}$$

The difference for two consecutive samples is

$$\begin{aligned} \Delta V &= e_{k+1}^T P_{z+1} e_{k+1} - e_k^T P_z e_k \\ &= \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix}^T \begin{pmatrix} -P_z & 0 \\ 0 & P_{z+1} \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} \end{aligned}$$

and choosing

$$M = \begin{pmatrix} 0 \\ H_{j,z} \end{pmatrix}$$

we have

$$\begin{pmatrix} -P_z & (*) \\ H_{j,z} A_{j,z} - L_{j,z} C_{j,z} & -H_{j,z} - H_{j,z}^T + P_{z+1} \end{pmatrix} < 0 \tag{2.9}$$

However, except for the relaxation brought by the nonquadratic Lyapunov function, this choice does not bring a significant improvement. The next step is choosing a different Lyapunov function for each subsystem, i.e., using

$$V(e_k) = e_k^T P_{j,z} e_k \tag{2.10}$$

for subsystem j.

With this, we obtain

$$\begin{aligned} \Delta V &= e_{k+1}^T P_{l,z+1} e_{k+1} - e_k^T P_{j,z} e_k \\ &= \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix}^T \begin{pmatrix} -P_{j,z} & 0 \\ 0 & P_{l,z+1} \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} \end{aligned}$$

where $[v_j, v_l]$ is an admissible path.

During the transition from j to l, the dynamics of the error system are described by

$$\begin{pmatrix} A_{j,z} - H_{j,z}^{-1} L_{j,z} C_{j,z} & -I \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} = 0$$

By choosing in Lemma 2.1

$$M = \begin{pmatrix} 0 \\ H_{j,z} \end{pmatrix}$$

we have

$$\begin{pmatrix} -P_{j,z} & (*) \\ H_{j,z}A_{j,z} - L_{j,z}C_{j,z} & -H_{j,z} - H_{j,z}^T + P_{l,z+1} \end{pmatrix} < 0 \tag{2.11}$$

and the following conditions can be formulated:

Theorem 2.8 *The error dynamics* (2.6) *is asymptotically stable, if there exist* $P_{j,k} = P_{j,k}^T > 0$, $H_{j,k}$, $(v_j, v_l) \in \mathscr{E}$, $k = 1, 2, \ldots, r$, *such that* (2.11) *holds for all admissible edges* $(v_j, v_l) \in \mathscr{E}$.

Although the conditions above are less conservative, it should be noted that they require that the error dynamics of each subsystem is asymptotically stable. However, as it has been shown on Example 2.5, this is not necessary.

Therefore, let us consider a switching Lyapunov function

$$V(e_k) = e_k^T P_{m,j,z} e_k \tag{2.12}$$

defined during the switches, i.e., on the edges of the associated graph $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, with $(v_m, v_j) \in \mathscr{E}$, instead of the subsystems. If a subsystem j may be active for several number of samples, the edge (v_j, v_j) is also considered.

Then, $e_k^T P_{m,j,z} e_k$ is active during the transition from vertex m to vertex j. The difference in the Lyapunov function for two consecutive samples is

$$\begin{aligned} \Delta V &= e_{k+1}^T P_{j,l,z+1} e_{k+1} - e_k^T P_{m,j,z} e_k \\ &= \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix}^T \begin{pmatrix} -P_{m,j,z} & 0 \\ 0 & P_{j,l,z+1} \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} \end{aligned}$$

where $[v_m, v_j, v_l]$ is an admissible path.

During the transition for j to l, the dynamics of the error system are described by

$$\begin{pmatrix} A_{j,z} - H_{j,z}^{-1} L_{j,z} C_{j,z} & -I \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} = 0$$

Using Lemma 2.1, the difference in the Lyapunov function is negative, if there exists M such that

$$\begin{pmatrix} -P_{m,j,z} & 0 \\ 0 & P_{j,l,z+1} \end{pmatrix} + M \begin{pmatrix} A_{j,z} - H_{j,z}^{-1} L_{j,z} C_{j,z} & -I \end{pmatrix} + (*) < 0$$

By choosing

$$M = \begin{pmatrix} 0 \\ H_{j,z} \end{pmatrix}$$

we have

$$\begin{pmatrix} -P_{m,j,z} & (*) \\ H_{j,z}A_{j,z} - L_{j,z}C_{j,z} & -H_{j,z} - H_{j,z}^T + P_{j,l,z+1} \end{pmatrix} < 0 \tag{2.13}$$

With this, the following conditions can be formulated:

Theorem 2.9 *The error dynamics* (2.6) *is asymptotically stable, if there exist* $P_{m,j,k} = P_{m,j,k}^T > 0$, $H_{j,k}$, $(v_m, v_j) \in \mathscr{E}$, $(v_j, v_l) \in \mathscr{E}$, $k = 1, 2, \ldots, r$, *such that* (2.13) *holds for all admissible paths* $\mathscr{P}(v_m, v_l) = [v_m, v_j, v_l]$, $v_m \in \mathscr{V}$.

Similarly to the previous cases, the condition (2.13) is nonlinear. However, LMI conditions can be easily formulated using e.g., Lemma 2.3, as follows:

Corollary 2.10 *The error dynamics* (2.6) *is asymptotically stable, if there exist* $P_{m,j,k} = P_{m,j,k}^T > 0$, $H_{j,k}$, $(v_m, v_j) \in \mathscr{E}$, $k, l = 1, 2, \ldots, r$, *such that*

$$\begin{aligned}
&\Gamma_{kk}^{m,j,l,\gamma} < 0 \\
&\frac{2}{r-1}\Gamma_{kk}^{m,j,l,\gamma} + \Gamma_{k\beta}^{i,j,l,\gamma} + \Gamma_{\beta k}^{i,j,l,\gamma} < 0 \\
&k, \beta, \gamma = 1, 2, \ldots, r
\end{aligned}$$

with

$$\Gamma_{k\beta}^{m,j,l,\gamma} = \begin{pmatrix} -P_{m,j,k} & (*) \\ H_{j,k}A_{j,\beta} - L_{j,k}C_{j,\beta} & -H_{j,k} + (*) + P_{j,l,\gamma} \end{pmatrix}$$

for all admissible paths $\mathscr{P}(v_m, v_l) = [v_m, v_j, v_l]$, $v_j \in \mathscr{V}$.

In what follows, let us illustrate on an example the conditions when using the different Lyapunov functions.

