Chapter 2
Fundamentals of Entransy and Entransy Dissipation Theory

Abstract The entransy is a new developed parameter that is effective in optimization of heat transfer. It can be used as an evaluation of the transport ability of heat. In this chapter, based on the energy conservation equation, the entransy balance equations for heat conduction and convective heat transfer are developed. The entransy dissipation extreme principle is developed. This extreme principle can be concluded into the minimum thermal resistance principle defined by entransy dissipation.

Keywords Energy conservation · Entransy dissipation · Extreme principle · Heat transfer · Thermal resistance · Conduction · Convection

In recent years, with the increasing of the living standard, the global increasing consumption of limited primary energy has become such a big concern not just for the lack of the primary energy, but for the depletion of the ozone layer and global warming. And also, it has been estimated that more than 80 % of the worldwide energy utilization involves the heat transfer process. Thermal engineering has for a long time recognized the huge potential for conserving energy and decreasing CO_2 emission so as to reduce the global warming effect through efficient heat transfer techniques. In general, approaches for heat transfer enhancement have been explored and employed over the full scope of energy generation, conversion, consumption, and conservation. Design considerations to optimize heat transfer have often been taken as the key for better energy utilization and have been evolving into a well-developed knowledge branch in both physics and engineering.

During the last several decades, a large number of heat transfer enhancement technologies have been developed, and they have successfully cut down not only the energy consumption, but also the cost of equipment itself. However, comparing with other scientific issues, engineering heat transfer is still considered to be an experimental problem and most approaches developed are empirical or semi-empirical with no adequate theoretical base. For instance, for a given set of constraints, it is nearly impossible to design a heat-exchanger rig with the optimal heat transfer performance so as to minimize the energy consumption.

J. Gu and Z. Gan, *Entransy in Phase-Change Systems*,
SpringerBriefs in Thermal Engineering and Applied Science,
DOI: 10.1007/978-3-319-07428-3_2, © The Author(s) 2014

Therefore, scientists developed several different theories and methods to optimize heat transfer, such as the constructal theory and the second law of thermodynamics theory in terms of entropy. The second law of thermodynamics is one of the most important fundamental laws in physics, which originates from the study of the efficiency of heat engine and places constraints upon the direction of heat transfer and the attainable efficiencies of heat engines. The concept of entropy introduced by Clausius for mathematically describing the second law of thermodynamics has stretched this law across almost every discipline of science. However, in the framework of the classical thermodynamics, the definition of entropy is abstract and ambiguous, which was noted even by Clausius. This has induced some controversies for statements related to the entropy. Recently, Bertola and Cafaro found that the principle of minimum entropy production is not compatible with continuum mechanics. Herwig showed that the assessment criterion for heat transfer enhancement based on the heat transfer theory contradicts the ones based on the second law of thermodynamics. The entropy generation number defined by Bejan is not consistent with the exchanger effectiveness, which describes the heat exchanger performance [1]. Shah and Skiepko found that the heat exchanger effectiveness can be maximum, minimum, or in between when the entropy generation achieves its minimum value for 18 kinds of heat exchangers, which does not totally conform to the fact that the reduction of entropy generation leads to the improvement of the heat exchanger performance. These findings signal that the concepts of entropy and entropy generation may not be perfect for describing the second law of thermodynamics for heat transfer.

Although there has been effort to modify the expression of the second law of thermodynamics and to improve the classical thermodynamics by considering the Carnot construction cycling in a finite time, the eminent position of entropy in thermodynamics has not been questioned. Recently, Guo et al. introduced the concepts of entransy and entransy dissipation to measure, respectively, the heat transfer capacity of an object or a system, and the loss of such capacity during a heat transfer process. Moreover, Guo et al. [2] defined two new physical quantities called entransy and entransy dissipation for describing the heat transfer ability and irreversibility of heat conduction, respectively. Guo et al. have introduced a dimensionless method for the entransy dissipation and defined an entransy dissipation number, which can serve as the heat exchanger performance evaluation criterion. Based on the concept of entransy dissipation, an equivalent thermal resistance of heat exchanger was defined, which is consistent with the exchanger effectiveness [3] and consequently, developed the minimum entransy dissipation-based thermal resistance principle to optimize the processes of heat conduction [2, 4], convective heat transfer [5–7], thermal radiation, and in heat exchangers [3].

