Einstein in Prague: Relativity Then and Now

Jiří Bičák

Abstract It was during his stay in Prague that Einstein started in earnest to develop his ideas about general relativity. I will recall those days in 1911 and 1912, discuss Einstein's papers on gravitation from that period and emphasize which new concepts and ideas he introduced. I also want to indicate how the main themes that preoccupied him then, the principle of equivalence, bending of light, gravitational redshift and frame dragging effects, are alive in contemporary relativity.

1 Introduction

I would like to start as I did in my talk at the conference, quoting what Einstein wrote soon after his arrival in Prague on April 3, 1911: "The city of Prague is very fine, so beautiful that it is worth a long journey for itself" (from the letter to his friend M. Besso on May 13, 1911); or, "I have a magnificent institute here in which I work very comfortably... By the way, Czechs are much more harmless than one thinks" (from the letter to M. Grossmann on April 27, 1911). I hope the conference participants, 100 years after Einstein in Prague, had a similar impression.

The quote from Hesiod's "Works and Days" from the seventh century BC—*The price of achievement is toil* ['Schinderei']; *and the gods have ruled that you must pay in advance*—enables me to give a brief summary: *Einstein paid much in Prague*. The days: April 1911–July 1912. The works: principle of equivalence, light bending, dragging of inertial frames; features of a future theory of gravity.

It was not until 1911, only after his arrival in Prague, that Einstein's interest in quantum theory started to diminish and his systematic concentration on the problems of a new theory of gravity began. There were specific issues which Einstein analyzed

---

J. Bičák (✉)
Institute of Theoretical Physics, Charles University in Prague, V Holešovičkách 2,
180 00 Prague, Czech Republic
e-mail: bicak.troja@gmail.com

J. Bičák and T. Ledvinka (eds.), *General Relativity, Cosmology and Astrophysics*,
Fundamental Theories of Physics 177, DOI: 10.1007/978-3-319-06349-2_2,

in Prague, for example that the bending of light due to the Sun's gravitational field is observable. However, as we shall see later, just before leaving Prague, on July 4, 1912, Einstein submitted a short paper to Annalen der Physik in which a number of fundamental features of the final general relativity were anticipated. Concerning new ideas and liberating oneself from old views, especially in this respect, "Einstein paid much in Prague". Before we turn to Einstein's work on gravity in Prague we shall recall his days here. We cannot describe the atmosphere in Prague of those days, with tensions between three main groups of its inhabitants, Czechs, Germans and Jews, on one hand, and with many intellectually inspiring aspects and interactions between them on the other hand, in any detail, but literature of a special interest will be mentioned.

2 Why and How He was Invited to Prague

Charles University in Prague, founded in 1348 as the first university "beyond the Alps", was originally one educational center for Czechs, Germans, Poles and various south-European nations. Nationalism led to its division in 1882 into the German and Czech parts.[1] In 1911 the Czech part had 4432 students, the German part 1844 students but, for example, Mach's lectures in the German part were visited by a number of Czech students. In the German part, the head of mathematical physics, Professor F. Lippich, was due to retire and the German university decided to have the Institute of Theoretical Physics that existed in the Czech University already. The chief advocate of the proposal to appoint Albert Einstein was Anton Lampa, professor of experimental physics, a great "Machian", a Czech by origin but an ardent supporter of Germanization who hoped that Einstein would develop further Mach's ideas.[2] Another member of the commission for choosing a candidate for theoretical physics was Georg Pick, a mathematician of broad interests, with whom Einstein made later, during his stay in Prague, a close friendship. In his recommendation letter from 1910, Max Planck wrote that if Einstein's theories were to be confirmed, Einstein would be considered the Copernicus of the twentieth century.

---

[1] In 1879/80 and 1883/84 Ernst Mach was elected Rector of the University. However, during his second Rectorship he resigned since he disapproved of the division of Prague University into Czech and German parts.

[2] Anton Lampa (1968–1938) was an experimental physicist, interested also in philosophy, history and cultural aspects of physics. Interestingly, he was the first to publish a paper on the appearance of a moving rod according to special relativity (Zeits. f. Phys., 1924). For the life and work of Lampa, see [1].

3 Days in Prague

No view of Einstein in Prague can be more original and immediate than that of Philipp Frank,[3] Einstein's successor as head of the Institute of Theoretical Physics in the German university in Prague until 1938. In 1929, Frank organized the famous first meeting on the Epistemology of the Exact Sciences in Prague. From 1939 till 1941 he wrote the biography of Einstein [3], which Einstein endorsed. Its whole Chapter IV, "Einstein at Prague", besides information on Einstein and the theory of relativity, paints amusingly also the atmosphere in Prague at the beginning of the twentieth century. Frank, when writing Einstein's biography, did not have manuscripts and other documents at his disposal, so for an earnest Einstein biographer, other sources are important. Nevertheless, Frank tells various entertaining stories which are probably not far from the truth. So, for example, it was the custom for a newly appointed professor to pay a visit to his colleagues. Einstein started to pay these visits and decided to go first to romantic old parts of Prague. But later he found that these calls, which numbered around forty, were really a waste of time and he stopped his visits. The professors whom he had not visited felt puzzled but in fact the main reason was that they did not live in interesting parts of the city or their name was far back in the alphabet. Another nice story supports our point that it was in Prague where Einstein for the first time started to concentrate on gravity rather than on quantum physics. Frank recalls how during his first visit of Einstein in Prague, Einstein took him to the window of his office from which they could overlook a large garden behind the wall (see Fig. 1, with the wall on the right). Einstein told Frank that he saw people there in deep meditation but also groups very vividly discussing. Only quite later he learned that the park belonged to the insane asylum of Bohemia. Einstein pointed to people walking there, turned to Frank and said: "Those are the madmen who do *not* occupy themselves with the quantum theory". This is not true today. The park and gardens still belong to the mental hospital, however, it is open to the public during the day. And physicists from our Faculty nearby quite often walk across it to get to the center of the city.

In Prague, Einstein associated himself with a group of Jewish intellectuals who gathered in the evenings at Berta Fanta's salon in the house "At the Unicorn" in the Old Town Square. There philosophy was discussed and music played. Einstein met an ardent Zionist, Hugo Bergmann, the son-in-law of Berta Fanta, but he was then not able to arouse Einstein's interest in his ideas. In the letter to Hedwig Born in 1916, Einstein wrote that Zionists in Prague are "a small troop of unrealistic people, harking back to the Middle Ages". Still later, after Bergmann became a Hebrew University professor in Jerusalem, Einstein called him "the serious saint from Prague". During

---

[3] P. Frank (1884, Vienna—1966, Cambridge, Mass.) was a theoretical physicist and logical positivist, a member of the Vienna Circle. His work in relativity is summarized in detail in the comprehensive article by Havas [2]. With R. von Mises, Frank published a book on differential and integral equations in physics; later he wrote several books on the philosophy of science. His booklet *Relativity–a richer truth*, with a foreword by Albert Einstein on the "Laws of science and the laws of ethics", published in 1951, is less known. It touches on a number of philosophical and ethical issues.

Fig. 1 The building of the former Faculty of Philosophy of the German University on Viničná 7 wherein Einstein had "an excellent Institute with a beautiful library"

the evenings Einstein perhaps met Franz Kafka, although Kafka, in fact, did not like to go there, but he certainly was acquainted with Max Brod, a writer and journalist who later, after Czechoslovakia was founded in 1918, played a significant role in promoting the Czech culture. Almost all of Einstein biographers, inspired by Philipp Frank, make the point that Max Brod drew on Einstein's character for his portrait of Kepler in his novel *Tycho Brahe's Way to God* and some even suggest that in this way certain egocentric features of Einstein's personality were disclosed. However, when reading Brod's autobiography *Streitbares Leben*, we discover that he was quite unhappy with Frank's interpretation and even wrote a letter to Einstein to explain that he never noticed any egocentric features in his behavior and, in any case, that it was rather the poet and writer Franz Werfel, Brod's friend, who contributed to the portrait of Kepler.

Einstein visited Prague once again in 1921 when he accepted an invitation from Urania, Prague's German Society, to give a lecture on the theory of relativity. Accompanied by Frank, he visited the Physical Institute of the Czech part of Charles University. "By this visit Einstein wanted to express his sympathy for the new Czechoslovak Republic and its democratic policy under Masaryk's leadership", writes Frank. Abraham Pais in his celebrated biography of Einstein [4] gives the list of people Einstein suggested for the Nobel Prize. Masaryk is among them, proposed by Einstein for the Peace Prize. Einstein was later in correspondence with Masaryk regarding the fate of a pacifist Přemysl Pitter.[4]

---

[4] Copies of their letters are available in "Einstein Archives Online"—see http://www.alberteinstein.info/

4 The Czech Culture and Science Responding to Einstein's Work

Karel Čapek (1890–1938), a humanist with encyclopedic knowledge, one of the best known writers and journalists of the Masaryk era, who was a close friend of Masaryk, wrote "philosophical" novels, charming detective stories, anti-war dramas, and also was a pioneer of the Czech science fiction. Alan J. Friedman and Carol C. Donley in their book *Einstein as myth and muse* (Cambridge University Press 1985) start their section "Approaches to relativity in fiction" emphasizing that from the use of the profoundly wrong aphorism "everything is relative", various authors explore intricate possibilities of Einstein and his theories. And they continue: "A remarkable early exposition of the possibilities appeared in 1924, with Karel Čapek's novel *Krakatit*. Čapek's awareness of science and technology was indicated by mentions... of the leading scientists of the day, including Einstein, Rutherford, Planck, Bohr and Millikan. The plot concerns an inventor who has discovered a way to release atomic energy... The technical details are as accurate as they could be in the early 1920s, and atomic energy is correctly seen as a possibility emerging from the radioactivity work of Becquerel and Rutherford, and not from Einstein's theories... The inventor, Prokop, is torn in the traditional struggle between God and the devil... Prokop's bewilderment, in the literal form of a fever, is described by the first metaphor from relativity: ... It appeared he was moving with velocity approaching velocity of light; in some way his heart was compressed. But that was only Fitzgerald-Lorentz contraction... " Prokop then finds himself in the closed Einstein universe... Curiously, Einstein's closed universe attracted also one of the most sophisticated Czech art critics and writers, F. X. Šalda. In 1928 already, he makes analogies between Einstein's conception of the finite, closed Universe and the conception of space in paintings by Cézanne.

We cannot continue here with more examples of the inspiring role which Einstein's theories exerted on the Czech culture. Let us just look at Fig. 2 where the portrait of Einstein by well-known Czech oil painter and graphic artist Max Švabinský (1873–1962) is shown. In a recent interview in the University magazine *Babylon*, Švabinský's son-in-law, originally a mathematician, declared that he sent one of the copies of this lithography to Robert Oppenheimer and it was hanging on the wall of his office in the Institute of Advanced Studies in Princeton.

4.1 Impact on Czech Physics and Astronomy

Relativity theory was popularized and even taught quite soon by the Czech physicists and astronomers. The first papers were written by A. Dittrich and A. Žáček in 1912. One of the main protagonists of Einstein's theories was professor of theoretical physics at the Czech part of the Charles University, František Záviška (1879–1945).