Example 2.11 Consider a TS system with three subsystems, each having two local models, as follows:

$$\begin{aligned}
A_{1,1} &= \begin{pmatrix} 0.73 & 0.52 \\ 0.66 & 0.74 \end{pmatrix} & A_{1,2} &= \begin{pmatrix} 0.28 & 0.27 \\ 1.50 & 1.38 \end{pmatrix} \\
C_{1,1} &= \begin{pmatrix} 1 & 0 \end{pmatrix} & C_{1,2} &= \begin{pmatrix} 1 & 0 \end{pmatrix} \\
A_{2,1} &= \begin{pmatrix} 1.05 & 0 \\ 0.5 & 0.1 \end{pmatrix} & A_{2,2} &= A_{2,1} \\
C_{2,1} &= \begin{pmatrix} 0 & 0 \end{pmatrix} & C_{2,2} &= C_{2,1} \\
A_{3,1} &= \begin{pmatrix} 0.86 & 0.72 \\ 0.23 & 0.76 \end{pmatrix} & A_{3,2} &= \begin{pmatrix} 0.25 & 1.51 \\ 1.94 & 1.83 \end{pmatrix} \\
C_{3,1} &= \begin{pmatrix} 1 & 0 \end{pmatrix} & C_{3,2} &= \begin{pmatrix} 0 & 0 \end{pmatrix}
\end{aligned}$$

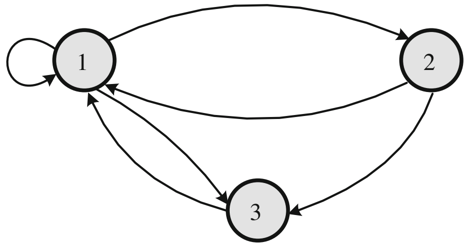

Fig. 2.5 Graph representation of the switching system in Example 2.11

Subsystem 1 may be active for at most five samples, while subsystems 2 and 3 are active only for one sample, i.e., $p_1^m = 1$, $p_1^M = 5$, $p_2^m = p_2^M = p_3^m = p_3^M = 1$. The possible switches are presented in Fig. 2.5. All the local models are unstable. Next to this, the second subsystem and the second local model of the third subsystem are unobservable.

Due to the unobservable and unstable local models, neither the conditions developed using a common quadratic Lyapunov function, nor those based on a common nonquadratic Lyapunov functions are feasible.

Using the conditions of Theorem 2.9, we obtain the gains[1]

$$H_{1,1} = \begin{pmatrix} 0.70 & -0.30 \\ -0.08 & 0.29 \end{pmatrix} \qquad H_{1,2} = \begin{pmatrix} 0.90 & -0.16 \\ -0.33 & 0.19 \end{pmatrix}$$

$$H_{2,1} = \begin{pmatrix} 0.58 & -0.35 \\ -0.10 & 0.51 \end{pmatrix} \qquad H_{2,2} = H_{2,1}$$

$$H_{3,1} = \begin{pmatrix} 0.58 & -0.39 \\ 0.05 & 0.37 \end{pmatrix} \qquad H_{3,2} = \begin{pmatrix} 0.61 & -0.37 \\ -0.37 & 0.24 \end{pmatrix}$$

$$L_{1,1} = \begin{pmatrix} 0.52 \\ 0.37 \end{pmatrix} \qquad L_{1,2} = \begin{pmatrix} 0.01 \\ 0.47 \end{pmatrix}$$

$$L_{3,1} = \begin{pmatrix} 0.28 \\ 0.37 \end{pmatrix} \qquad L_{3,2} = \begin{pmatrix} 0.25 \\ 0.85 \end{pmatrix}$$

Since the second subsystem is not observable, there are no gains $L_{2,1}$ and $L_{2,2}$. Trajectories of the states and the error are presented in Fig. 2.6a and b, respectively. The true initial states were $x_0 = [-1,\ 1]^T$ and the estimated initial states were $\hat{x}_0 = 0$. The corresponding switching law is given in Fig. 2.6c. For testing, the membership functions are assumed to depend on an exogenous measured signal, with h_1 being presented in Fig. 2.6d.

Using the conditions developed with a nonquadratic Lyapunov function for the subsystems, but taking into account the possible switches (notably that subsystems 2 and 3 are active only for one sample), we obtain the gains

[1] Throughout this chapter, computed values are truncated to two decimal places.

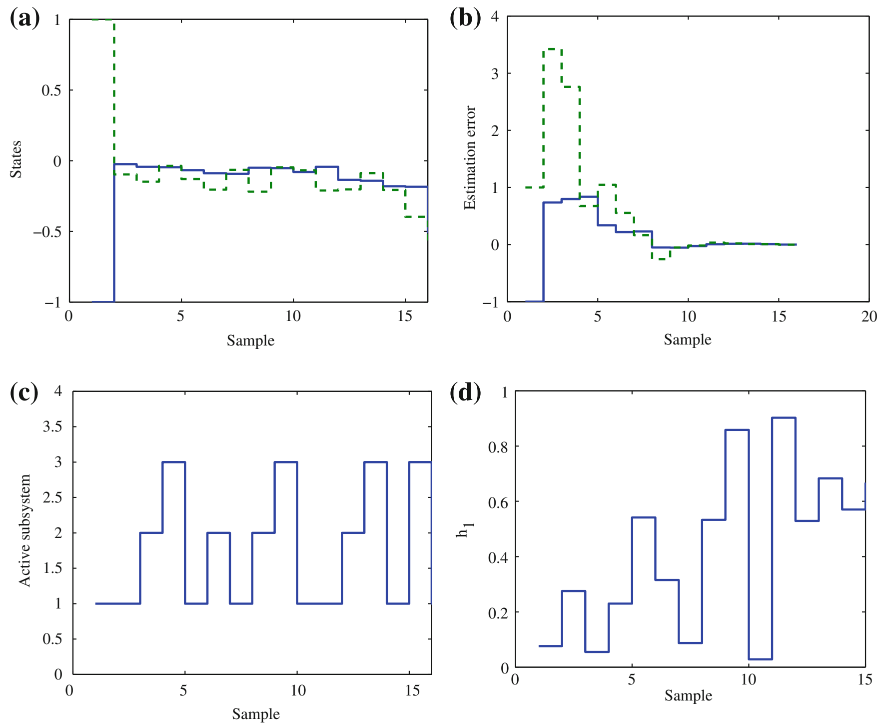

Fig. 2.6 Simulation results for Example 2.11. **a** A trajectory of the switching system. **b** Trajectory of the error. **c** The switching law used. **d** Membership function

$$
\begin{aligned}
H_{1,1} &= \begin{pmatrix} 0.66 & -0.30 \\ -0.09 & 0.31 \end{pmatrix} & H_{1,2} &= \begin{pmatrix} 0.95 & -0.16 \\ -0.36 & 0.20 \end{pmatrix} \\
H_{2,1} &= \begin{pmatrix} 0.58 & -0.36 \\ -0.01 & 0.36 \end{pmatrix} & H_{2,2} &= H_{2,1} \\
H_{3,1} &= \begin{pmatrix} 0.59 & -0.41 \\ 0.06 & 0.35 \end{pmatrix} & H_{3,2} &= \begin{pmatrix} 0.65 & -0.39 \\ -0.38 & 0.25 \end{pmatrix} \\
L_{1,1} &= \begin{pmatrix} 0.47 \\ 0.42 \end{pmatrix} & L_{1,2} &= \begin{pmatrix} 0.08 \\ 0.5 \end{pmatrix} \\
L_{3,1} &= \begin{pmatrix} 0.28 \\ 0.37 \end{pmatrix} & L_{3,2} &= \begin{pmatrix} 0.23 \\ 0.83 \end{pmatrix}
\end{aligned}
$$

which are quite close to those obtained by Theorem 2.9. It should be noted that without taking into account that subsystems 2 and 3 are active only for one sample, the conditions using a nonquadratic Lyapunov function for each subsystem are unfeasible.

