Chapter 2
Overview of Numerical Simulations on Accretion Processes and Our Objectives

Abstract We give an overview of the past work done on numerical simulation for accretion flows around black holes. We start with non-viscous cases. We also point out some important simulations of viscous accretion discs. In the last section of this chapter, we present the precise goals for this thesis work.

2.1 Introduction

We turn now to present an overview of various simulations in astrophysics using numerical methods to solve the basic conservation equations. Numerical simulations of fluid flows in the vicinity of strongly gravitating compact objects require a three-dimensional description because of their complex nature and lack of symmetry. To provide a physical basis for the interpretative framework that the source is accretion driven, a detailed description of the relevant hydrodynamical (and radiative) process is required. One of the most convincing ways to study whether a solution is stable or not is to perform time-dependent numerical simulations.

Prendergast (1960) was the first to carry out numerical simulation of gaseous flows, while ignoring the pressure. His model expressed two constituent stars as two mass points and ignored either release or accretion of the gas (see also, Huang 1965, 1966). These drawbacks were corrected by Prendergast and Taam (1974), who used the beam scheme and with mass-accreting star of large size, were unable to find the formation of any accretion disc. Biermann (1971) conducted simulations by the characteristic line method for models close to wind accretion rather than accretion discs. Sorensen et al. (1974, 1975) made calculations using fluid in cell method (FLIC) and Cartesian coordinates. They took the mass-losing star and the mass-accreting star into consideration and assumed the latter to be of sufficiently small size to allow formation of an accretion disc. The results of their calculation showed a gas stream from the L1 point flowing towards the compact star and formation of an accretion disc. However, Wilson (1972) first investigated numerically time-dependent accretion of inviscid matter onto a rotating (Kerr) black hole. This was the first problem to which his formulation of the hydrodynamic equations was applied.

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Nowadays, there is a large body of numerical investigations in the literature dealing with hydrodynamical integrations in static background space-times. Most of these are based on the Wilson formulation of hydrodynamic equations and use schemes based on finite differences with some amount of artificial viscosity. More recently, researchers have started to use conservative formulations of the equations, and their characteristic information, in the design of numerical schemes.

Satisfactory numerical solutions of hydrodynamical equations were obtained for spherical accretion Kylafis and Lamb (1979) and for steady-state flow onto magnetic white dwarfs (Wada et al. 1980). In 1981, Langer et al. presented the first time-dependent numerical solutions to hydrodynamical equations for accretion onto a white dwarf. They demonstrated that the shock height underwent a periodic limit cycle due to thermal instability in the bremsstrahlung dominated cooling. In their next paper (1981), they described in detail their method of solution and considered a wide range of accretion rates and white masses and radii of the white dwarfs. The effect of cyclotron emission on the same work was included by Chanmugam et al. (1985). Sawada et al. (1986a, b), Spruit et al. (1987) made a second attempt on the same problem that Sorenson et al. (1976) had worked on earlier. They worked using state-of-the-art techniques such as the Osher upwind finite difference method with second-order accuracy, generalised curvilinear coordinates and a supercomputer of vector type. The Osher upwind difference method can run the calculation stably while suppressing the artificial viscosity at a low level and is a predecessor of the Total Variation Diminishing (TVD) method, which is a representative modern computational fluid dynamics scheme. As a result, they discovered in accretion disc the presence of spiral shocks, the very feature that was never discovered using other schemes with more dissipations. Since then, authors have been carrying out two-dimensional simulation for accretion discs by various methods and they all obtained spiral shocks (Spruit et al. 1987; Rozyczka and Spruit 1989; Matsuda et al. 1990; Savonije et al. 1994; Godon 1997).

In the following, we present a summary of illustrative time-dependent accretion simulations in hydrodynamics. We concentrate on the progress of multidimensional simulations around black holes.