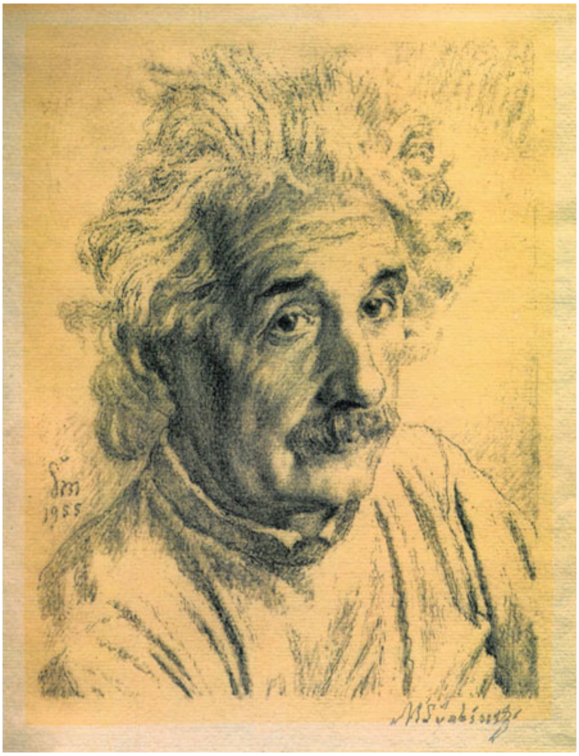


Fig. 2 Einstein's portrait from 1955 by Max Švabinský

In 1925, his semi-popular book *Einstein's principle of relativity and theory of gravity* including basic principles of general relativity appeared. He was in good relations with Philipp Frank and translated Frank's book *Das Ende der mechanischen Physik* (The end of mechanistic Physics), published in 1935, into Czech. It is a criticism of the totalitarian (Nazi) philosophy from the point of view of the theory of knowledge and philosophy of science. An interesting impact of Einstein's prediction of light bending on a Czech astronomer, F. Link, will be discussed below. At present there is a rather extensive literature on Einstein's influence on culture and science in the Czech lands between World Wars I and II available, and on Einstein's work done during the Prague stay.[5]

---

[5] To give some examples, we quote a booklet [5] published on the occasion of Einstein's centenary in 1979 (in Czech and partially in German), the 3rd number of the Czechoslovak Journal of Physics from the same year dedicated to Einstein and the comprehensive article on his route to general relativity, concentrated primarily on the Prague period [6]. Two articles in English [7, 8] are texts of talks about Einstein's Prague papers on gravity given at the Conference of the European Physical Society in Prague in 1984 and at the Marcel Grossmann meeting in Perth in 1988. And a very recent detailed work [9] by Těšínská, a historian of science (containing 57 references) concentrates

After World War II Miroslav Brdička (1912–2007) wrote one of the first original papers on general relativity ("On gravitational waves", Proc. Roy. Irish Acad., 1951) after his stay 1948–1949 as a scholar in the Dublin Institute for Advanced Studies. It has been curious to see this work quoted and Brdička's name observed in the title of the very recent paper by G. Gibbons and C. Rugina.[6] The communist upheaval in Czechoslovakia in 1948 led to a large wave of emigration. Within this there was also a geometer Václav Hlavatý who became professor at Indiana University and started to work extensively in relativity, in particular on Einstein's unified field theories. In another wave of emigration, after August 1968, Karel Kuchař came first to Princeton, following the invitation of John Wheeler, and then became professor in Salt Lake City. His influential work in the quantum theory of covariant systems, canonical quantum gravity and the issue of time is well known to contemporary relativists, including some of participants of this conference.

Finally, I am glad to say that at present there are several groups active in relativity, relativistic astrophysics and cosmology in the Czech Republic—at the Faculty of Mathematics and Physics (Theoretical Physics, Astronomy), in the Academy of Sciences (Institute of Astronomy, Mathematical Institute) in Prague, and at the Silesia University in Opava and Masaryk University in Brno. A volume *Gravitation: following the Prague inspiration* [10] contains comprehensive essays by 14 Czechoslovak relativists and astrophysicists about their work. Some most recent results are included in the Proceedings [11] of this conference, containing contributions based on oral and poster presentations.

Before I finish this "Czech intermezzo" let me present the Fig. 3 from a two-day celebratory meeting on the occasion of Einstein's centenary which took place in the Carolinum–the same place as our conference–on February 26 and 27, 1979. Among more than 200 participants there were several distinguished guests from abroad, including two associated directly with Einstein—P. G. Bergmann and J. A. Wheeler. Peter Bergmann met his wife Margot when they both were students of Physics at Prague German University.[7]

5 Lectures, Seminars and Papers of Albert Einstein in Prague

Below in Table 1 we see that the lectures Einstein gave in Prague were on classical subjects like mechanics, thermodynamics and molecular theory of heat. It is not known whether in the seminars he organized more modern topics were included.

(Footnote 5 continued)
on Einstein's call to Prague and on development of theoretical physics at the Czech university in Prague in relation to Einstein's work.

[6] See Coryacher-Chaplygin, Kovalevskaya, and Brdička-Eardley-Nappi-Witten pp-waves spacetimes with higher rank Stäckel-Killing tensor, J. Math. Phys **52**, 122901 (2011).

[7] Unfortunately they did not recall where Einstein's or even Frank's office was. However, their faces turned into a fine smile when we went through the big door to see the Einstein memorial tablet in the entrance hall inside because they could clearly identify the door into the building in Fig. 1.

Fig. 3 During Einstein's centenary celebrations outside Carolinum in February 1979: John Archibald Wheeler, Andrzej Trautman, Mrs. Melcher, Ernst Schmutzer, Jiří Langer, Margot Bergmann, Peter Bergmann, and Horst Melcher (from *left* to *right*). The present conference took place in the same place... (Photograph taken by the author)

Table 1 Einstein's lectures and seminars in Prague

Period	Title	No. of students
20.4.1911–31.7.1911	Mechanik diskreter Massenpunkte (3h)	13
	Thermodynamik (2h)	12
	Seminar	6
19.10.1911–27.3.1912	Mechanik diskreter Massenpunkte (3h)	12
	Wärmelehre (2h)	13
	Seminar	7
12.4.1912–31.7.1912	Mechanik der Kontinua (2h)	10
	Molekulartheorie der Wärme (3h)	11
	Seminar	7

During the first year, the lectures were given in Klementinum in the centre of the historical part of Prague, then they moved to Viničná 7.

Concerning the Prague papers of Albert Einstein, five were on thermodynamics, radiation theory and quantum theory, among them also just brief notes. An exception was the review on the problem of specific heats which Einstein presented at the first Solvay Congress in November 1911 where he and Friedrich Hasenöhrl from Vienna were the only representatives of the Austro-Hungarian Empire. Since we are concerned primarily with gravity, we shall not analyze those papers, but we give now the *complete list of the works of Albert Einstein in Prague on the theory of relativity and gravitation*.[8] The first three papers appeared in 1911, and the following four in 1912. The titles are here given in English, the exact German title and bibliographical details are given in the References.

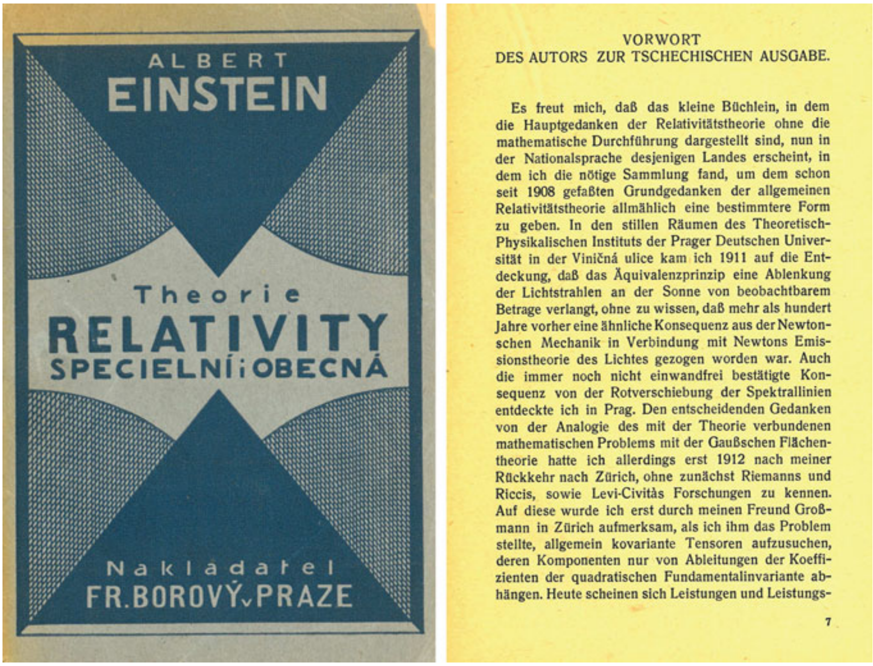
1. The Theory of Relativity [12].
2. On the Ehrenfest Paradox. Comment on Varičak's Paper [13].
3. On the Influence of Gravitation on the Propagation of Light [14].
4. The Speed of Light and the Statics of the Gravitational Field [15].
5. On the Theory of the Static Gravitational Field and Note Added in Proof [16].
6. Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction? [17].
7. Relativity and Gravitation. Reply to a Comment by M. Abraham [18].

The first paper, assigning Prague as Einstein's address already, is based on a lecture given in Zurich in January 16, 1911, before Einstein's arrival in Prague. The lecture, in which Einstein used the term "Relativity Theory" in a title for the first time, was followed by the discussion published in April 1912 (see "The collected papers of Albert Einstein", Vol. 3). The second paper is a short note on a paradox involving moving measuring rigid rods in special relativity. Our main attention will be paid to five papers on the development of a new theory of gravity. Before we turn to them in detail, we shall first present a document, valuable in connection with Einstein's stay in Prague, though largely unknown. As a document of historical importance it was first published and commented upon in 1979 in [5], then quoted in the biography of Pais [4], and later also elsewhere. The document concerns the 1923 Czech translation of Einstein's little book *About the Special and General Theory of Relativity in Plain Terms*.[9] Einstein wrote a special foreword to the Czech edition; this appeared in both the German original and the Czech translation. In the foreword he recalls what he did during his Prague stay: "*I am pleased that this small book, in which the main ideas of the theory of relativity are explained without mathematical elaboration, should now appear in the native language of the country in which I found the necessary concentration for developing the basic idea of the general theory of relativity which I had already conceived in 1908. In the quiet rooms of the Institute of Theoretical Physics of Prague's German University in Viničná Street, I discovered that the principle of equivalence implies the deflection of light rays near the Sun by an observable amount, without at that time knowing that a similar result*

---

[8] Einstein's long review on the Special Theory of Relativity for the *Handbuch der Radiologie* was started in Prague at the beginning of 1912, and continued after Einstein's move to Zurich (the quality of ink and paper improved after the move). The First World War interrupted the publication. Einstein was later unwilling to add material on general relativity or even revise the existing manuscript. Still, it is an extraordinarily precious document since it is the earliest and most significant of the surviving scientific manuscripts written by Einstein before World War I. A fine facsimile was published in 1996 by George Braziller, Inc., in association with J. Safra Foundation and the Israel Museum, Jerusalem. For more details on the manuscript, see the Collected papers of Albert Einstein, Volume 4.