In order to reduce the conservativeness by exploiting the knowledge available of the switching sequence, one can also use the observer

$$\begin{aligned} \widehat{x}_{k+1} &= A_{j,z}\widehat{x}_k + B_{j,z}u_k + H_{m,j,z}^{-1}L_{m,j,z}(y_k - \widehat{y}_k) \\ \widehat{y}_k &= C_{j,z}\widehat{x}_k \end{aligned} \tag{2.14}$$

for the j-th subsystem, if the last switch has been from vertex m to vertex j.

The error dynamics $e_k = x_k - \widehat{x}_k$ using this observer, can be written as

$$\begin{aligned} e_{k+1} &= A_{j,z}e_k - H_{m,j,z}^{-1}L_{m,j,z}(y_k - \widehat{y}_k) \\ &= (A_{j,z} - H_{m,j,z}^{-1}L_{m,j,z}C_{j,z})e_k \end{aligned} \tag{2.15}$$

Following the same steps as above, and in Lemma 2.1 choosing

$$M = \begin{pmatrix} 0 \\ H_{m,j,z} \end{pmatrix}$$

we have

Corollary 2.12 *The error dynamics (2.15) is asymptotically stable, if there exist $P_{m,j,k} = P_{m,j,k}^T > 0$, $H_{m,j,k}$, $(v_m, v_j) \in \mathscr{E}$, $(v_j, v_l) \in \mathscr{E}$, $k = 1, 2, \ldots, r$, such that*

$$\begin{pmatrix} -P_{m,j,z} & (*) \\ H_{m,j,z}A_{j,z} - L_{m,j,z}C_{j,z} & -H_{m,j,z} + (*) + P_{j,l,z+1} \end{pmatrix} < 0 \tag{2.16}$$

for all admissible paths $\mathscr{P}(v_m, v_l) = [v_m, v_j, v_l]$, $v_m \in \mathscr{V}$.

This result can actually be used if the switching sequence is known at least one switch in advance and represents the case when the observer gains are specified for each switch instead of each subsystem.

The conditions of Theorem 2.9 can be further extended to take into account longer sequences of switches. However, this comes with added computational cost and will eventually lead to considering all possible switching trajectories. To avoid this, but still reduce the conservativeness of the approach, consider the α-sample variation of the Lyapunov function. As proven by Kruszewski and Guerra [34], a system is asymptotically stable, if the associated Lyapunov function decreases every α samples, $\alpha \geq 1$, instead of every sample. Thus, for observer design, let us consider the error dynamics (2.6) and the Lyapunov function (2.12), defined on the edges of the associated graph, with $P_{v_i,v_j,z}$ being active during the transition from vertex i to vertex j. The difference in the Lyapunov function for α consecutive samples is

$$\begin{aligned} \Delta V &= e_{k+\alpha}^T P_{v_\alpha,v_{\alpha+1},z+\alpha} e_{k+\alpha} - e_k^T P_{v_0,v_1,z} e_k \\ &= \begin{pmatrix} e_k \\ e_{k+\alpha} \end{pmatrix}^T \begin{pmatrix} -P_{v_0,v_1,z} & 0 \\ 0 & P_{v_\alpha,v_{\alpha+1},z+\alpha} \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+\alpha} \end{pmatrix} \end{aligned}$$

where $[v_0, v_1, \dots, v_{\alpha+1}]$ is an admissible path.

Along the switching sequence $[v_0, v_1, \dots, v_{\alpha+1}]$, the error dynamics are described by

$$\begin{pmatrix} G_1 & -I & 0 & \dots & 0 \\ 0 & G_2 & -I & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -I \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \\ \vdots \\ e_{k+\alpha} \end{pmatrix} = 0$$

with $G_i = A_{v_i,z+i-1} - H^{-1}_{v_i,z+i-1} L_{v_i,z+i-1} C_{v_i,z+i-1}$.

Following the same steps as in the proof of Theorem 2.9, using Lemma 2.1, the difference in the Lyapunov function is negative, if there exists M such that

$$\begin{pmatrix} -P_{v_0,v_1,z} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & P_{v_\alpha,v_{\alpha+1},z+\alpha} \end{pmatrix} + M \begin{pmatrix} G_1 & -I & 0 & \dots & 0 \\ 0 & G_2 & -I & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -I \end{pmatrix} + (*) < 0$$

with G_i defined as above.

By choosing

$$M = \begin{pmatrix} 0 & 0 & \dots & 0 \\ H_{v_1,z} & 0 & \dots & 0 \\ 0 & H_{v_2,z+1} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & H_{v_\alpha,z+\alpha-1} \end{pmatrix}$$

we have

$$\begin{pmatrix} -P_{v_0,v_1,z} & (*) & \dots & (*) \\ \Omega_1 & -H_{v_1,z} + (*) & \dots & (*) \\ 0 & \Omega_2 & \dots & (*) \\ \vdots & \vdots & \dots & \begin{pmatrix} -H_{v_\alpha,z+\alpha-1} + (*) \\ +P_{v_\alpha,v_{\alpha+1},z+\alpha} \end{pmatrix} \end{pmatrix} < 0 \tag{2.17}$$

with $\Omega_i = H_{v_i,z+i-1} A_{v_i,z+i-1} - L_{v_i,z+i-1} C_{v_i,z+i-1}$, leading to the following theorem:

Theorem 2.13 *The error dynamics (2.6) is asymptotically stable, if there exist* $\alpha \in \mathbb{N}^+$, $P_{i,j,k} = P^T_{i,j,k} > 0$, $H_{j,k}$, $(v_i, v_j) \in \mathscr{E}$, $k = 1, 2, \dots, r$, *such that (2.17) holds for all admissible paths* $\mathscr{P}(v_0, v_{\alpha+1}) = [v_0, v_1, \dots, v_{\alpha+1}]$.

The conditions above are illustrated on the following example.