2.1 The Definition of Entransy and Entransy Dissipation

Guo et al. found that all transport processes contain two different types of physical quantities due to the existing irreversibility: the conserved ones and the nonconserved ones, and the loss or dissipation in the nonconserved quantities can then be used as the measurements of the irreversibility in the transport process. Taking an electric system as an example, although both the electric charge and the total energy are conserved during an electric conduction, the electric energy, however, is not conserved and it is partly dissipated into the thermal energy form due to the existence of the electrical resistance. Consequently, the electrical energy dissipation rate is often regarded as the irreversibility measurement in the electric conduction process. Similarly, for a viscous fluid flow, both the mass and the momentum of the fluid, transported during the fluid flow, are conserved, whereas the mechanical energy, including both the potential and kinetic energies, of the fluid is turned partially into the thermal energy form due to the viscous dissipation. As a result, the mechanical energy dissipation is a common measure of irreversibility in a fluid flow process. The above two examples show that the mass, or the electric quantity, is conserved during the transport processes, while some form of the energy associated with them is not. This loss or dissipation of the energy can be used as the measurement of irreversibility in these transport processes. However, an irreversible heat transfer process seems to have its own particularity, for the heat energy always remains constant during transfer and it does not appear to be readily clear what the nonconserved quantity is in a heat transfer process. Based on the analogy between electrical and heat conductions, Guo et al. made a comparison between electrical conduction and heat conduction as shown in Table 2.1 [2].

It could be found in the table that there is no corresponding parameter in heat conduction for the electrical potential energy in a capacitor, and hence they defined an equivalent quantity, G, that corresponding to the electrical potential energy in a capacitor, which is called "**entransy.**" They further derived Eq. (2.1) according to the similar procedure of the derivation of the electrical potential energy in a capacitor. Entransy was originally referred to as the heat transport potential capacity in an earlier paper by the Guo et al. [2].

$$G = \frac{1}{2} QT[\mathrm{JK}] \tag{2.1}$$

where Q is, mc_vT, the thermal energy or stored heat in a body at constant volume kept at temperature T. It is equivalent to potential electrical energy in a capacitor, which makes a current (heat flow) between two objects connected with a resistance together at two different potential levels. It is also possible to explain a heat transfer process through the analogy as depicted in Fig. 2.1.

Entransy represents the heat transfer ability of an object. It possesses both the nature of "energy" and the transfer ability. If an object is put in contact with an infinite number of heat sinks that have infinitesimally lower temperatures, the total quantity of "potential energy" of heat, whose output can be $\frac{1}{2}QT$. Biot suggested a

Table 2.1 Electrical and thermal energy variable and equation similarities

Electrical energy						
Stored charge in a capacitor	Current	Resistance	Capacitance	Electrical potential (Voltage)	Current density	Ohm's law
Q	I	R	C	U^a	$\dot{q}$	$\dot{q} = -K\frac{dU}{dn}$
C	$\frac{C}{s}$ (or A)	Ω	F	V	$\frac{C}{m^2 s}$	
Thermal energy						
Stored heat in a body	Heat flow	resistance	Heat capacity	Thermal potential (Temperature)	Heat flux	Fourier law
$Q = mc_v T$	$\dot{Q}$	R	$C = \frac{Q}{T}$	T^a	$\dot{q}$	$\dot{q} = -K\frac{dT}{dn}$
J	$\frac{J}{s}$ (or W)	$\frac{K}{W}$	$\frac{J}{K}$	K	$\frac{J}{m^2 s}$	

[a] Voltage and temperature are potential parameters and need a reference for measurement ($T = \Delta T$ and the difference measured from 0 K)

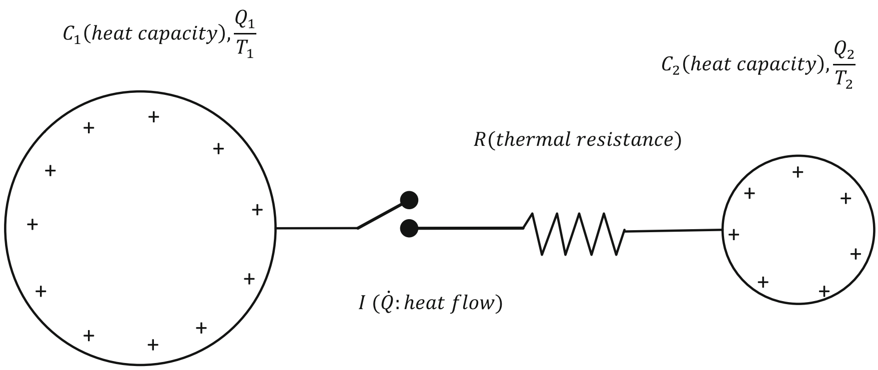

Fig. 2.1 Descriptive analogy between heat flow and electrical current with corresponding parameters and terminology in heat transfer (e.g., capacitor and resistance)

similar concept in the derivation of the differential conduction equation using the variation method. Eckert and Drake [8] pointed out that, Biot in a series of papers beginning in 1955, formulated from the ideas of irreversible thermal dynamics, a variational equivalent of the heat conduction equation that constituted a thermodynamical analogy to Hamilton's principle in mechanics and led to a Lagrangian formulation of the heat conduction problem in terms of generalized coordinates…, Biot defines a thermal potential as $E = {}^1\!/_2 \iiint_\Omega \rho c T^2 \mathrm{d}V\ldots$ The thermal potential "E" plays a role analogous to a potential energy…