[9] The book was translated by an excellent physics teacher at a highly regarded Czech Gymnasium in Prague, V. Štíbr. Incidentally, after World War II Štíbr was the physics teacher of Karel Kuchař.



VORWORT
DES AUTORS ZUR TSCHECHISCHEN AUSGABE.

Es freut mich, daß das kleine Büchlein, in dem die Hauptgedanken der Relativitätstheorie ohne die mathematische Durchführung dargestellt sind, nun in der Nationalsprache desjenigen Landes erscheint, in dem ich die nötige Sammlung fand, um dem schon seit 1908 gefaßten Grundgedanken der allgemeinen Relativitätstheorie allmählich eine bestimmtere Form zu geben. In den stillen Räumen des Theoretisch-Physikalischen Instituts der Prager Deutschen Universität in der Viničná ulice kam ich 1911 auf die Entdeckung, daß das Äquivalenzprinzip eine Ablenkung der Lichtstrahlen an der Sonne von beobachtbarem Betrage verlangt, ohne zu wissen, daß mehr als hundert Jahre vorher eine ähnliche Konsequenz aus der Newtonschen Mechanik in Verbindung mit Newtons Emissionstheorie des Lichtes gezogen worden war. Auch die immer noch nicht einwandfrei bestätigte Konsequenz von der Rotverschiebung der Spektrallinien entdeckte ich in Prag. Den entscheidenden Gedanken von der Analogie des mit der Theorie verbundenen mathematischen Problems mit der Gaußschen Flächentheorie hatte ich allerdings erst 1912 nach meiner Rückkehr nach Zürich, ohne zunächst Riemanns und Riccis, sowie Levi-Civitàs Forschungen zu kennen. Auf diese wurde ich erst durch meinen Freund Großmann in Zürich aufmerksam, als ich ihm das Problem stellte, allgemein kovariante Tensoren aufzusuchen, deren Komponenten nur von Ableitungen der Koeffizienten der quadratischen Fundamentalinvariante abhängen. Heute scheinen sich Leistungen und Leistungs-

7

Fig. 4 The cover of the Czech edition of Einstein's popular book "About the Special and General Theory of Relativity in Plain Terms" from 1923 and the foreword in original German which Einstein wrote for the Czech edition

had been derived from Newton's mechanics and his corpuscular theory of light. In Prague I also discovered the shift of spectral lines towards the red which is not yet completely confirmed. However, the decisive idea of the analogy between the mathematical formulation of the theory and the Gaussian theory of surfaces came to me only in 1912 after my return to Zürich, without being aware at that time of the work of Riemann, Ricci, and Levi-Civita. This was first brought to my attention by my friend Grossmann when I posed to him the problem of looking for generally covariant tensors whose components depend only on derivatives of the coefficients of the quadratic fundamental invariant. It now appears that it is already possible to evaluate the achievements and limitations of the whole theory. It gives a deep knowledge of the physical nature of space, time, matter and gravity; however, it does not provide sufficient means for solving the problems of quanta and of the atomic constitution of the elementary electric units of which matter is composed."

In Fig. 4, the cover and the German foreword are displayed.

During talks about Einstein's Prague period a question often arises whether the idea of using Riemannian geometry in building a new theory of gravity emerged during the Prague stay already, or only after his return to Zurich when he started to collaborate with his friend Marcel Grossmann. In the foreword above, Einstein mentions that the work of Riemann, Ricci, and Levi-Civita was brought to his attention

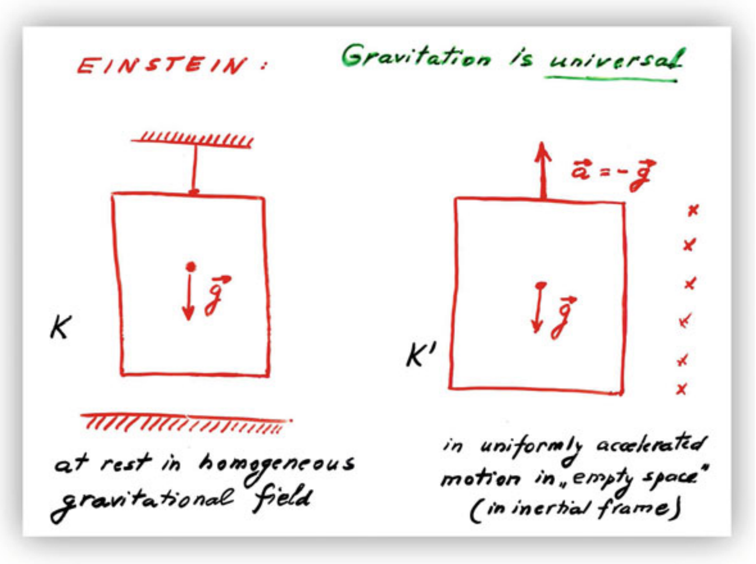


Fig. 5 All physical phenomena will proceed in the same way in the systems K and K'—principle of equivalence

first by Grossmann. Nevertheless, Einstein did not care much about giving precise historical statements; for example, he writes that in Prague he discovered the shift of spectral lines towards red although this effect is contained in his paper from 1907 already in which he tackled the problem of gravity for the first time and then left it until the beginning of the Prague period. That the idea of using the Riemannian geometry was conceived in Prague already as a result of discussions with his friend mathematician Georg Pick is stated in the biography by Frank [3], as well as in the "Personal Reminiscence" by Reinhold Fürth who studied at the German University of Prague during 1912–1916 and became then an important member of the Faculty.[10]

6 The Principle of Equivalence

At the beginning of his first paper on gravity [14] the equivalence of the systems K and K' indicated in Fig. 5 is discussed. To paraphrase Einstein's words in English

---

[10] Fürth says that "Pick, professor of mathematics at Prague and a fellow violin player, drew his [Einstein's] attention to the Italian, Levi-Civita, and his absolute differential calculus… (see *Einstein—the first hundred years*, ed. by M. Goldsmith et al. Pergamon Press 1980). Fürth became a member of Max Born's group in Edinburgh in 1939 but still in October 1938, after the Munich agreement, he was in Prague and published the text "Der Streit um die Deutung der Relativitaetstheorie" (The struggle about the meaning of the relativity theory) in which he defends relativity against philosophers like Prof. Kraus in Prague who wanted to find inconsistencies of the theory by "metaphysical" arguments.

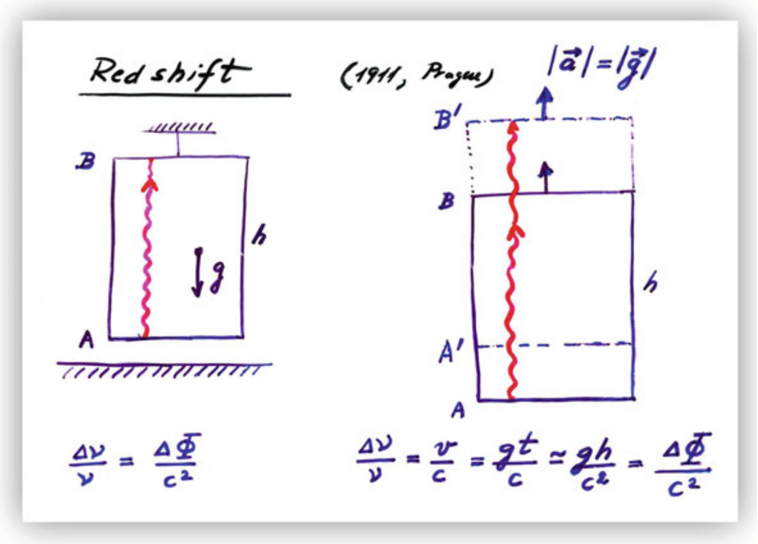


Fig. 6 The principle of equivalence implies the gravitational redshift

he clearly states that the equivalence of both systems is certain as far as we restrict ourselves to purely mechanical phenomena, but it will acquire a deeper meaning if we extend the equivalence to *all laws of nature*. Then we have a principle which has a great heuristic meaning. In his biography *A. Einstein: Creator and Rebel* Banesh Hoffmann, Einstein's direct collaborator, writes: "In the paper of 1907... Einstein had already begun his attack on the problem of acceleration, and he returned to it in his Prague paper of 1911. His arguments, particularly in its 1911 form, must rank as one of the most remarkable in the history of science." The principle of equivalence is then applied to derive the gravitational redshift—for the first time in a beautifully pedagogical way (see Fig. 6). As Mark Twain writes: "The nice thing about Science is that one gets such wholesale returns of conjecture from such a trifling investment of fact"...

What is the present-day formulation of the (weak) equivalence principle? Employing Cliff Will's formulation from his Living Reviews article [19]:

- Test bodies fall with the same acceleration independently of their structure or composition.
- The outcome of any local non-gravitational experiment is independent of: (a) the velocity of the local inertial frame in which it is performed, (b) where and when in the universe it is performed.

From the time of Newton and Eötvös it has been a continuing effort to measure a possible violation of the first item. After the 1950s, it is connected with the names of Dicke in Princeton, Braginskij in Moscow and, most recently, with the group at the University of Washington which used a torsion balance tray to study the accelerations

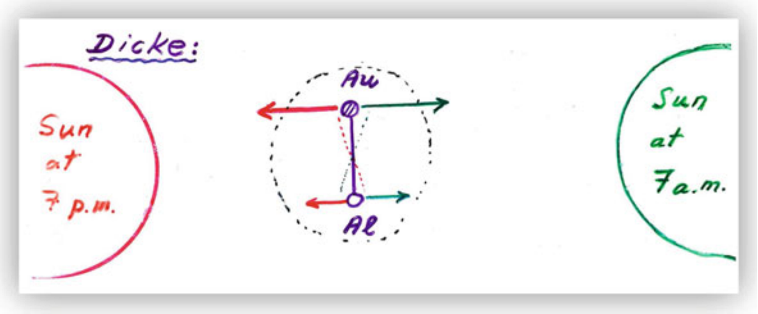


Fig. 7 Dicke used the field of the Sun to put the limits on difference in acceleration of various materials. (The suspension of the torsion balance is perpendicular to the picture)

of various materials toward local masses, towards the Sun (as Dicke did—see Fig. 7) and even towards the Galaxy.

The present best limits on the fractional difference in acceleration of various materials/bodies A and B are

$$\eta = \frac{a_A - a_B}{\frac{1}{2}\,(a_A + a_B)} \qquad \begin{cases} (0.3 \pm 1.8) \times 10^{-13}, & \text{Eöt-Wash} \\ (-1.0 \pm 1.4) \times 10^{-13}, & \text{LLR} \end{cases} \tag{1}$$

where the first result is given by experiments at the University of Washington, whereas the second comes from Lunar laser ranging. See [19] for more details.