Example 2.14 Consider the switching system—actually a periodic switching system—illustrated in Fig. 2.7.

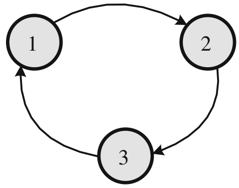

Fig. 2.7 Periodic switching system for Example 2.14

Assuming that each subsystem is active for only one sample, i.e., $p^m = p^M = 1$, the graph is $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, with $\mathscr{V} = \{1, 2, 3\}$ and $\mathscr{E} = \{(1, 2), (2, 3), (3, 1)\}$. Consider the following local models of the TS system above:

$$
\begin{aligned}
A_{1,1} &= \begin{pmatrix} 1.26 & 0.64 \\ 1.92 & 0.61 \end{pmatrix} & A_{1,2} &= \begin{pmatrix} 0.40 & 0.64 \\ 0.09 & 0.10 \end{pmatrix} \\
C_{1,1} &= \begin{pmatrix} 1 & 0 \end{pmatrix} & C_{1,2} &= \begin{pmatrix} 1 & 0 \end{pmatrix} \\
A_{2,1} &= \begin{pmatrix} 1.05 & 0 \\ 0.50 & 0.80 \end{pmatrix} & A_{2,2} &= A_{2,1} \\
C_{2,1} &= \begin{pmatrix} 0 & 0 \end{pmatrix} & C_{2,2} &= \begin{pmatrix} 0 & 0 \end{pmatrix} \\
A_{3,1} &= \begin{pmatrix} 0.40 & 0.45 \\ 0.30 & 0.84 \end{pmatrix} & A_{3,2} &= \begin{pmatrix} 1.67 & 0.43 \\ 1.42 & 0.33 \end{pmatrix} \\
C_{3,1} &= \begin{pmatrix} 0 & 1 \end{pmatrix} & C_{3,2} &= \begin{pmatrix} 0 & 0 \end{pmatrix}
\end{aligned}
$$

The second subsystem is linear, but it is unstable and unobservable. The second local model of the third subsystem is again unstable and unobservable. Due to this, methods available in the literature yield unfeasible LMIs. However, using Lemma 2.2 to formulate LMI conditions for Theorem 2.9, we obtain

$$
\begin{aligned}
H_{1,1} &= \begin{pmatrix} 0.66 & -0.06 \\ 0.03 & 0.56 \end{pmatrix} & H_{1,2} &= \begin{pmatrix} 0.70 & -0.08 \\ 0.06 & 0.55 \end{pmatrix} \\
H_{2,1} &= \begin{pmatrix} 0.46 & -0.09 \\ -0.03 & 0.34 \end{pmatrix} & H_{2,2} &= H_{2,1} \\
H_{3,1} &= \begin{pmatrix} 0.41 & -0.29 \\ -0.33 & 0.58 \end{pmatrix} & H_{3,2} &= \begin{pmatrix} 0.35 & -0.32 \\ -0.38 & 0.54 \end{pmatrix} \\
L_{1,1} &= \begin{pmatrix} 0.98 \\ 1.39 \end{pmatrix} & L_{1,2} &= \begin{pmatrix} 0.57 \\ 0.14 \end{pmatrix} \\
L_{3,1} &= \begin{pmatrix} -0.03 \\ 0.35 \end{pmatrix} & L_{3,2} &= \begin{pmatrix} 0.02 \\ 0.39 \end{pmatrix}
\end{aligned}
$$

Consequently, this observer is able to estimate the states of the switching system above. Since the second subsystem on its own is not observable, there are no observer gains $L_{2,1}$ and $L_{2,2}$. Trajectories of the states and of the error dynamics, together with the membership function h_1, are presented in Fig. 2.5. The true initial states were $x_0 = [-1;\ 1]^T$, while the estimated initial states were $\widehat{x}_0 = [0,\ 0]^T$.

Let us now discuss the conditions developed for the α-sample variation of the Lyapunov function. The previous conditions required that the Lyapunov function

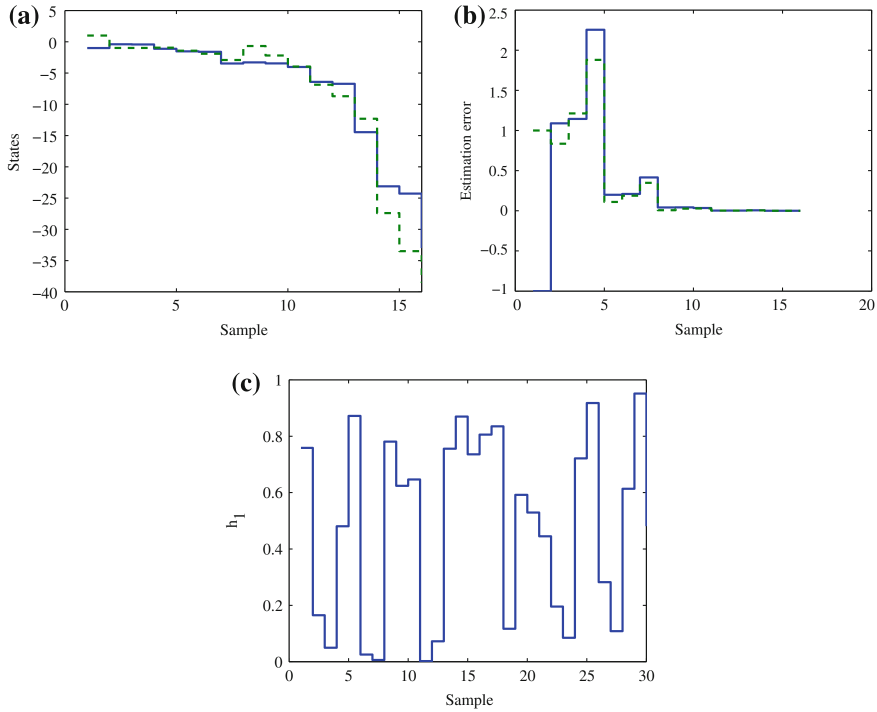

Fig. 2.8 Simulation results for Example 2.14. **a** A trajectory of the switching system. **b** The trajectory of the error. **c** Membership function

decreases with every switch/every sample, i.e., for all $(v_i, v_j) \in \mathscr{E}$. That is, we had the conditions:

$$
\begin{pmatrix} -P_{1,2,z} & (*) \\ H_{2,z}A_{2,z} - L_{2,z}C_{2,z} & -H_{2,z} + (*) + P_{2,3,z+1} \end{pmatrix} < 0
$$
$$
\begin{pmatrix} -P_{2,3,z} & (*) \\ H_{3,z}A_{3,z} - L_{3,z}C_{3,z} & -H_{3,z} + (*) + P_{3,1,z+1} \end{pmatrix} < 0
$$
$$
\begin{pmatrix} -P_{3,1,z} & (*) \\ H_{1,z}A_{1,z} - L_{1,z}C_{1,z} & -H_{1,z} + (*) + P_{1,2,z+1} \end{pmatrix} < 0
$$