2.2 Simulation of Inviscid Flows

There are a variety of astrophysical situations in which one expects to find fluid accreting onto a black hole. Among these are the stellar collapse to a black hole, a black hole in a binary system and a supermassive black hole in active galactic nuclei. Pioneering numerical efforts in the study of black hole accretion made use of the so-called frozen star paradigm of a black hole. The first numerical attempt to study the behaviour of matter around black holes was made nearly three decades ago (Hawley and Smarr 1983). They developed a 2D axisymmetric, general relativistic, Eulerian, first-order backward space difference technique inviscid hydrodynamic accretion flow in a fixed Kerr black hole gravitational field. It was shown that the large angular momentum accretion is accompanied by shock waves that travel outwards. These simulations also confirm the results of Wilson (1978) that non-steady shock waves are formed which travel outward. A series of important simulations were carried out with this code to show that thick accretion discs can indeed form in inviscid flows (Hawley et al. 1984a, b). In these simulations, a shock wave was found but it was not steady at all as it travelled outwards. Furthermore, theoretical models were not mature enough (no radial velocity included, for example). Fryxell et al. (1987) devoted their attention to the hydrodynamical aspects of the problem in adiabatic approximation. They presented the results of a number of numerical simulations to describe the dependence of hydrodynamical flows on the boundary conditions at the surface of the gravitating object. In the above simulations, no standing shock waves were found as the parameter space (spanned by energy and angular momentum) for which standing shock may form was not used at the outer boundary. Livio et al. (1991) showed that the shock cone exhibited a side-to-side motion, which was termed the 'flip-flop' instability. The flip-flop motion was accompanied by the episodic accretion of material with high specific angular momentum of opposite signs. With a three-dimensional extension of the axisymmetric code of Hawley et al. (1984a, b), Hawley (1991) studied the global hydrodynamic non-axisymmetric instabilities in thick, constant angular momentum accretion gas tori, orbiting around a Schwarzschild black hole. Matsuda et al. (1992) were the first to show that 2D adiabatic flows are unstable to 'flip-flop' instability even in the case of accretion from a homogeneous medium so that the density or velocity gradients merely provide an initial perturbation. Ishii et al. (1993) presented the results of two- and three-dimensional numerical hydrodynamical calculations of accretion flows of an isothermal gas past a gravitating compact object. They found that 2D isothermal flows exhibit 'flip-flop' instability both in homogeneous and non-homogeneous cases. In contrast to the finite difference method, Lin and Pringle (1976) and Hensler (1982) performed calculations with a particle method, the former in particular using the sticky particle method, which can be called a predecessor of the Smoothed Particle Hydrodynamics (SPH) method. All these calculations, having incorporated an artificial viscosity to stabilise the calculation, could not reveal the detailed structure of the inside of an accretion disc. The SPH discretization approach satisfies that the available finite number of computational nodes or grid points follow the fluid (Lagrangian description). So, it (SPH) had become a useful computational tool for complex two- and three-dimensional problems (Benz 1990), and it owes its popularity mostly to its computational simplicity. Monaghan (1988) derived SPH equations for relativistic fluids moving in a static metric. Kheyfets and Zurek (1990) devloped a formulation of SPH compatible with the principles of general relativity in which the contact interactions are modelled by spatial smoothing functions constructed explicitly in the local frame co-moving with fluid. Laguna et al. (1993) took a logical step in the progression and applied the SPH techniques in developing a computational tool to study the three-dimensional dynamics of a relativistic ideal fluid in a static curved space-time geometry. Steinmetz and Mueller (1993) compared one-dimensional plane-parallel hydrodynamic shock solutions and spherical cloud collapse and suggested some improvements in the SPH technique to minimise dissipation. Davis et al. (1993) compared the collisions of a

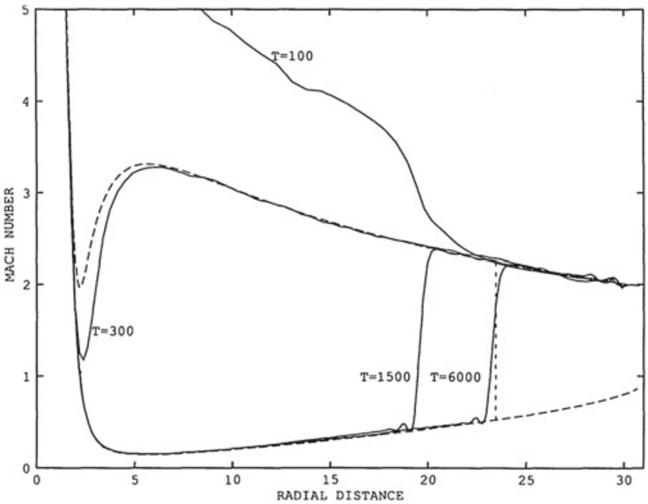

Fig. 2.1 An example of the simulation of a thin accretion disc which includes a standing shock wave. Mach number of the flow is plotted against the radial distance. Results at different times are shown as *solid curves*. Analytical solution is shown as a *dashed curve* with vertical shock transition. The flow forms the shock at the predicted location. This figure has been taken from Chakrabarti and Molteni (1993)