An endeavor to improve the limit on η will undoubtedly continue since some theories, inspired primarily by string theory, predict the violation of the weak equivalence principle due to the presence of dilatons. That is why a *Satellite Test of Equivalence Principle* (*"STEP"*) has been conceived to improve the limit on η by about five orders of magnitude to $\eta = 10^{-17} - 10^{-18}$. It is a drag-free satellite consisting of an outer shell around an inner test mass [20].

6.1 Gravitational Redshift Today

Concerning the gravitational redshift, the first reliable experiment was the Pound-Rebka-Snider experiment in 1960 using the Mössbauer effect to measure precisely the frequency shift of γ-rays in a 22.6 m tower at Harvard. These experiments, yielding an accuracy of the order of 10 %, were followed by clocks placed in an aircraft and a rocket; the clock rates were compared with the same clocks on the ground. An accuracy of 7×10^{-5} in determining $\Delta\nu$ was obtained by using a hydrogen maser clock in the rocket by Vessot et al. in 1980. Only 30 years later this accuracy was claimed to have been improved upon by laboratory experiments based on quantum interference of atoms [21] which yielded an accuracy of 7×10^{-9}. However, the interpretation was criticized in the work [22] which considers the experiment as a test of the universality of free fall but not the redshift effect. An intriguing idea for producing and measuring a "force-free gravitational redshift" (a gravitational analogue of the Aharonov-Bohm effect) imagines two atomic matter waves which serve as clocks at different gravitational potentials and the redshift can be caused just by a potential difference although a force vanishes. This suggestion was published 100 years after Einstein was going to leave Prague, just two weeks before our conference in [23]. Therein the authors address also the criticism raised in [22]. Anyway, the best known "common" proof of gravitational redshift we currently have available is, of course, the daily successful operation of the Global Positioning System (GPS).

What is the present situation concerning the gravitational shift of the spectral lines from the surface of the Sun relative to the corresponding laboratory lines which, in his Prague paper [12], Einstein predicted to be $\Delta\lambda = 2.1 \times 10^{-6}\,\lambda$, which in the velocity scale is approximately 600 ms^{-1}? The measurements are complicated because there are various sources of wavelength shifts, such as radial currents different at different levels, convection, inner asymmetries of spectral lines etc. The first convincing measurements appeared only in 1991 by using the infrared oxygen triplet [24]. Converted to velocities, the three chromospheric oxygen lines yielded $627 \pm 10\ \mathrm{ms}^{-1}$ which is 0.99 ± 0.02 of the value predicted by the principle of equivalence. Precise measurements of the solar redshift are planned [25] which should reach an accuracy of 10^{-6} and so test the second order relativistic effects. For the most recent pioneering work on the gravitational redshift of galaxies in clusters, see [26].

7 Bending of Light

After the formulation of the principle of equivalence and its application in deriving the formula for redshift, Einstein turned in the same paper [14] to the problem of light propagation in a gravitational field. The equivalence of the systems K' and K in Fig. 6 implies the frequency shift, i.e., different rates of clocks located at different gravitational potentials Φ in the case of a *homogeneous* gravitational field. However, typically for his grasp of the laws of nature, Einstein assumes that the same effect takes place in a *general* static field.[11] As a consequence, the velocity of light, measured by clocks influenced by a gravitational potential, depends on the value of the potential. Denoting the velocity of light at origin of coordinates by c_0, then at a place in which gravitational potential with respect to the origin is Φ, the velocity of light is given by the relation $c = c_0(1 + \Phi/c^2)$. Employing a simple picture based on Huygens

---

[11] Einstein notices the effect of gravity on the light propagation in his paper from 1907 already in which, for the first time, the gravitational field and equivalence principle are mentioned. However, only a homogeneous field is analyzed and the effect considered in the field of the Earth is found to be unobservable.

principle, Einstein then derives the formula for an angle α by which a light ray will be deflected in the direction n' along any trajectory s:

$$\alpha = -\frac{1}{c^2} \int \frac{\partial \Phi}{\partial n'}\, ds. \tag{2}$$

According to this formula a ray propagating around a celestial body with mass M deflects towards the body by the value given in Fig. 8—the picture of the final page of Einstein's best known paper from the Prague period. Under the formula the resulting value of 0.83 arcseconds for a light ray passing close to the Sun is given, half of the value following from the final form of general relativity. Einstein promptly contacted various observatories in the world and tried to encourage observation of the effect during a total eclipse.

The only response came from Erwin Freundlich (1885–1965), an assistant in the Royal Observatory in Berlin.[12] Freundlich took part in a solar eclipse expedition in 1914 in Crimea but World War I broke and he was interned in Russia. It was only just after the war, in 1919, when the effect was measured for the first time by the British expeditions led by Eddington and Dyson. There is much literature on these observations, some of which attributed the confirmation of the relativistic value to Eddington's high regard of general relativity and also his wish, of a pacifist, to show how despite the war English astronomers confirm a theory by a German physicist. It is true, difficulties during the expeditions appeared, but in a recent article, D. Kennefick carefully re-analyzed the case and showed that Eddington et al. had good reasons for the claim that general relativity was confirmed, and Newtonian theory was not (see Fig. 9).

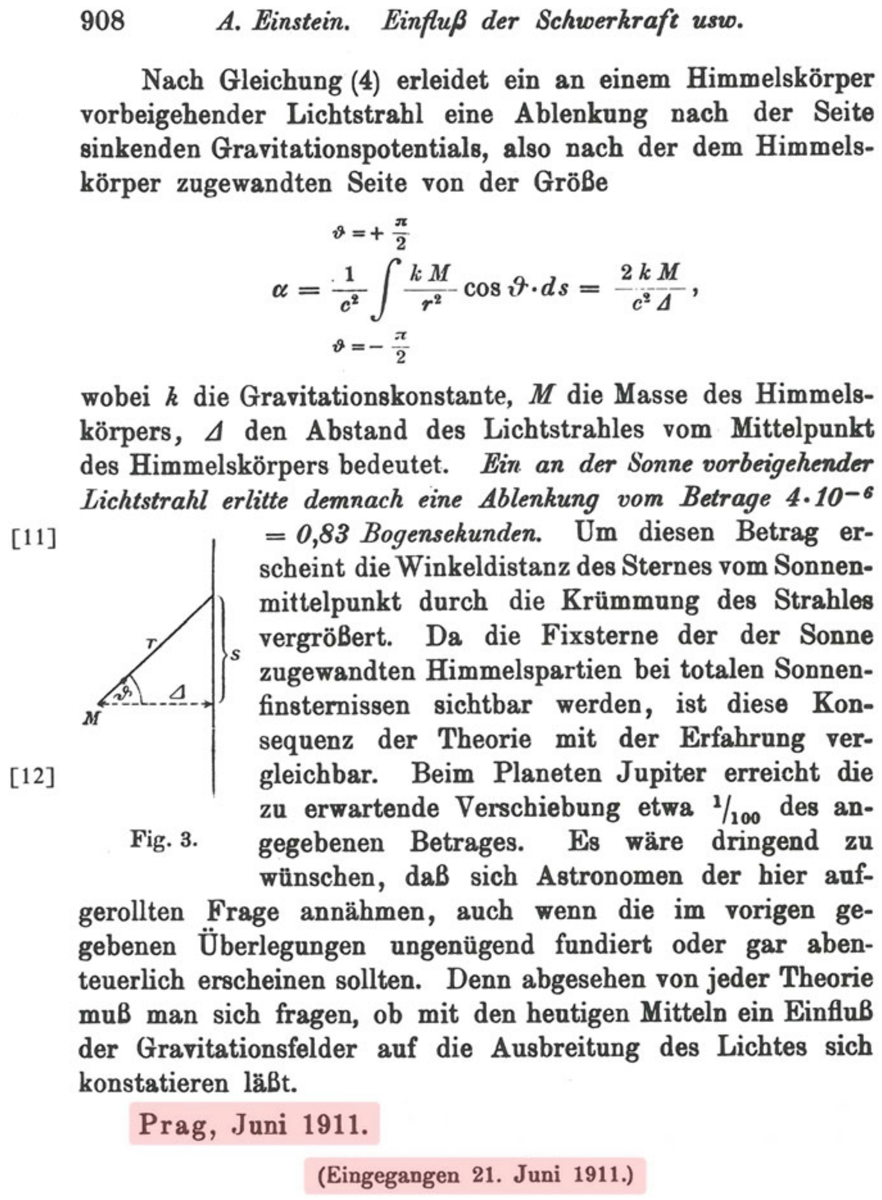
The light deflection is one of the finest examples of the continuing success of general relativity as the best gravity theory available. When one allows a more general form of a static spherically symmetric vacuum metric as it can follow from an alternative theory of gravity by inserting parameters β and γ into the Schwarzschild metric (see Fig. 10), one finds that it is γ which enters the formula for the deflection of waves.

If general relativity is correct, $\gamma = 1$. Let us have a look at Fig. 11 from Cliff Will's Living Reviews in Relativity article [19] on "The Confrontation between General Relativity and Experiment".[13] A 2004 analysis of ≈ 2 million VLBI observations of 541 radio sources at 87 VLBI sites imply $\gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$!

---

[12] For a detailed account on Freundlich and tests of relativity theory, see [27]. In fact, the first attempt to measure light deflection was already made during an eclipse in October 1912 by C. D. Perrine from Cordoba, Argentina, who was inspired by Freundlich. However, rain frustrated all efforts. (I thank Jorge Pullin for pointing this out in the discussion after my lecture in the conference.) Freundlich later wrote one of the first books on Einstein's gravitation theory (Springer 1916, Cambridge 1920). From the "Prague perspective" it is interesting to notice that in 1937 he was appointed professor of astronomy at the Charles University in Prague, however, was forced to leave because of Hitler's policies towards Czechoslovakia in January 1939.

[13] Will's reviews on the verification of Einstein's theory are well known. Perhaps less recognized is his "Resource Letter" on the tests of gravity [28] which gives many references on both current literature as well on some of historical papers.