A 2-sample variation means that the Lyapunov function has to decrease along paths of length 2, i.e., we have the conditions:

$$
\begin{pmatrix} -P_{1,2,z} & (*) & (*) \\ \begin{pmatrix} H_{2,z}A_{2,z} \\ -L_{2,z}C_{2,z} \end{pmatrix} & -H_{2,z} + (*) & (*) \\ 0 & \begin{pmatrix} H_{3,z+1}A_{3,z+1} \\ -L_{3,z+1}C_{3,z+1} \end{pmatrix} & \begin{pmatrix} -H_{3,z+1} + (*) \\ +P_{3,1,z+2} \end{pmatrix} \end{pmatrix} < 0
$$

$$\begin{pmatrix} -P_{2,3,z} & (*) & (*) \\ \begin{pmatrix} H_{3,z}A_{3,z} \\ -L_{3,z}C_{3,z} \end{pmatrix} & -H_{3,z} + (*) & (*) \\ 0 & \begin{pmatrix} H_{1,z+1}A_{1,z+1} \\ -L_{1,z+1}C_{1,z+1} \end{pmatrix} & \begin{pmatrix} -H_{1,z+1} + (*) \\ +P_{1,2,z+2} \end{pmatrix} \end{pmatrix} < 0$$

$$\begin{pmatrix} -P_{3,1,z} & (*) & (*) \\ \begin{pmatrix} H_{1,z}A_{1,z} \\ -L_{1,z}C_{1,z} \end{pmatrix} & -H_{1,z} + (*) & (*) \\ 0 & \begin{pmatrix} H_{2,z+1}A_{2,z+1} \\ -L_{2,z+1}C_{2,z+1} \end{pmatrix} & \begin{pmatrix} -H_{2,z+1} + (*) \\ +P_{2,3,z+2} \end{pmatrix} \end{pmatrix} < 0$$

Furthermore, 3-sample variation means that the Lyapunov function has to decrease along paths of length 3, which, in this case, is equivalent to the whole period of switching.

2.3.2 Periodic TS Systems

As shown above, in case of periodic systems, a lesser number of edges have to be considered and the conditions become circular. Therefore, in what follows, we consider periodic systems:

$$\begin{aligned} x_{k+1} &= A_{j,z}x_k + B_{j,z}u_k \\ y_k &= C_{j,z}x_k \end{aligned}$$

Since the system is periodic, it is assumed that the subsystems are activated in a sequence $\underbrace{1,\ 1,\ldots,\ 1}_{p_1},\ \underbrace{2,\ 2,\ldots,\ 2}_{p_2},\ldots,\ \underbrace{n_\mathrm{s},\ n_\mathrm{s},\ldots,\ n_\mathrm{s}}_{p_{n_\mathrm{s}}},\ \underbrace{1,\ 1,\ldots,\ 1}_{p_1}$, etc., where p_i denotes the number of samples for which the ith subsystem is active. In what follows, we will refer to p_i as the period of the ith subsystem.

Recall that using the same switching observer as above, i.e.,

$$\begin{aligned} \widehat{x}_{k+1} &= A_{j,z}\widehat{x}_k + B_{j,z}u_k + H_{j,z}^{-1}L_{j,z}(y_k - \widehat{y}_k) \\ \widehat{y}_k &= C_{j,z}\widehat{x}_k \end{aligned}$$

the error dynamics $e_k = x_k - \widehat{x}_k$ can be written as

$$e_{k+1} = (A_{j,z} - H_{j,z}^{-1}L_{j,z}C_{j,z})e_k \tag{2.18}$$

which, due to the periodicity of the switches, in this case will also be periodic.

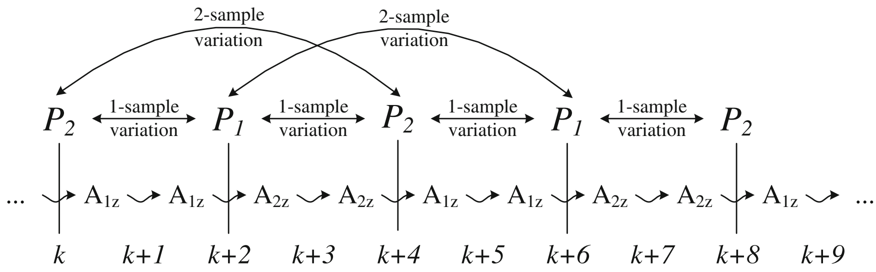

Fig. 2.9 Switches in the system and in the Lyapunov function

Although all the results presented in Sect. 2.3.1 apply for this case, to further reduce the conservativeness of the conditions, we will use a periodic Lyapunov function defined only when a switching takes place. As long as the periodicity of each subsystem is known and constant, the active subsystems' dynamic can easily be written for more than one sample.

To illustrate the periodicity of the system and the definition of the Lyapunov function, consider the following example.

Example 2.15 Consider a periodic TS model consisting of two subsystems, each with period 2, i.e., we have:

$$x_{k+1} = \begin{cases} \sum_{i=1}^{r} h_i(z_k) A_{1i} x_k \text{ if } k = 4m, 4m+1 \\ \sum_{i=1}^{r} h_i(z_k) A_{2i} x_k \text{ if } k = 4m+2, 4m+3 \end{cases} \tag{2.19}$$

The switching in the system and in the Lyapunov function are depicted in Fig. 2.9. As can be seen, the Lyapunov function (with matrices P_1 and P_2) is defined only in the moments when there is a switching in the system: from $A_{1,z}$ to $A_{2,z}$ or from $A_{2,z}$ to $A_{1,z}$, respectively. A 1-sample variation of the Lyapunov function corresponds to the difference between two consecutive values of the Lyapunov function. A 2-sample variation corresponds to the difference after 2 samples of the Lyapunov function, etc.

Therefore, to develop observer design conditions, consider the periodic Lyapunov function defined only in the instants when a switching takes place in the system:

$$V(e_k) = e_k^T P_{j,z} e_k$$

if the active subsystem was j, $j = 1, 2 \ldots, n_s$.

The difference in the Lyapunov function is

$$V(e_{k+p_{\underline{j+1}}}) - V(e_k) = \begin{pmatrix} e_k \\ e_{k+p_{\underline{j+1}}} \end{pmatrix}^T \begin{pmatrix} -P_{j,z} & 0 \\ 0 & P_{\underline{j+1},z+p_{\underline{j+1}}} \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+p_{\underline{j+1}}} \end{pmatrix}$$

where $\underline{j} = j \mod n_s$.