However, Biot did not further explain the physical meaning of thermal potential and its application was not found later except in the approximate solutions of anisotropic conduction problems. Accompanying the electric charge, the electric energy is transported during electric conduction. Similarly, along with the heat, the entransy is transported during heat transfer too. Furthermore, when a quantity of heat is transferred from a high temperature to a low temperature, the entransy is reduced and some of it is dissipated during the heat transport. The lost entransy is called entransy dissipation. Entransy dissipation is an evaluation of the irreversibility of heat transport ability.

For a transient heat conduction process without any heat transfer with ambient environment, the thermal energy conservation equation can be expressed as

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) \tag{2.2}$$

where ρ, c_p, λ are density, constant pressure, specific heat and thermal conductivity, respectively. The entransy equilibrium equation can be obtained by multiplying both sides by temperature T:

$$\rho c_p T \frac{\partial T}{\partial t} = \nabla \cdot (\lambda T \nabla T) - \lambda |\nabla T|^2 \tag{2.3}$$

where the left side represents the time variation of the entransy stored per unit volume, the first term on the right side is the entransy transfer from one object to another associated with heat transfer, while the second term is the local entransy dissipation rate due to heat conduction. This is similarly like electric energy dissipation during an electric conduction process or mechanical energy dissipation during mechanical moving process. Since electrical energy and mechanical energy dissipations are both irreversibility measure of their respective process, entransy dissipation rate is hence a measure of the irreversibility in heat transfer process and can be written as

$$\dot{G} = \lambda|\nabla T|^2. \tag{2.4}$$

In a similar manner, the thermal energy conservation equation for a steady-state convective heat transfer process with no heat source can be expressed,

$$\rho \mathrm{c_p} \mathrm{U_f} \cdot \nabla \mathrm{T} = \nabla \cdot (\lambda \nabla \mathrm{T}), \tag{2.5}$$

where U_f is the velocity vector of the fluid. Similarly, the entransy equilibrium equation for the convective heat transfer can be derived by multiplying both sides by temperature T,

$$U_f \cdot \nabla \left(\frac{\rho c_p T^2}{2} \right) = \nabla \cdot (\lambda T \nabla T) - \lambda |\nabla T|^2 \tag{2.6}$$

The left side of this equation express the entransy transferred associated with the fluid particles motion, while the right side of this equation is in the same form as the heat conduction process, which include the entransy diffusion within the fluid due to temperature gradient and the local entransy dissipation rate.

By integrating Eq. (2.6) over the entire domain, transforming the volume integral to the surface integral on the domain boundary, and ignoring the heat diffusion in the flow direction at both inlets and outlets, we can obtain

$$\left(\frac{1}{2} \rho \dot{V} c_p T^2 \right)_{\mathrm{out}} - \left(\frac{1}{2} \rho \dot{V} c_p T^2 \right)_{\mathrm{in}} = \iint_{\Gamma} \vec{n} \cdot \lambda T \nabla T \mathrm{d}A - \iiint_{\Omega} \lambda |\nabla T|^2 \mathrm{d}V \tag{2.7}$$

where the first term on the left side describes the entransy flowing out of the domain, while the second term is the entransy flowing into the domain. On the right side, the first term indicates the entransy flow rate induced by heat transfer through the domain boundary, while the second term can be viewed as the total entransy dissipation rate

$$\dot{G} = \iiint_{\Omega} \lambda |\nabla T|^2 \mathrm{d}V. \tag{2.8}$$

Similar to electrical resistance, the entransy dissipation-based thermal resistance of a heat exchanger is defined as

$$R_{ex} = \frac{\dot{G}}{Q^2}, \tag{2.9}$$

where R_{ex} is the entransy dissipation-based thermal resistance, $\dot{G}$ is the entransy dissipation rate during the heat transfer process and Q is the total heat transfer rate.