908 *A. Einstein. Einfluß der Schwerkraft usw.*

Nach Gleichung (4) erleidet ein an einem Himmelskörper vorbeigehender Lichtstrahl eine Ablenkung nach der Seite sinkenden Gravitationspotentials, also nach der dem Himmelskörper zugewandten Seite von der Größe

$$\alpha = \frac{1}{c^2}\int_{\vartheta=-\frac{\pi}{2}}^{\vartheta=+\frac{\pi}{2}} \frac{k\,M}{r^2}\cos\vartheta\cdot ds = \frac{2\,k\,M}{c^2\,\Delta},$$

wobei *k* die Gravitationskonstante, *M* die Masse des Himmelskörpers, *Δ* den Abstand des Lichtstrahles vom Mittelpunkt des Himmelskörpers bedeutet. *Ein an der Sonne vorbeigehender Lichtstrahl erlitte demnach eine Ablenkung vom Betrage* $4\cdot 10^{-6}$
[11] *= 0,83 Bogensekunden.* Um diesen Betrag erscheint die Winkeldistanz des Sternes vom Sonnenmittelpunkt durch die Krümmung des Strahles vergrößert. Da die Fixsterne der der Sonne zugewandten Himmelspartien bei totalen Sonnenfinsternissen sichtbar werden, ist diese Konsequenz der Theorie mit der Erfahrung ver-
[12] gleichbar. Beim Planeten Jupiter erreicht die zu erwartende Verschiebung etwa $^1/_{100}$ des angegebenen Betrages. Es wäre dringend zu wünschen, daß sich Astronomen der hier aufgerollten Frage annähmen, auch wenn die im vorigen gegebenen Überlegungen ungenügend fundiert oder gar abenteuerlich erscheinen sollten. Denn abgesehen von jeder Theorie muß man sich fragen, ob mit den heutigen Mitteln ein Einfluß der Gravitationsfelder auf die Ausbreitung des Lichtes sich konstatieren läßt.

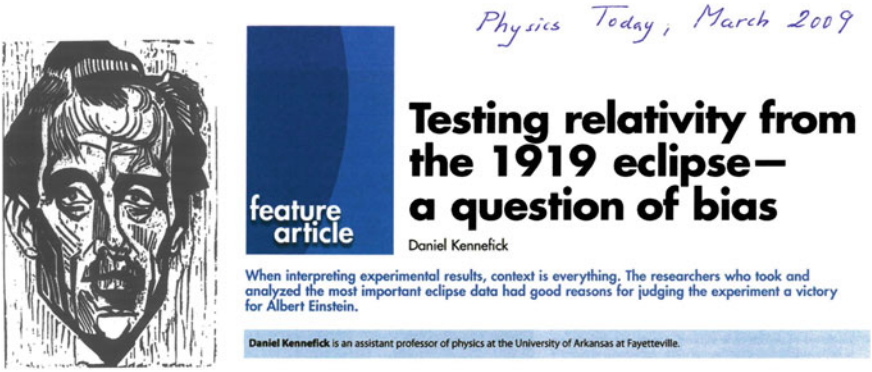
Fig. 3.

Prag, Juni 1911.

(Eingegangen 21. Juni 1911.)

Fig. 8 The last page of Einstein's most famous Prague paper on deflection of light

For example, scalar-tensor theories must have parameter $\omega > 40000$ (which their proponents like R. Dicke considered to be around 6), to be compatible with the light deflection observations.



Physics Today, March 2009

feature article

Testing relativity from the 1919 eclipse—a question of bias

Daniel Kennefick

When interpreting experimental results, context is everything. The researchers who took and analyzed the most important eclipse data had good reasons for judging the experiment a victory for Albert Einstein.

Daniel Kennefick is an assistant professor of physics at the University of Arkansas at Fayetteville.

Fig. 9 Portrait of Erwin Freundlich by Max Pechstein from 1918. The heading of a recent article which showed undoubtedly that Eddington's and Dyson's expeditions of 1919 to measure light bending, despite difficulties they encountered, could state safely that their observations ruled out Newton theory and confirmed general relativity

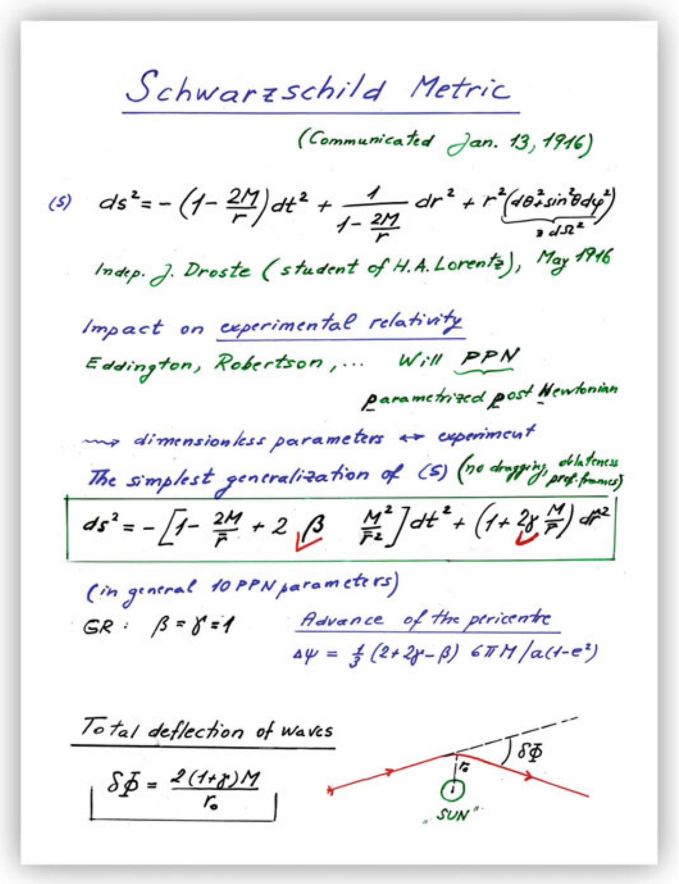
A fundamental consequence of the influence of gravity on the light propagation is the existence of black holes formed by gravitational collapse. I dare to state this commonly known fact only because I recall how in Erice in 1974 Roger Penrose drew a picture similar to Fig. 12. He only "forgot" to write "Prague 1911" beneath the left picture.

8 Gravitational Lensing

In contemporary astronomy, nowhere is light deflection so extensively and profitably used as in the phenomenon of gravitational lensing. Going now from the present to the past we may look at just two well-known examples: the images of two galaxy clusters after collision (Fig. 13) indicating the existence of dark matter in the clusters; it moves further after the collision (its distribution, shown by green counters, being known thanks to gravitational weak-lensing) than ionized gas (yellow-red in the right part observed in X-rays); and a Horseshoe Einstein Ring from Hubble telescope (Fig. 14).

Here I wish to make just a few remarks on the history of discovering the effect, in which Prague played a significant role, unrecognized generally yet. It was long thought that the effect was first described by Einstein in a paper published in *Science* on December 4, 1936, after his interaction with a Czech amateur scientist Rudi Mandl. Later Einstein's Scratch Notebook from 1912 (when he was still in Prague) was found and it became evident that the effect was found by him then already. The complete history of this finding and its relation to the 1936 paper is comprehensively described in several publications by Renn and Sauer; see, for example, [30].

It is well-known that the effect was also discussed by Eddington in 1920 and by Chwolson in 1924. However, the role of a Czech astronomer, an expert in the eclipse phenomena of the Moon and planets, František Link (1906–1984), remains



Schwarzschild Metric

(Communicated Jan. 13, 1916)

$$(S)\quad ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{1}{1-\frac{2M}{r}}dr^2 + r^2\underbrace{(d\theta^2 + \sin^2\theta\, d\varphi^2)}_{\equiv d\Omega^2}$$

Indep. J. Droste (student of H.A. Lorentz), May 1916

Impact on experimental relativity

Eddington, Robertson, … Will PPN

parametrized post Newtonian

⇝ dimensionless parameters ↔ experiment

The simplest generalization of (S) (no dragging, oblateness, pref. frames)

$$\boxed{ds^2 = -\left[1-\frac{2M}{\bar r} + 2\beta\,\frac{M^2}{\bar r^2}\right]dt^2 + \left(1+2\gamma\frac{M}{\bar r}\right)d\bar r^2}$$

(in general 10 PPN parameters)

GR : $\beta = \gamma = 1$

Advance of the pericentre

$$\Delta\psi = \tfrac{1}{3}(2+2\gamma-\beta)\,6\pi M/a(1-e^2)$$

Total deflection of waves

$$\delta\Phi = \frac{2(1+\gamma)M}{r_0}$$

Fig. 10 The generalization of the Schwarzschild metric to include alternative theories of gravity

apparently unknown outside his country.[14] In Fig. 15 it is seen that Link published his first paper on using Einstein's bending in photometry in Comptes Rendus in March 1936, more than eight months before Einstein's paper appeared in *Science*. In this paper already, his expression for $\rho_\infty = \sqrt{Kk\alpha_1}$ corresponds to the angular size of the Einstein ring, usually denoted as Θ_E. A much more detailed work on the "Photometric consequences of the Einstein deflection" by Link appeared in Bulletin Astronomique in 1937. In that time Link did not notice Einstein's paper in *Science*

---

[14] On the occasion of the centenary of Link's birth there was a seminar organized by astronomers and historians of science at Charles University on November 29, 2006, where gravitational lensing was extensively discussed and the role of Link, the first director of the Astronomical Institute of the Academy, recalled. Incidentally, after my talk was prepared, including the part on lensing and Link, a preprint appeared in the arXiv:1206.1165v1 [physics.hist-ph] by D. Valls-Gabaud in which the pioneering role of Link in the origins of gravitational lensing is emphasized.

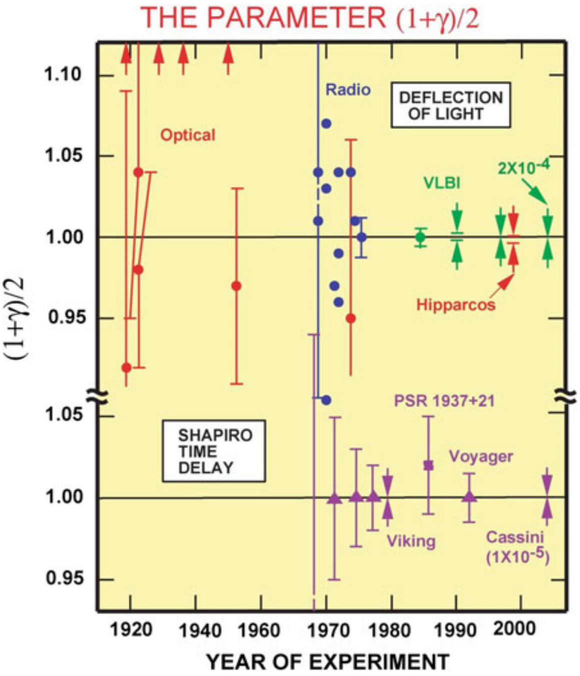


Fig. 11 Limits on parameter γ from measurements of the light deflection. $\gamma = 1$ in general relativity. Taken from [19]

but he gave a very detailed account of the history of lensing and many mathematical details later in his monograph [31].

9 Prague Works on Gravitation from 1912

These are four papers: two on the static gravitational field, [15, 16], including non-linear field equations and the motion of test particles in a given field, the first work on a new effect–on linear gravitational dragging [17], and last but not least, the reply to a comment by Abraham [18] in which Einstein outlines the main features a future theory of gravity should possess.