The error dynamics during the $p_{\underline{j+1}}$ samples are

$$\begin{pmatrix} \Upsilon_{j+1,1} & -I & \dots & 0 & 0 \\ 0 & \Upsilon_{j+1,2} & -I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \Upsilon_{j+1,p_{\underline{j+1}}-1} & -I \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \\ \vdots \\ e_{k+p_{\underline{j+1}}} \end{pmatrix} = 0$$

with

$$\Upsilon_{j+1,i} = A_{\underline{j+1},z+i} - H^{-1}_{\underline{j+1},z+i} L_{\underline{j+1},z+i} C_{\underline{j+1},z+i}$$

for $i = 1,\ 2 \dots,\ p_{\underline{j+1}} - 1$.

Choosing $M = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ H_{\underline{j+1},z+1} & 0 & \dots & 0 & 0 \\ 0 & H_{\underline{j+1},z+2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & H_{\underline{j+1},z+p_{\underline{j+1}}-1} & 0 \\ 0 & 0 & \dots & 0 & H_{\underline{j+1},z+p_{\underline{j+1}}} \end{pmatrix}$ and applying Lemma 2.1 leads to

$$\begin{pmatrix} -P_{j,z} & (*) & \dots & (*) & (*) \\ \Omega_{j+1,1} & -H_{\underline{j+1},z+1} - (*) & \dots & (*) & (*) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \Omega_{j+1,p_{\underline{j+1}}} & -H_{\underline{j+1},z+p_{\underline{j+1}}} - (*) + P_{j+1,z+p_{\underline{j+1}}} \end{pmatrix} < 0$$

with $\Omega_{j+1,i} = H_{\underline{j+1},z+i} A_{\underline{j+1},z+i-1} - L_{\underline{j+1},z+i-1} C_{\underline{j+1},z+i-1}$ for $i = 1,\ 2,\ \dots,\ p_{\underline{j+1}} - 1$.

Then, the following result can be formulated:

Theorem 2.16 *The error dynamics* (2.18) *is locally asymptotically stable if there exist* $P_{j,i}$, $H_{j,i}$, $j = 1, 2 \dots n_s$, $i = 1, 2, \dots r$ *such that:*

$$\begin{pmatrix} -P_{j,z} & (*) & \dots & (*) & (*) \\ \Omega_{j+1,1} & -H_{\underline{j+1},z+1} - (*) & \dots & (*) & (*) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \Omega_{j+1,p_{\underline{j+1}}} & -H_{\underline{j+1},z+p_{\underline{j+1}}} - (*) + P_{j+1,z+p_{\underline{j+1}}} \end{pmatrix} < 0$$

The result above can easily be extended using α-sample variation [35]. Recall that the Lypunov function is only defined in the switching instants, and the α-difference in the Lyapunov function corresponds to α consecutive switches in the system.

Since the Lyapunov function is only defined in the switching instants, the α-difference *in the Lyapunov function* is

$$V(x_{k+t}) - V(x_k) = \begin{pmatrix} e_k \\ e_{k+t} \end{pmatrix}^T \begin{pmatrix} -P_{j,z} & 0 \\ 0 & P_{\underline{j+\alpha},z+t} \end{pmatrix} \begin{pmatrix} e_k \\ x_{k+t} \end{pmatrix}$$

where $t = \sum_{i=1}^{\alpha} p_{\underline{j+1}}$.

The error dynamics during the t samples corresponding to the α switches in the system are

$$\Gamma_{j+1:j+\alpha} \begin{pmatrix} e_k \\ e_{k+1} \\ \vdots \\ e_{k+t} \end{pmatrix} = 0$$

with

$$\Gamma_{j+1:j+\alpha} = \begin{pmatrix} G_{1,0} & -I & \dots & 0 & 0 \\ 0 & G_{1,1} \dots & 0 & 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & G_{\alpha,t-1} & -I \end{pmatrix}$$

where $G_{i,j} = A_{\underline{j+i},z+j} - H^{-1}_{\underline{j+i},z+j} L_{\underline{j+i},z+j} C_{\underline{j+i},z+j}$, $i = 1, 2, \dots, \alpha$, $j = 1, 2 \dots, t-1$.

Using Lemma 2.1, the difference in the Lyapunov function is negative definite, if there exists M such that

$$\begin{pmatrix} -P_{j,z} & 0 \dots & 0 \\ 0 & 0 \dots & 0 \\ \vdots & \vdots \ \vdots & \vdots \\ 0 & 0 \dots & P_{\underline{j+1},z+t} \end{pmatrix} + M\Gamma_{j+1:j+\alpha} + (*) < 0$$

Choosing $M = \begin{pmatrix} 0 & 0 & \dots & 0 \\ H_{\underline{j+1},z} & 0 & \dots & 0 \\ 0 & H_{\underline{j+1},z+1} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & H_{\underline{j+\alpha},z+t-1} \end{pmatrix}$ leads directly to

$$\begin{pmatrix} -P_{j,z} & (*) & \dots & (*) & (*) \\ \Omega_{\underline{j+1},z} & -H_{\underline{j+1},z} + (*) & \dots & (*) & (*) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \Omega_{\underline{j+\alpha},z+t-1} & \Omega_{j+\alpha,j+\alpha} \end{pmatrix} < 0 \tag{2.20}$$

where $t = \sum_{i=1}^{\alpha} p_{\underline{j+1}}$, and $\Omega_{j+i,j+k} = H_{\underline{j+i},z+k} A_{\underline{j+i},z+k} - L_{\underline{j+i},z+k} C_{\underline{j+i},z+k}$ and $\Omega_{j+\alpha,j+\alpha} = -H^T_{\underline{j+\alpha},z+t-1} - H^T_{\underline{j+\alpha},z+t-1} + P_{\underline{j+\alpha},z+t-1}$.

Theorem 2.17 *The periodic error dynamics* (2.18) *with periods* $p_1, p_2, \dots, p_{n_s}$ *is asymptotically stable, if there exist* $P_{ji} = P_{ji}^T > 0$, H_{ji}, $j = 1, 2, \dots, n_s$, $i = 1, 2, \dots, r_j$, $l = 1, 2, \dots, \alpha$, *such that* (2.20) *is satisfied.*

Let us illustrate the results for periodic TS systems on the following example.