main-sequence star with a white dwarf and showed satisfactory performance of SPH as well as finite difference code. Laguna et al. (1994) compared a general relativistic SPH code for one-dimensional plane-parallel shock solutions and spherical inflows with and without pressure. They found good agreement with the analytical solution. Several numerical works were done by various authors using SPH code written in axisymmetric coordinate system using Pseudo-Newtonian potential. Chakrabarti and Molteni (1993) presented the results of numerical simulations of thin accretion discs and winds. They showed that their simulation agrees very well with the theoretical work (Chakrabarti 1990a) on shock formation. The most significant conclusion was that shocks in an inviscid flow were extremely stable. Figure 2.1 shows an example of the simulation of a thin accretion disc which includes standing shock waves (Chakrabarti and Molteni 1993). Mach number of the flow is plotted against the radial distance (in units of Schwarzschild radius of the central black hole). Solid curves are simulation results and dashed curves are the supersonic and subsonic branches, respectively. Two vertical dashed lines indicate locations of analytically predicted shock transitions, the outer one being stable. After a transient phase, a shock forms near the inner edge which then travels outward till it reaches the outer stable shock. Molteni et al. (1994) simulated the formation of a thick disc. For a large number of cases they also found the formation of strong winds which are hot and subsonic when originating from the disc surface very close to the black hole, but become supersonic within a few tens of Schwarzschild radii of the black hole. They also showed that in the case of higher angular momentum, the black hole accretes very less amount of

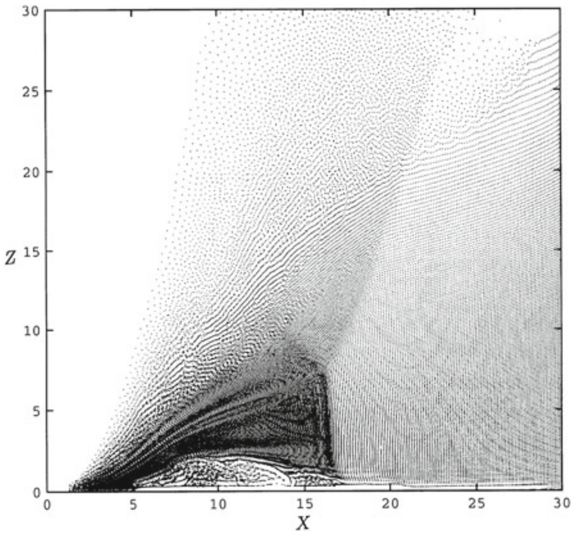

Fig. 2.2 Pseudo-particles in a smoothed particle hydrodynamic simulation of a two-dimensional sub-Keplerian angular momentum flow. The figure has been taken from Molteni et al. (1994)

matter and most of the matter is driven outwards as a strong wind. Figure 2.2 shows the particle distribution in which the standing shock at X 16 is clearly visible. The presence of oblique shock is also visible.

Sponholz and Molteni (1994) studied the shock formation around a Kerr black hole and found differing shock locations in co-rotating and contra-rotating flows. Molteni et al. (1996) extended their earlier numerical simulation (MLC96) of accretion discs with shock waves when cooling effect are also included. They considered bremsstrahlung and other power-law processes to mimic cooling in the simulation. They observed that for a given angular momentum of the flow, the shock wave undergoes a steady, radial oscillation with the period roughly equal to the cooling time. As a result of oscillations, the energy output from the disc also varies quasi-periodically.