In the first paper, an inertial frame $\mathscr{I}_0$ is first considered and a uniformly accelerated frame $\mathscr{K}$ with an acceleration a with respect to $\mathscr{I}_0$ (Fig. 16). Denoting coordinates in these frames by $\mathscr{I}_0(\tau, \xi, \eta, \zeta)$ and $\mathscr{K}(t, x, y, z)$, Einstein assumed the following approximate form of the transformation between the frames: $\xi = \lambda(x) + \alpha(x)t^2 + \mathscr{O}\left(t^3\right)$, $\tau = \beta(x) + \gamma(x)t + \delta(x)t^2 + \mathscr{O}\left(t^3\right)$, $\eta = y$, $\quad \zeta = z$, where functions $\lambda(x), \alpha(x), \ldots$ are to be determined as follows. First assume that at $t = 0$ the origins coincide, $\xi = 0,\ x = 0$. Consider the line elements in $\mathscr{I}_0$ and $\mathscr{K}$ to have the form

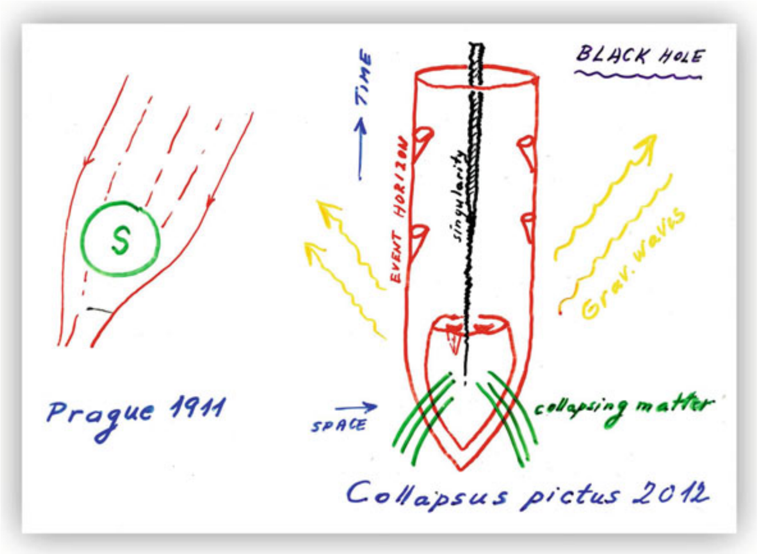


Fig. 12 In the strong-field case matter can act on light so that a black hole is formed. This is indicated in the spacetime diagram of gravitational collapse on the right

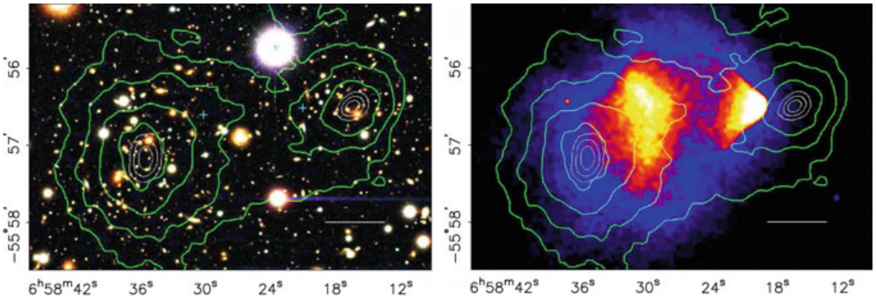


Fig. 13 "The bullet cluster"—the collision of two galaxy clusters (1E0657–558) provides the best current evidence for the nature of dark matter (taken from [29]). See the text for details

$$\mathrm{d}s^2_{\mathscr{I}} = -c_*^2 \mathrm{d}\tau^2 + \mathrm{d}\xi^2 + \mathrm{d}\eta^2 + \mathrm{d}\zeta^2\,, \quad c_* = 1$$
$$\mathrm{d}s^2_{\mathscr{K}} = -c^2(x)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2.$$

Requiring then that $\mathrm{d}s^2_{\mathscr{I}_0} = 0 \;\Leftrightarrow\; \mathrm{d}s^2_{\mathscr{K}} = 0$, one finds functions $\lambda(x), \alpha(x), \ldots$ in the transformation and arrives at the following final form of the transformation between the inertial and uniformly accelerated frames:

$$\xi = x + \tfrac{1}{2}\,act^2,\ \tau = ct,\ c = c_0 + ax. \tag{3}$$

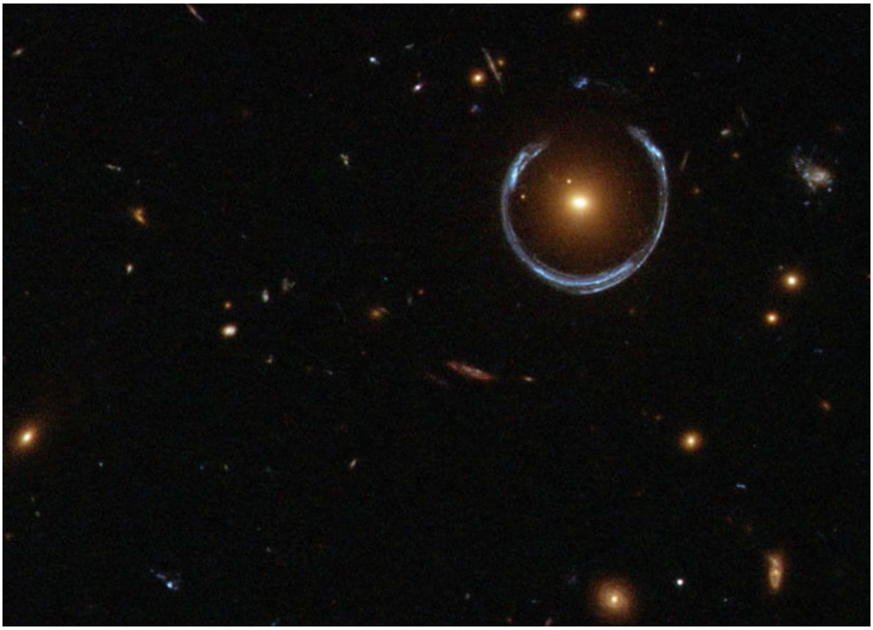
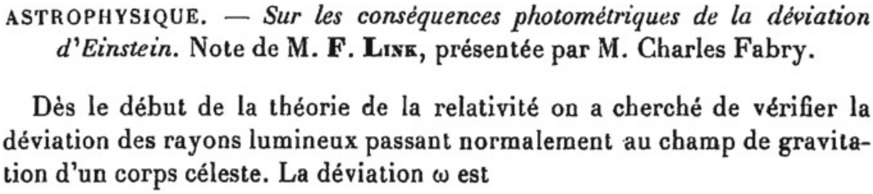


Fig. 14 A Horseshoe Einstein Ring (Credit: ESA / Hubble & NASA)



ASTROPHYSIQUE. — *Sur les conséquences photométriques de la déviation d'Einstein.* Note de M. **F. Link**, présentée par M. Charles Fabry.

Dès le début de la théorie de la relativité on a cherché de vérifier la déviation des rayons lumineux passant normalement au champ de gravitation d'un corps céleste. La déviation ω est

Fig. 15 The first publication on gravitational lensing: Link F., Comptes Rendus **202** (16 Mar 1936), 917–919. Notice that the paper was communicated by C. Fabry

9.1 *Intermezzo*

Today we know that the transformation to the rigid uniformly accelerated ("Rindler") frame reads

$$\xi = \frac{1}{a}\,(\cosh at - 1) + x \cosh at, \tag{4}$$

$$\tau = \frac{1}{a} \sinh at + x \sinh at. \tag{5}$$

For small t (neglecting $\mathcal{O}\left(t^3\right)$) Einstein's "Prague transformation" above immediately follows from these two relations. The spacetime orbits of uniformly accelerated particles are hyperbolas; hyperbolas (timelike at $z^2 > t^2$, spacelike at $z^2 < t^2$) are

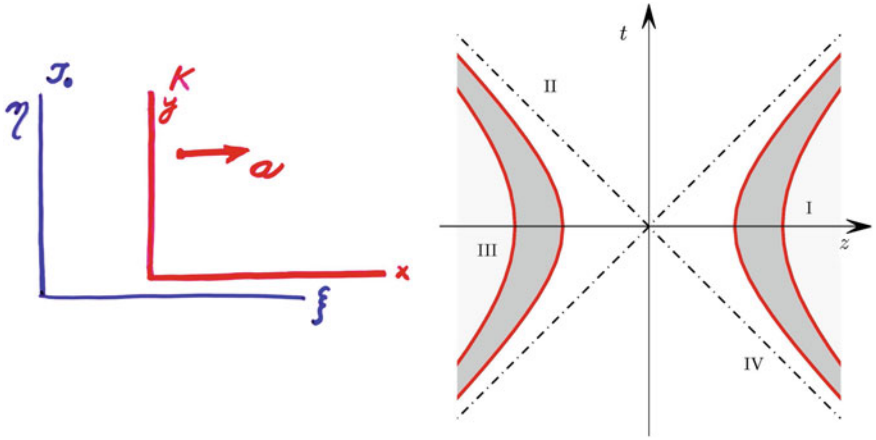


Fig. 16 The system $\mathscr{K}$, considered by Einstein in [15], is "uniformly accelerated" (in a relativistic sense) with respect to inertial frame $\mathscr{I}_0$; and a schematic spacetime diagram of the C-metric representing two *black* holes uniformly accelerated in opposite directions

orbits of the boost Killing vector. Spacetimes with axial/rotational symmetry and boost symmetry, so called boost-rotation symmetric spacetimes, play an important role in general relativity: they are radiative, with a non-vanishing news function, have a plausible Newtonian limit, admit global null infinity; they have been used as test-beds in numerical relativity. One of the best known examples is the C-metric, representing two uniformly accelerated black holes in opposite directions—a schematic diagram is on the right in Fig. 16. In the Rindler-type coordinates (which can be introduced in the quadrants in which sources occur and the boost Killing vector is timelike) the spacetime is static but in the other two quadrants it is dynamical (locally of the Einstein-Rosen type).[15]

Returning back to 1912, we notice that the velocity of light in a uniformly accelerated frame, Eq. 3, satisfies Laplace's equation. Now invoking the equivalence principle, Einstein postulates that the field equation even for a *general* static gravitational field will read

$$\Delta c = 0 \;\; \text{(in vacuum)}, \qquad \Delta c = kc\rho \;\; \text{(in matter)}.$$

However, he soon realized that one gets contradictions with conservation of energy and momentum ($\int \mathbf{f}\, \mathrm{d}V \neq 0$, with $\mathbf{f} = -\rho \operatorname{grad} c$). Hence, in a shortly published following paper [16] he avoided the inconsistency by modifying the field equation into

---

[15] I dared to make this intermezzo since our group in Prague has been devoting quite an effort to understand these spacetimes (see, e.g., contribution by Bičák and Kofroň in the Proceedings [11] and the literature quoted therein).