Example 2.18 Consider the periodic fuzzy system with two subsystems as follows:

$$x_{k+1} = \sum_{i=1}^{2} h_i(z_k) A_{1i} x_k$$
$$y_k = \sum_{i=1}^{2} h_i(z_k) C_{1i} x_k$$

with

$$A_{11} = \begin{pmatrix} -0.44 & -0.26 \\ -0.65 & 0.62 \end{pmatrix} \quad A_{12} = \begin{pmatrix} 1.1 & -0.2 \\ 0.53 & -0.27 \end{pmatrix}$$
$$C_{11} = \begin{pmatrix} 0 & 0 \end{pmatrix} \quad C_{12} = C_{11}$$

$$x_{k+1} = \sum_{i=1}^{2} h_i(z_k) A_{2i} x_k$$
$$y_k = \sum_{i=1}^{2} h_i(z_k) C_{2i} x_k$$

with

$$A_{21} = \begin{pmatrix} 0.02 & 0.6 \\ -0.22 & -0.44 \end{pmatrix} \quad A_{22} = \begin{pmatrix} 0.32 & -0.15 \\ -1 & 0.8 \end{pmatrix}$$
$$C_{21} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad C_{22} = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

The local models A_{12} and A_{22} are unstable, and unobservable. The periods of the subsystems are $p_1 = 1$ for the first subsystem and $p_2 = 2$ for the second subsystem.

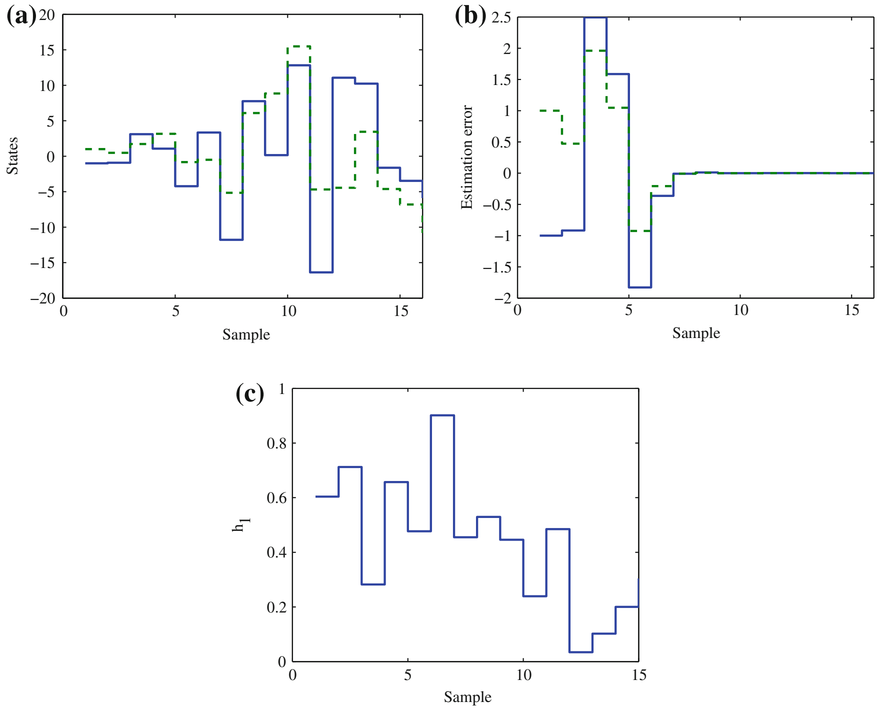

Fig. 2.10 Simulation results for Example 2.18. **a** A trajectory of the switching system. **b** The trajectory of the error. **c** Membership function

Using the conditions of Theorem 2.16 we obtain the observer gains

$$H_{1,1} = \begin{pmatrix} 0.14 & -0.28 \\ -0.23 & 0.54 \end{pmatrix} \qquad H_{1,2} = \begin{pmatrix} 0.15 & -0.26 \\ -0.29 & 0.55 \end{pmatrix}$$
$$H_{2,1} = \begin{pmatrix} 0.28 & -0.24 \\ -0.36 & 0.42 \end{pmatrix} \qquad H_{2,2} = \begin{pmatrix} 0.19 & -0.23 \\ -0.28 & 0.40 \end{pmatrix}$$
$$L_{2,1} = \begin{pmatrix} -0.50 \\ 0.67 \end{pmatrix} \qquad L_{2,2} = \begin{pmatrix} -0.19 \\ 0.40 \end{pmatrix}$$

Consequently, this observer is able to estimate the states of the switching system above. Since the first subsystem on its own is not observable, there are no observer gains $L_{1,1}$ and $L_{1,2}$. Trajectories of the states and of the error dynamics, together with the membership function h_1, are presented in Fig. 2.10. The true initial states were $x_0 = [-1;\ 1]^T$, while the estimated initial states were $\widehat{x_0} = [0,\ 0]^T$.

For this periodic system, it is not possible to find either a quadratic or a non-quadratic Lyapunov function, common for both subsystems, as the corresponding LMIs are unfeasible.

2.3.3 Controlled Switches

Finally, let us consider the case when an observer has to be designed while the switches may be chosen. Therefore, our goal is to "stabilize" the error dynamics by switching among the subsystems. The switching observer considered,

$$\begin{aligned} \widehat{x}_{k+1} &= A_{j,z}\widehat{x}_k + B_{j,z}u_k + H_{j,z}^{-1}L_{j,z}(y_k - \widehat{y}_k) \\ \widehat{y}_k &= C_{j,z}\widehat{x}_k \end{aligned} \tag{2.21}$$

is the same as before, but next to the matrices $H_{j,i}^{-1}$, $L_{j,i}$, $j = 1, 2\ldots, n_s$, $i = 1, 2, \ldots, r$, the switching sequence also has to be determined.

The error dynamics is again

$$e_{k+1} = A_{j,z}e_k - H_{j,z}^{-1}L_{j,z}(y_k - \widehat{y}_k) = (A_{j,z} - H_{j,z}^{-1}L_{j,z}C_{j,z})e_k \tag{2.22}$$

Since the switching sequence can be chosen, if there exists a subsystem with an asymptotically stable error dynamics that can be active for an infinite number of samples, the problem can be reformulated as finding a path—a classical graph theoretical problem—from each subsystem to this stable subsystem. If such a path exists, then the switching law is given by this path, and the problem is solved. Therefore, we consider the case when none of the subsystems may be infinitely active. Next to this, for the easier development of the conditions, we assume that the associated graph is strongly connected, i.e., there exists a path between any two vertices.

Recall that we consider the associated directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where the vertices $\mathcal{V} = \{v_1, v_2, \ldots, v_{n_s}\}$ correspond to the subsystems, and each edge $e_{i,j} = (v_i, v_j) \in \mathcal{E}$ corresponds to an admissible transition.

In what follows, we build a weight-adjacency matrix, that assigns to each admissible transition, including self-transitions, a weight. By convention, if $(v_i, v_j) \notin \mathcal{E}$, for $i \neq j, i, j = 1, 2, \ldots, n_s$ the corresponding weight $w_{i,j} = \infty$, and if $(v_i, v_i) \notin \mathcal{E}$, $i = 1, 2, \ldots, n_s$, then the corresponding weight $w_{i,i} = 1$. For all other edges, the corresponding weight will be given by an upper bound on the increase of a Lyapunov function associated to the error dynamics.