In 1982, Ami Harten published a groundbreaking paper that became the basis of Computational Fluid Dynamics (CFD) research for many years to come. Under the title ‘High Resolution Schemes for Hyperbolic Conservation Laws’, Harten introduced the term total variation non-increasing (TVNI), which was later shortened by other researchers to TVD. The details of the TVD scheme are discussed in the next chapter. A series of numerical simulations of accretion flows around compact objects have been done using TVD scheme. Ryu et al. (1993) described an explicit second-order finite difference code based on a TVD scheme for self-gravitating cosmological hydrodynamics systems. This code was developed to follow correctly the adiabatic changes of extremely supersonic pre-shock flows with Mach number larger than 100 as well as very strong shocks. Various numerical experiments proved that the code could handle the expanding low density regions very well as well as conserve the total energy accurately. The details of the original TVD scheme are described in detail in that paper. In continuation of the previous work, Ryu et al. (1995) analysed the steady-state flow structure around a central object obtained from numerical sim-

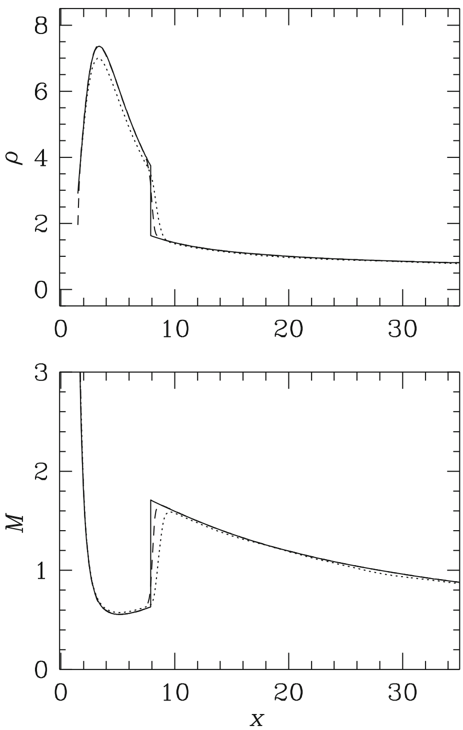

Fig. 2.3 Comparison of analytical and numerical results for 1D. Here, $\mathcal{E} = 0.036$ and $\lambda = 1.80$. The *long*- and *short-dashed curves* are the results of the TVD and SPH simulations, respectively. The *solid curve* is the analytical result for the same parameters. *Upper panel* is the mass density in arbitrary units, and the *lower panel* is the Mach number of the flow. Here the flow passes through the outer sonic point (at $x_o = 27.9$), then through a shock (at 7.89) and finally through the inner sonic point (at 2.563). This figure has been taken from Molteni et al. (1996)

ulations in the case with finite flow thickness. The calculation discussed in that paper showed good qualitative agreement with similar calculations done previously in Molteni et al. (1994). They suggested a possible explanation for the unstable behaviour of the thin accretion flow with vanishing thickness based on 1.5D model.

A comparative study between the results of TVD method and SPH method was made by Molteni et al. (1996). They compared the results of numerical simulations of thin and quasi-spherical accretion with existing analytical solutions. They showed that in one-dimensional thin flows, the result of both simulations (with or without shock) agrees well with each other and also with analytical solutions. Comparisons of analytical and numerical solutions in one-dimensional case are given in Fig. 2.3. See the caption for details. However, for two-dimensional thick flows, there was some variation between the two results. With the confidence that their codes were reasonably good to study time-dependent flows, they presented more complex behaviour of time-dependent accretion flows in their next paper (Ryu et al. 1997). In that paper Ryu et al. (1997), they characterised the nature of thin, axisymmetric, inviscid accretion flows of cold adiabatic gas with zero-specific energy in the vicinity of a black hole by the specific angular momentum. They showed that when the flow has small angular momentum ($\lambda \leq \lambda_{\rm mb}$, where $\lambda_{\rm mb}$ is marginally bound value), most of the material is accreted into the black hole and when the flow has large angular momentum ($\lambda > \lambda_{\rm mb}$) almost no accretion into the black hole occurs. Igumenshchev and Beloborodov (1997) performed two-dimensional relativistic hydrodynamical simulations of inviscid transonic disc accretion onto a rotating (Kerr) black hole. In the next section, we discuss only the recent progress on simulations of viscous flows.