$$\Delta c = k\left[c\rho + \frac{1}{2k}\frac{\mathrm{grad}^2 c}{c}\right].$$

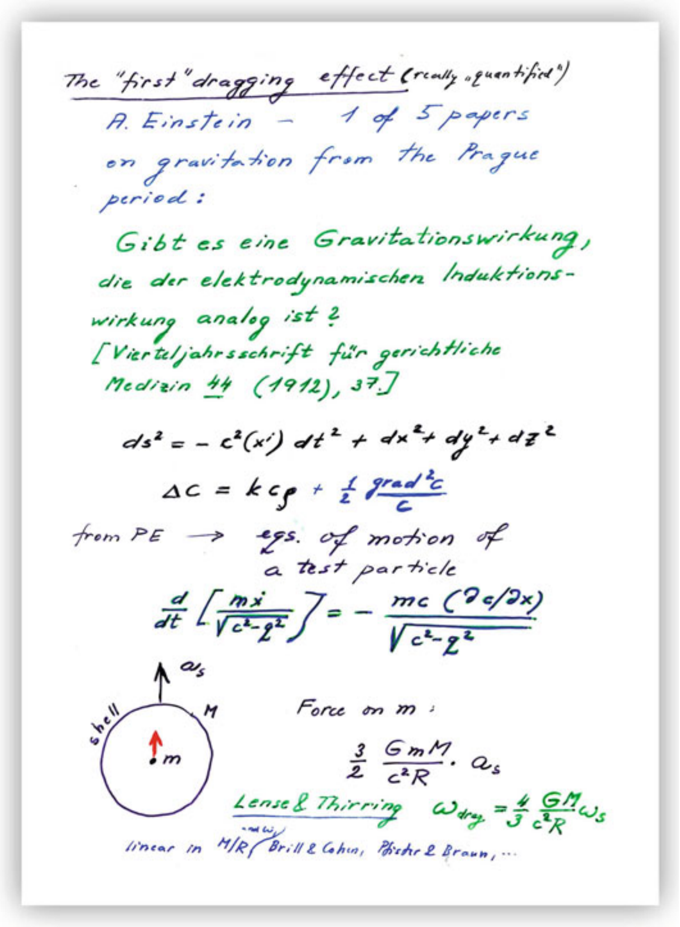
The field equation becomes *nonlinear*; and the additional term on the right-hand side describes the energy density of the gravitational field which thus itself contributes as a source to the field. (An interesting discussion of this theory of gravity from a purely Newtonian point of view was presented in this conference by Domenico Giulini—see [11].) In the same paper, a local ("pocket") temperature is introduced and shown to depend on c, i.e., on a local gravitational potential; and in the Appendix the equations of motion for a test particle in a given gravitational field are derived from the variational/Hamilton principle–the foreshadow of a variational principle for geodesics. Still more remarkably, Einstein introduces, for the first time, a "local view" on the equivalence principle since the nonlinear field equation above is not satisfied by $c = c_0 + ax$. The local transformation between the two frames in a small neighborhood is generalized to

$$\xi = x + \frac{1}{2}c\frac{\mathrm{d}c}{\mathrm{d}x}t^2, \quad c(x) \text{ arbitrary.}$$

9.2 Dragging of Inertial Frames

The following paper from the Prague period is not directly connected to a construction of a new theory of gravity. However, it fits well to the Prague physics since it is related to Ernst Mach–"the fact is that Mach exercised a great influence upon our generation through his historical-critical writings... ", wrote Einstein in 1916. By using equations of motion derived in the preceding paper, Einstein considers a shell of matter and its influence on a mass point placed in its center; as the shell starts to accelerate, a force on the point mass appears and its inertial mass increases (see Fig. 17).

In the "Machian spirit" Einstein puts forward an idea that perhaps the inertia of a point mass is *fully* determined by the action of all other masses. We know today that this does *not* arise in general relativity, however, it is so that *inertial frames* are, under suitable conditions, determined by averages over matter distributions. However, Einstein's considerations from Prague show, for the first time, that an inertial frame will be dragged by moving masses. The effect was soon exhibited also for rotating masses by Einstein and Thirring and has been studied extensively until the present, including recently the dragging effects due to gravitational waves (see contributions by Pfister, by Lynden-Bell and Katz, and by Bičák, Katz, Ledvinka and Lynden-Bell in the Proceedings [11] of this conference).



The "first" dragging effect (really "quantified")

A. Einstein – 1 of 5 papers on gravitation from the Prague period:

Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?
[Vierteljahrsschrift für gerichtliche Medizin 44 (1912), 37.]

$$ds^2 = -c^2(x^i)\,dt^2 + dx^2 + dy^2 + dz^2$$

$$\Delta c = k c \rho + \frac{1}{2}\frac{\mathrm{grad}^2 c}{c}$$

from PE $\longrightarrow$ eqs. of motion of a test particle

$$\frac{d}{dt}\left[\frac{m\dot{x}}{\sqrt{c^2-q^2}}\right] = -\frac{mc\,(\partial c/\partial x)}{\sqrt{c^2-q^2}}$$

Force on m:

$$\frac{3}{2}\frac{GmM}{c^2R}\cdot a_s$$

Lense & Thirring $\omega_{drag} = \frac{4}{3}\frac{GM}{c^2R}\omega_s$

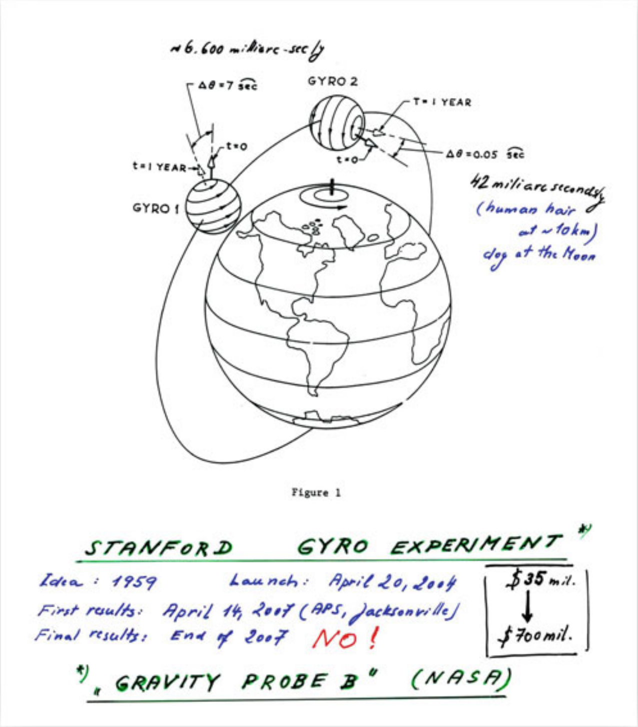
linear in M/R (and ω_s) Brill & Cohen, Pfister & Braun, …

Fig. 17 From his "Prague theory of gravity" Einstein discovered a (linear) dragging effect

The planning and eventually launching of the Stanford gyroscopic experiment, later called *Gravity Probe B*, started in 1959 and ended in 2011.[16] The spacecraft with gyros were orbiting the Earth during a mission that ran from April 2004 to September 2005; however, many simulations had to be performed after the measurements finished. There were eventually four superconducting gyros in the satellite and both the geodetic effect (cf. the "left" gyroscope in Fig. 18) and (much smaller) dragging effect due to the rotation of the Earth were measured. In an article published in Physical Review Letters from May 31, 2011, "the authors reported analysis

---

[16] It started thus contemporaneously with my studies at the Faculty of Mathematics and Physics in Prague and closed close to my retirement; but I enjoyed this long time span because the experiment is interesting and well explainable to a general public.



42 miliarc seconds/y
(human hair at ~10km)
dog at the Moon

STANFORD GYRO EXPERIMENT *)

Idea: 1959 Launch: April 20, 2004
First results: April 14, 2007 (APS, Jacksonville)
Final results: End of 2007 NO!

\$35 mil. ↓ \$700 mil.

*) „GRAVITY PROBE B" (NASA)

Fig. 18 Stanford gyro experiment–“Gravity Probe B”–measured both a geodetic precession due to curvature around Earth (gyro 1) and a dragging effect due to the rotation of Earth (gyro 2)

of the data from all four gyroscopes results in a geodetic drift rate of -6601.8 ± 18.3 milliarcsecond/year ($\equiv$ mas/yr; 1 mas = 4.84×10^{-9} rad) and a frame-dragging drift rate of -37.2 ± 7.2 mas/yr, to be compared with the GR predictions of -6606.1 mas/yr and -39.2 mas/yr, respectively”. An extensive literature on science, technology, sociology and politics associated with the experiment is available and interesting to read (see, e.g., http://physics.aps.org/articles/v4/43). Principally, however, despite the doubts about the initial results (burdened by some unexpected phenomena like, for example, random patches of electrostatic potentials on both gyros and their housing) it is now firmly established that dragging effects, or as many relativists like to say—gravitomagnetic effects, were measured and so general relativity is confirmed by using stationary rather than just static gravitational fields. Another confirmation of the dragging (Lense-Thirring) effect came from the laser tracking of two Earth-orbiting LAGEOS satellites, analyzed in a number of papers by I. Ciufolini and his collaborators.

9.3 *Relativität and Gravitation. Erwiderung...*

In his last paper from the Prague period "Relativity and Gravitation. Reply to a Comment by M. Abraham" received by *Annalen der Physik* on July 4, 1912, just a few days before he left Prague, Einstein was inspired by Abraham's criticism to summarize the views on the contemporary state of the theory of relativity and its description of gravity. From Einstein's summary it is evident that he learned many fundamental issues during his Prague stay. This was also the reason we decided to give the name to the conference following this work. The main points which Einstein expressed in his summary are as follows:

- Local significance of equivalence principle.
- Equations of motion for point masses (variational principle).
- Equations of the electromagnetic field when gravity is present.
- Nonlinear field equation for gravity (energy density of gravitational field itself as a source).
- All equations must be form invariant with respect to a larger group than the Lorentz group.
- *Spacetime coordinates lose their simple physical meaning*.

Let us quote the last point in more detail, in Einstein's own words: "Man sieht schon aus dem bisher behandelten, daß die Raum-Zeit-Koordinaten ihre einfache physikalische Deutung einbüßen werden, und es ist noch nicht abzusehen, welche Form die allgemeinen raumzeitlichen Transformationsgleichungen haben könnten..."

In Prague it was thus for the first time that Einstein realized that spacetime coordinates need not determine the distances between spacetime points. Later, in his *Autobiographical Notes*,[17] he comments "Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that coordinates must have an immediate metrical meaning". At the end of his Prague stay, Einstein was on the right track towards general relativity. Important features of the new theory were understood, but gravitation was described wholly by one function only, the variable velocity of light.

As we discussed above, Einstein might have learned about Riemannian geometry in Prague already from Georg Pick, however, this does not seem to be of great importance since, in any case, a few months after he left Prague for Zurich, he started to collaborate with Marcel Grossmann and, with some exaggeration perhaps, one may say that they had general relativity almost in hand:

---

[17] In Albert Einstein-Philospher and Scientist, The Library of the Living Philosphers Vol. VII, ed. P. A. Schild, Open Court Publ. Co. 1949.