Consider the Lyapunov function

$$V(e_k) = e_k^T P_{i,z} e_k$$

with $P_{i,z} = P_{i,z}^T > 0$, for the ith subsystem, $i = 1, 2, \ldots, n_s$.

Before constructing the weighting matrix, let us find constants $\delta_{i,j} > 0$ so that $V(e_{k+1}) \leq \delta_{i,j} V(e_k)$, if the transition is (v_i, v_j), i.e., upper bounds on the increase of the Lyapunov function in one sample.

For any $(v_i, v_j) \in \mathcal{E}$ we have

$$V(e_{k+1}) - \delta_{i,j} V(e_k) < 0$$

$$e_{k+1}^T P_{j,z+1} e_{k+1} - \delta_{i,j} e_k^T P_{i,z} e_k < 0$$

$$\begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix}^T \begin{pmatrix} -\delta_{i,j} P_{i,z} & 0 \\ 0 & P_{j,z+1} \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} < 0$$

At the same time, the system's dynamics can be written as

$$\begin{pmatrix} A_{i,z} - H_{i,z}^{-1} L_{i,z} C_{i,z} & -I \end{pmatrix} \begin{pmatrix} e_k \\ e_{k+1} \end{pmatrix} = 0$$

Using Lemma 2.1, we have $V(e_{k+1}) - \delta_{i,j} V(e_k) < 0$ if there exists $\overline{M}_{i,j,z}$ so that

$$\begin{pmatrix} -\delta_{i,j} P_{i,z} & 0 \\ 0 & P_{j,z+1} \end{pmatrix} + \overline{M}_{i,j,z} \begin{pmatrix} A_{i,z} - H_{i,z}^{-1} L_{i,z} C_{i,z} & -I \end{pmatrix} + (*) < 0$$

Choosing $\overline{M}_{i,j,z} = \begin{pmatrix} 0 \\ H_{i,j,z} \end{pmatrix}$ we obtain the sufficient conditions

$$\begin{pmatrix} -\delta_{i,j} P_{i,z} & (*) \\ H_{i,j,z} A_{i,z} - L_{i,z} C_{i,z} & -H_{i,j,z} - H_{i,j,z}^T + P_{j,z+1} \end{pmatrix} < 0 \tag{2.23}$$

To find all $\delta_{i,j}$, one has to solve (2.23) for all $(v_i, v_j) \in \mathscr{E}$.

Now, define the weight matrix as $\mathscr{W} = [w_{i,j}]$ with

$$w_{i,j} = \begin{cases} \delta_{i,i}^{p_i^m - 1} & \text{if } \delta_{i,i} > 1 \\ \delta_{i,i}^{p_i^M - 1} & \text{if } \delta_{i,i} < 1 \\ \delta_{i,j} & \text{if } i \neq j \end{cases}$$

Assume that in the weight matrix constructed above, there exists a subunitary cycle, i.e., there exists $\mathscr{C}_n = \{v_{c1}, v_{c2}, \ldots, v_{cp}, v_{c1}\}$ such that the product of the edges and nodes in this cycle is subunitary, and let this product be denoted by δ_n. For any subsystem i such that $v_i \in \mathscr{C}_n$, we have $V(x_{k+n_c}) < \delta_n V(x_k)$, i.e., after a full cycle, the corresponding Lyapunov function increases at most δ_n times, with $\delta_n < 1$. Consequently, the periodic switching law $\mathscr{C} = [c_1, c_2, \ldots, c_p, c_1]$ stabilizes the periodic error dynamics given by this cycle.

Let us now see the subsystems (if any) that are not in $\mathscr{C}_n$. Since we assumed that $\mathscr{G}$ is strongly connected, for all $v_i \notin \mathscr{C}$, there exists a path $\mathscr{P}(v_i, v_j)$ from the ith subsystem to a subsystem j on the cycle. Consider the switching law $\mathscr{P}(v_i, v_j)\mathscr{C}_n$, i.e., first a switching law that leads to the cycle and then the periodic switching law corresponding to the cycle. Although during the switches corresponding to $\mathscr{P}(v_i, v_j)$ the Lyapunov function might increase, during the periodic switching, it will eventually decrease and converge to zero.

Based on the explanation above, the following result can be stated:

Theorem 2.19 *The error dynamics* (2.22) *is asymptotically stable along a switching law, if its associated graph* $\mathscr{G} = \{\mathscr{V}, \mathscr{E}, \mathscr{W}\}$ *contains a subunitary cycle* $\mathscr{C}_n$*. Furthermore, for the* i*th initial subsystem,* $i = 1, 2, \ldots, n_s$*, the switching law that stabilizes the error dynamics is given by* $\mathscr{P}(v_i, v_j)\mathscr{C}_n$*, where* v_i *denotes the vertex corresponding to the initial subsystems, and* $\mathscr{P}(v_i, v_j)$ *is a path to vertex* v_j*, with* $v_j \in \mathscr{C}_n$*.*

It should be noted that we assumed that the associated graph is strongly connected, because in this case the existence of a periodic switching law—which is not necessarily stabilizing—is guaranteed. If the graph is not strongly connected, the possibility of developing an observer depends on the starting subsystem. Therefore, in order to design an observer, each strongly connected subgraph has to be analyzed.

2.4 Summary

In this chapter, we considered observer design for switching nonlinear systems represented by TS fuzzy models. We have developed conditions that, when satisfied, guarantee that the estimation error converges to zero. The conditions were derived by taking into account the number of samples a subsystem may be active and the possible switches in the system. We have considered three cases.

The first case is general switching systems, where the switching sequence is not known in advance and it cannot be directly influenced. Due to this assumption, the conditions require that the estimation error dynamics decreases along the trajectory of the subsystems and the switches. This is the worst-case assumption, i.e., all possible combinations of switches between the estimation error subsystems have to be taken into account. The second case, for which the conditions can be relaxed, is when the switching is periodic. Although the conditions developed for the general case apply, due to the reduced number of switches, a smaller number of conditions are necessary. Finally, we considered the case that next to designing the observer gains, the switching sequence can be chosen.

The developed conditions have been extended to the α-sample variation of the Lyapunov function, in order to reduce their conservativeness. Other possibilities to relax the conditions represents the usage of double sums in the Lyapunov function, or using delayed Lyapunov functions or observers.

A shortcoming of the conditions is the computational complexity required to generate the possible combinations of switches and subsystems. In particular for large-scale switching systems, when considering α-sample variation, the number of conditions may be exponential in the number of subsystems, and in consequence, a large number of LMIs that has to be solved.

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