2.3 Simulation of Viscous Flows

The simulation of viscous accretion discs around black holes in astrophysics is currently an active field of research. The question of the two-dimensional structure of flow patterns in accretion discs was first raised nearly three decades ago. On the theoretical side, since the pioneering work on SS73 thin disc models, parameterized by the so-called α-viscosity in which the gas rotates with Keplerian angular momentum and transported radially by viscous stress, have been applied successfully to many models. All black hole accretion flow models require that angular momentum be removed from the flow in some way so that the material can flow inward. It has long been suspected that diffusion of angular momentum through an accretion flow is driven by turbulence. The α-model (Shakura and Sunyaev 1973) introduced a phenomenological shear stress into the equations of motion to model the effects of this turbulence. Pringle (1981) pointed out that in presence of viscosity, most of the matter of the disc accretes into the black hole while most of the angular momentum is taken farther away by very little matter. Using α viscosity Urpin (1984) obtained the flow structure by applying first- and second-order corrections to the standard one-dimensional approximations of the equations of hydrodynamics.

Full two-dimensional simulations reported by Robertson and Frank (1986) followed the viscous evolution of an accretion disc around a white dwarf. The first two-dimensional hydrodynamical calculations with radiation transport for an accretion disc around black holes was apparently those of Eggum et al. (1987). Meanwhile, work had begun with turbulent viscosity by convection in accretion discs in various approaches. Papaloizou and Lin (1988) and Ryu and Goodman (1992) studied convective instabilities in thin gaseous discs and confirmed that angular momentum transport can be supported by convective turbulence. Goldman and Wandel (1995) investigated accretion discs where viscosity is solely given by convection and where the energy transport is maintained by radiation and convection. They found the resulting viscosity too low by a factor of 10 to 100.

It has been shown by Chakrabarti (1989) that low angular momentum, non-dissipative flows produce axisymmetric standing shock waves in tens of Schwarzschild radii, but the presence of a large viscosity (Chakrabarti 1990b) will remove the shock wave since the Rankine-Hugoniot relation is not satisfied in highly dissipative flow. The general conclusion was that the stable shock (X_{s3} in the notation of Chakrabarti 1990b) is weaker and forms farther away as the viscosity parameter is increased. When the viscosity is very high, shocks do not form at all. A confirmation of such an assertion, originally made in the context of isothermal flows, came through both numerical simulations (Chakrabarti and Molteni 1995) and theoretical studies of flows with more general equation of states (Chakrabarti 1996; Chakrabarti and Das 2004; Das and Chakrabarti 2004).

Chakrabarti and Molteni (1995) studied on numerical evolution of viscous isothermal discs, with particular emphasis on the nature of shocks in flows close to a black hole. They showed that, if the transport of angular momentum follows Shakura and Sunyaev (1973) α-viscosity prescription, a shock must form where a jump of angular momentum takes place. When the viscosity is very small, they show that the transport rates of angular momentum on both sides of the shock could match and the shock can remain steady. In this case, the shock is weaker and forms farther away from the black hole. When the viscosity parameter is increased, the shock wave is driven outwards and the disc in the post-shock flow becomes Keplerian. They also find that for high viscosity, the shock disappears and the disc becomes almost Keplerian except very close to the inner edge of the disc. In Fig. 2.4a, b, the Mach number and angular momentum variations in viscous (solid) and inviscid (dashed) flows are shown. Igumenshchev et al. (1996) studied two-dimensional flows but concentrated only on the inner region of the disc, namely the region less than $20r_g$. The main