2.2 Entransy Analysis in Conduction Heat Transfer

The conduction heat transfer equation with no heat source available in a domain is

$$\rho c_v \frac{\mathrm{d}T}{\mathrm{d}t} = \nabla \cdot (K \nabla T) \tag{2.10}$$

By multiplying both sides by T, the entransy equation is

$$\rho c_v T \frac{\mathrm{d}T}{\mathrm{d}t} = -\nabla \cdot (TK\nabla T) + K\nabla T \cdot \nabla T, \tag{2.11}$$

or

$$\frac{\mathrm{d}G}{\mathrm{d}t} = -\nabla \cdot \dot{G} - G_{\varphi}, \tag{2.12}$$

where G is entransy density, $\frac{1}{2}\rho c_v T^2$, G_{φ} is entransy dissipation as $\mathrm{K}(\nabla \mathrm{T})^2$, and $\dot{G}$ is entransy flux. The presence of temperature gradient to the power of two in local entransy dissipation resembles viscose dissipation term in entropy generation in fluid flow, which is similar to electrical dissipation. For all these squared terms, energy should be paid (consumed) to maintain the corresponding flow (e.g., heat or mass). For instance, to maintain a viscous flow, pumping power should be provided to have the fluid flowing. For heat transfer, energy should be delivered to maintain the heat flow as there is always a resistance present and entransy dissipation. It looks like a connecting concept between heat transfer and thermodynamics and furthermore between different energy and heat transfer modes.

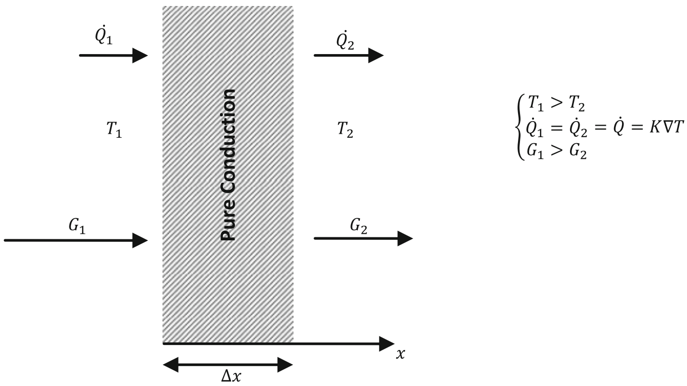

Fig. 2.2 Steady heat conduction in an object with very large depth and height (one-dimensional conduction)

2.3 Equivalent of Thermal Resistance in Heat Convection and Entransy Dissipation

In a one-dimensional or infinitesimal differential heat conduction case, thermal resistance at a point is easy to define as $R = \Delta T/\dot{Q}$ On the other hand, for multi-dimensional heat conduction case, it is not straightforward to calculate it and needs some definitions and averaging procedure. Now, it is possible to define an equivalent thermal resistance based on entransy dissipation, which occurs in thermal resistance

$$R = \frac{G_\varphi}{\dot{Q}^2} \tag{2.13}$$

where $G_\varphi = \int_V K(\nabla T)^2 \mathrm{d}V$ is the rate of entransy dissipation over a volume has a temperature gradient (thermal potential gradient) with a heat flow rate of $\dot{Q}$ caused by ∇T established in the volume in heat conduction mode. The equivalent thermal resistance can be found in any arbitrary shape in a three-dimensional and general case with no limitations. In a one-dimensional conduction case, it will be trivial as

$$R = \frac{\dot{Q}\,\Delta T}{\dot{Q}^2} = \frac{\Delta T}{\dot{Q}} \tag{2.14}$$

The temperature gradient in a one-dimensional Fourier heat conduction case across a very long and wide solid bar with a thickness of Δx can be found where a constant heat flux of $\dot{Q}$ transfer thermal heat energy from higher temperature to the colder side across the object shown in Fig. 2.2. Energy is conserved and $\dot{Q}$ is constant for this special one-dimensional case, but there is entransy dissipation as the thermal resistance is available to facilitate the heat flux (equivalent to current in electrical energy). Entransy leaving the colder side, G_2 is less than G_1 as a result of entransy dissipation G_φ which can be found as follows:

$$G_1 - G_2 = G_\varphi = -\int_0^{\Delta x} K\nabla T.\nabla T \mathrm{d}x = -\int_0^{\Delta x} \dot{Q}\frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x = \dot{Q}(T_1 - T_2) \tag{2.15}$$

The heat flow direction is always in an entransy dissipation mode, which means G_φ should be positive (another interpretation of the second law of thermodynamics).

2.4 Conclusions

Entransy is a parameter developed in recent years. It is effective in optimization of heat transfer. Entransy is an evaluation of the transport ability of heat. Both the amount of heat and the potential contribute to the entransy. Entransy will be lost during heat transportation from a high to a low temperature, and entransy dissipation will be produced.

In this chapter, based on the energy conservation equation, the entransy balance equations for heat conduction and convective heat transfer have been developed. The entransy dissipation extreme principles are also developed, that is, the maximum entransy dissipation corresponds to the maximum heat flux for prescribed temperature difference and the minimum entransy dissipation corresponds to minimum temperature difference for prescribed heat flux. This extreme principle can be concluded into the minimum thermal resistance principle defined by entransy dissipation.

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Entransy in Phase-Change Systems
Gu, J.; Gan, Z.
2014, XIII, 56 p. 27 illus., 19 illus. in color., Softcover
ISBN: 978-3-319-07427-6