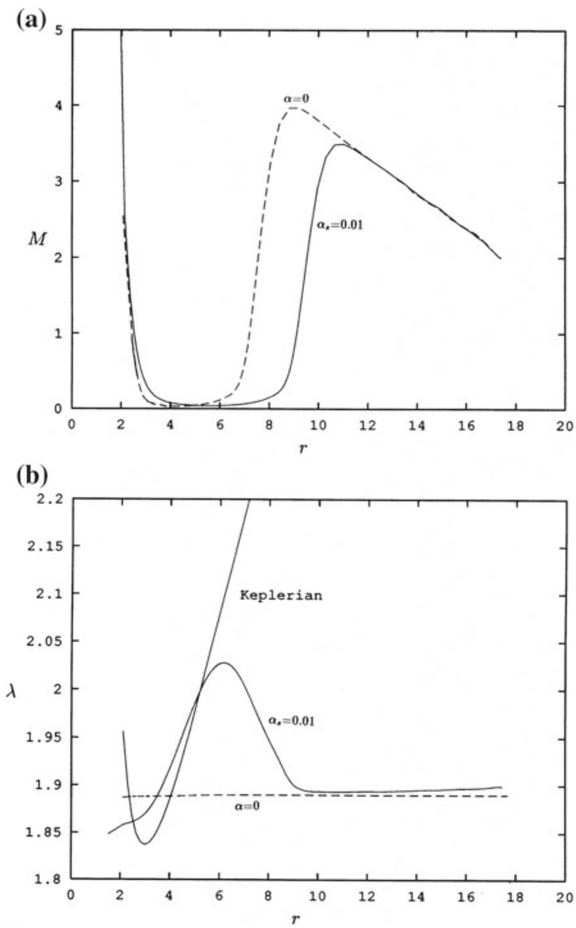

Fig. 2.4 Comparison of **a** Mach number and **b** angular momentum variations in viscous (*solid*) and inviscid (*dashed*) isothermal thin accretion discs. The viscosity parameter $\alpha_s = 0.01$ is chosen everywhere in the simulation. Note that the shock in the viscous disc forms farther out and is weaker and wider. The *solid curve* in (**b**) marked 'Keplerian' is the Keplerian distribution plotted for comparison. Due to the inefficiency of transfer of angular momentum in the pre-shock flow, a mixed-shock forms with higher angular momentum in the post-shock flow (Chakrabarti and Molteni 1993)

interest was to study the transonic nature of the flow just before the matter enters into the horizon. They find that a torus-like structure forms close to the black hole and the angular momentum increases outward. Igumenshchev and Beloborodov (1997) used the finite difference method and allowed the heat generated by viscosity heating to be radiated away or absorbed totally. The computational box was up to $300r_g$, but the outer boundary condition was that of a near-Keplerian flow having no radial velocity. The inner boundary was kept at $3r_g$. Thus, the possibility of having a shock or the inner sonic point was excluded. The disc was found to be stable for very high viscosity parameter and less stable for lower α. Igumenshchev and Abramowicz (2000) extended the earlier work by studying dependence on the polytropic index γ which varied from 4/3 to 5/3 as well as viscosity parameter and again found that the stability of the solutions depends on these parameters.

Using SPH code (Lanzafame et al. 1998), studied the behaviour of sub-Keplerian viscous transonic flows when flow is neither isothermal nor restricted only to the equatorial plane as in CM95. They found that even two-dimensional thick discs, shocks form and the steady shock location increase with viscosity as in one-dimensional study of CM95. They also showed that beyond the critical viscosity when a steady shock was not expected, the flow forms an unsteady shock which periodically evacuates the discs. Figure 2.5a–d show the drifting of steady shock location in a two-

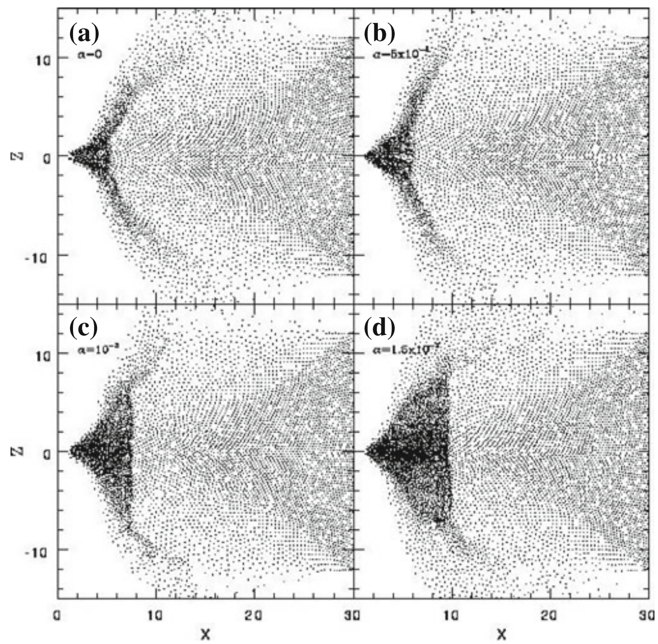

Fig. 2.5 Drifting of the steady shock location in a two-dimensional, axisymmetric accretion flows as the viscosity parameters. Here, the specific angular momentum $\lambda = 1.6$ and energy $\mathcal{E} = 0.001955219$. (**a**) is for the inviscid flow. **b**–**d** shows the effect of the introduction of viscosity in the flow. The viscosity parameter α is 5×10^{-4} in (**b**), 10^{-3} in (**c**) and 1.5×10^{-3}, respectively. This figure has been taken from Lanzafame et al. (1998)

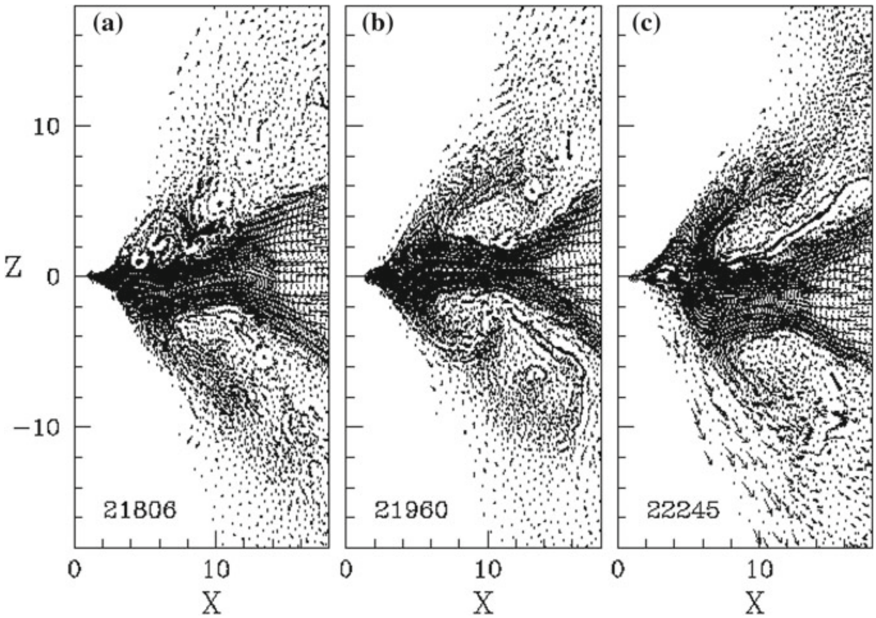

Fig. 2.6 Snapshots of simulations of accretion discs around a $10^8 M_\odot$ black hole by Smoothed Particle Hydrodynamics for three different times in (**a**), (**b**) and (**c**). The *dots* are the particle locations and *arrows* are drawn for every fifth particle for clarity. In (**a**), (**b**) and (**c**) times (in units of r_g/c) is marked in each box. Note the vertical as well as radial oscillation of the accretion shock wave located at $\sim 13r_g$ (from Chakrabarti et al. 2004)

dimensional, axisymmetric accretion flow as the viscosity parameter is increased towards the critical value. Clearly, as in the one-dimensional case, higher viscosity causes higher differential angular momentum transport between the pre-shock and post-shock solutions and as a result the shock is drifted away in the radial direction until the momentum balance is reached.

Chakrabarti et al. (2004) presented the results of several numerical simulations of two-dimensional axisymmetric accretion flows around black holes using Smoothed Particle Hydrodynamics (SPH) in the presence of cooling effects. They consider both stellar black holes and supermassive black holes. They showed examples of shock formation which exhibit radial and/or vertical oscillations. The result of their simulation is shown in Fig. 2.6. In recent times, a few simulations of disc-outflow coupling have been done (Nishikawa et al. 2007; McKinney and Narayan 2007). However, the results are strongly dependent on the initial conditions (Ustyugova et al. 1999) and it is difficult to simultaneously simulate the disc and the outflow regions because the timescales of the accretion and outflow are in general very different. Moreover, in these simulations, how the matter gets deflected from the equatorial plane has been studied largely in the context of Keplerian disc regime.

2.4 Goals of the Thesis

There exist a few dominating factors that determine the morphology of an accretion flow. A significant factor is how fast the angular momentum of accreting matter can be eliminated. The accreting matter invariably can have too much angular momentum if the matter is supplied through Roche lobe overflow, which prohibits itself to fall directly onto the black hole. Therefore, there must be a mechanism by which accreting matter can rapidly transport a large amount of angular momentum. Most of the time, some kind of effective viscosity is generally adopted for the purpose, but the physical cause is not fully understood. On the other hand, if accretion is through winds, the problem of transport of angular momentum is not so accurate. In this case, the opposite problem, namely how to produce a Keplerian disc through transport of angular momentum from a low angular momentum flow is more relevant. In our work, viscosity in accreting matter is described as in the Shakura and Sunyaev (1973) prescription. Viscosity is also an important parameter that controls the ability of the accreting disc to produce dissipative heat from kinetic energy. Besides, the processes of radiative cooling and optical thickness are significant. They determine whether the disc will be stable as a single component or behave differently at different regions as, for example, in two component advective flows (Chakrabarti and Titarchuk 1995).

In this thesis work, we extend the numerical work of Chakrabarti and Molteni (1995), Molteni et al. (1996) and Lanzafame et al. (1998) and provide a model that closely resembles realistic astrophysical accretion discs around black holes, while being restricted to two dimensions. In particular, the hydrodynamic evolution of the disc is followed with a grid-based Total Variation Diminishing (TVD) code, which approximates a gas with a finite difference method (Harten 1983). The evolution in phase space of this gas is determined by the Eulerian form of the hydrodynamic equations. We start our work by studying the time variation of the evolution of the inviscid accretion discs around black holes, and their properties. We also study the change in the pattern of the flows when the strength of the shear viscosity is varied. However, for a given set of incident flow parameters (e.g. specific energy, specific angular momentum and viscosity), the flow may possess multiple critical points (Chakrabarti 1989, 1990a, b). This suggests the possible existence of global solutions in one dimension with standing and oscillating shocks located between two critical points. Our objective is to verify these theoretical predictions through numerical simulations under different input parameters. For a two-dimensional flow, a completely self-consistent theoretical solution is not possible. Thus, a numerical simulation is necessary to reveal the queries. The classifications were made using three different models of the flow, namely a disc of constant thickness, a disc with conical wedge cross section and a disc in vertical equilibrium (Chakrabarti and Das 2001). Our motivation was to study whether a simulated result reproduces theoretical models or not. It has been shown by C89 that low angular momentum, non-dissipative flows produce axisymmetric standing shock waves in tens of Schwarzschild radii, but the presence of a large viscosity in Chakrabarti (1990b) will remove the shock wave since the Rankine-Hugoniot relation is not satisfied in highly dissipative flow.

Our aim is to verify these theoretical results through numerical simulations in two-dimensional system. However, the viscous flow must also be cooled in order that not only the flow has Keplerian angular momentum distribution but also its temperature is cold enough so that a standard (Shakura and Sunyaev 1973) disc is produced. Thus, our goal is also to form a Keplerian disc using numerical simulations that include viscosity and cooling. In my thesis, the effect of the shear viscosity on the evolution of non-self-gravitating discs is investigated when a simple power-law cooling takes place inside the flow.

2.5 Some Remarks on Units and Dimensions

The radius of a non-rotating black hole $r_g = 2GM/c^2$ is only 3.0 km if $M = M_\odot$. A stellar mass black hole generally has $M > 3M_\odot$. Hence, the radius is around 9 km. For a supermassive black hole, one can scale these numbers depending on the mass of the black hole. However, the physical processes in accretion flows generally have length scales of the order of the Schwarzschild radius and thus, it is convenient to choose this as the unit of length. Similarly, it is well known that the velocity of in-falling matter through the horizon is equal to the velocity of light (Chakrabarti 1996). Thus, it is expected that matter and sound velocities would be of this order and thus the units may be chosen accordingly.

Keeping these in mind, we choose, $2G = c = M = 1$. In this case, the unit of velocity would be c, the unit of distance would be $2GM/c^2$ (the Schwarzschild radius), the unit of time would be $2GM/c^3$ and the unit of angular momentum would be $2GM/c$. In this unit system, the pseudo-Newtonian potential is written as $-\frac{1}{2(r-1)}$.

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