- Gravitation described by ten components of the metric tensor $g_{\mu\nu}$.
- Correct variational principle for the geodesic equation.
- Maxwell equations in general gravitational field covariant under general coordinate transformations.
- The source of gravity $T^{\mu\nu}$ covariantly conserved, $\nabla_\nu T^{\mu\nu} = 0$.
- The field equations $\Gamma_{\mu\nu} = \kappa T_{\mu\nu}$ where $\Gamma_{\mu\nu}$ is formed from the metric tensor and its derivatives.

Although in 1913 they first considered the Ricci tensor at the left-hand side of the field equations (on the right-hand side of which was a symmetrical energy-momentum tensor of matter $T^{\mu\nu}$), they wrongly concluded that they do not get a correct Newtonian limit, i.e., in vacuum the Laplace equation for the Newtonian potential.[18] And Einstein went astray from his route to a correct theory by assuming that the field equations, in order to give a meaningful Newtonian limit, cannot be generally covariant... The world had to wait another three years to see the equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}$. About their Genesis, see the monumental survey quoted in [32].

9.4 Coda

On Saturday, May 15, 2004, the Prague daily newspaper “Lidové Noviny” published an article “Einstein through the eyes of Johanna from Czechia”; see Fig. 19. It was just in those days when Princeton University published the diary of Johanna Fanta, the daughter-in-law of Berta Fanta whose salon Einstein liked to visit during his stay in Prague in 1911–1912. Einstein and Johanna knew each other from Einstein’s Berlin period already, but became quite close friends, spending often time on the Lake Carnegie in a boat, when Einstein was in Princeton and Johanna was employed in the library of the University. During the last two years Johanna made regular notes of what Einstein said or wrote. This is most interesting, often touching and sad reading. Let us quote the notes from just three days. April 13, 1954: “Expresses annoyance at Oppenheimer for letting the McCarthy and Atomic Energy Commission affairs bother him so much. Already told the press that he has great respect for Oppenheimer, both as a human being and as a scientist.” October 24, 1954: “He calculated like crazy again today but accomplished nothing.” April 10, 1955, 8 days before Einstein’s death: “He tried all day to compose a radio message on behalf of Israel and did not succeed in finishing it. He claims he is totally stupid—that he has always thought so, and that only once in a while was he able to accomplish something.”

In a rather poor, hilly part of south-western Moravia, close to the place where Gustav Mahler was born, there lived various “country-type” sages interested in literature, art, music, religion, and also science. In the left side of the last Fig. 20 there is the

---

[18] For a detailed analysis of the “Zurich notebook” which Einstein began to write soon after leaving Prague, see [32].

20 | Sobota 15. května 2004 | LIDOVÉ NOVINY

Zajímavosti týdne

Einstein očima Johanny z Čech

V těchto dnech by Princetonská univerzita měla začít plnit to, co před měsícem slíbila: odstartovat sérii článků o vztahu mezi Albertem Einsteinem a Johannou Fantovou. Jedním z podkladů jsou i nedávno objevené zápisky, které si fyzikova přítelkyně vedla v posledních dvou letech jeho života.

Marie Homolová

Ty zápisky se našly šťastnou náhodou. Princetonská univerzita se rozhodla připravit seriál o slavných párech, které se vyskytovaly v její historii. A protože se vědělo, že Einstein, který za amerického pobytu v Princetonu působil, a Fantová, která tam pracovala v knihovně, měli k sobě blízko, otevřeli autoři seriálu letos v únoru její dokumentaci. Kromě jiného tu ke svému překvapení našli i deník, obsahující na 62 stránkách více než dvě stovky záznamů.

UDÁLOST TÝDNE

„V 90. letech se mě řada lidí dotazovala, zda existuje,“ říká Don Skemer, kurátor zdejší sbírky rukopisů. „Odkazoval jsem je na experty a sbírky po celých Stá-

„Chtěla jsem k tomu, jak Einsteina chápeme, přidat nový pohled,“ napsala Fantová v úvodu. „Nezachytit ho jako žijící legendu ani jako věhlasného vědce, ale jako člověka.“

Z Prahy do Princetonu

Johanna Fantová se narodila roku 1901 v severních Čechách jako Johanna Bobaschová. Její muž Otto byl synem manželů Fantových, kteří před první světovou válkou vedli v pražském domě U bílého jednorožce slavný salon, kde se scházel pražský kulturní svět. Mezi hosty se objevoval i Franz Kafka, Max Brod a... Albert Einstein, který tehdy v letech 1911 a 1912 přednášel v Praze na německé univerzitě fyziku.

Johanna, které tehdy bylo deset a běhala na zcela jiné adrese, se s Einsteinem sešla až v roce 1929 v Berlíně, kde jí nabídl, aby mu uspořádala jeho rozsáhlou, ale chaotickou knihovnu. Ačkoliv je dělilo dvaadvacet let věku, velmi se spřátelili, a nakonec ji přizval i ke své nejmilejší zábavě, jízdě na plachetnici. Když v roce 1939 odjela do Spojených států, znovu se k Einsteinovi, který tam už několik let žil v emigraci, přihlásila.

Einstein už měl za sebou dvě manželství. V roce 1902 se oženil se srbskou studentkou fyziky Milevou, měli spolu dva syny, ale časem se rozvedli. Druhá manželka, sestřenice Elsa, ho doprovázela i do Ameriky, kam emigroval

Albert Einstein a Johanna Fantová spolu v Americe rádi jachtařili na Lake Carnegie Repro LN

týdně spolu dlouze telefonovali, ona mu stříhala vlasy, on jí psal básně, obnovili i příjemné chvíle na pla-

„Unavený z dlouhého ticha chci ti tímto jasně ukázat, jak silné myšlenky na tebe budou vždy sídlit v jedné

že všichni jsme malé Země, ale každý si myslí, že je jejich středem...

A tak dále.

mera, který vedl americký program vývoje atomové bomby. Když se Oppenheimer, kterého si Einstein velmi považoval, ocitl před McCarthyovým Výborem pro vyšetřování neamerické činnosti, Fantová si zaznamenala, že se to u nich stalo častým předmětem konverzace. „Politická perzekuce jeho přítele byla pro něj hořkým vystřízlivěním,“ zapsala si.

Soucítil s Oppenheimerovým zklamáním, ale o sobě říkal, že se „narodil se sloní kůží“ a nikdo mu nemůže ublížit. „Každá kritika po mně steče jako voda.“

Přesto si nikdy nepřestal vyčítat, že je svým způsobem odpovědný za tragédie způsobené atomovou bombou. Nejen proto, že k bombě dal jistý podnět svými vědeckými závěry, ale především za druhé světové války napsal prezidentu Rooseweltovi dopis, ve kterém ho varoval, že Němci jsou blízko k sestrojení a je důležité předběhnout je. Ten dopis byl jedním z impulsů, které způsobily, že se americký projekt opravdu rozjel na plné obrátky. Odpovědnost za to, co následovalo, „ho velmi stresovala“, zapsala si Fantová.

Zápisy dosvědčují, jak se trápil vývojem světa i tím, že už ho nemůže ovlivnit. Sovětům vrátil mírovou cenu, nechtěl nést nálepku bolševika. A zároveň odmítl varovat před preventivní válkou s Čínou, protože věděl, že by jeho slova padla do prázdna. Zlobil se, že jméno Einstein „zna-

Fig. 19 An article about Einstein and his friend Johanna Fanta in Lidové noviny, May 2004

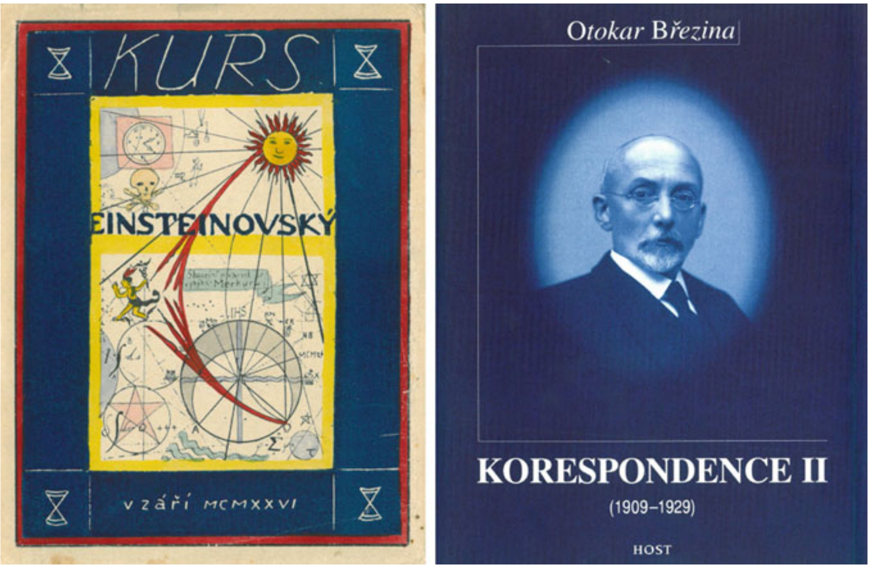


Fig. 20 Covers of two books: Einstein's course published in Moravian village Stará Říše in September 1926, containing the translation of parts of Eddington's popular book *Space, Time and Gravitation* and the correspondence of Otokar Březina, poet and essayist, who created almost all of his works in a nearby village Nová Říše

cover of the “Einsteinian course” published in September 1926 by Josef Florian in the village Stará Říše. It contains a very fine translation of Eddington’s famous book with conversation between an experimental physicist, a pure mathematician. and a relativist who advocates the newer conceptions of time and space in physics. On the cover one can see symbols of Noah, Moses, Jesus, etc but, at the left low corner, there is also the correct expression for the variational principle for geodesics! On the right side there is the cover of the second volume of the correspondence of Otokar Březina, the top poet of Czech symbolism who lived and taught in a neighboring village Nová Říše (cf. Wikipedia). In his correspondence he mentions Planck’s quantum theory, Bohr, Minkowski and Einstein in the 1920s. In a letter to the professor of classical literature, František Novotný in Brno, who sent him his monumental work about Plato, Březina wrote encouraging words which I used both at the end of my talk in June and in [5] on the occasion of our Einstein centenary celebrations in 1979 in Karolinum as the present conference took place:

“There will always be minds who, by the united power of knowledge and dreams, science and poetry, will strive for a unified picture of all phenomena in the Universe, an image that in equal measure corresponds both to the eternal longing of the human spirit for harmony and beauty and to the thirst of the heart for justice.” Is not then the example of Albert Einstein a reason for an enduring optimism?

Acknowledgments I am grateful to David Kofroň for the help with the manuscript, especially with the figures, and for his suggestions improving the text. I also thank the Albert Einstein Institute, Golm, where most of this text was written after the conference, for the kind hospitality, Lars Andersson for comments and Ian Hinder for correcting some of my Czechisms. The partial support from the Czech Science Foundation No 14-37086G (A. Einstein Center) is also acknowledged.

References

1. Kleinert, A.: Anton Lampa und Albert Einstein-Die Neubesetzung der physikalischen Lehrstühle an der deutschen Univesrität Prag 1909 und 1910. Gesnerus **32**, 285 (1975)
2. Havas, P.: Einstein, relativity and gravitation research in Vienna before 1938. In: Goenner, H., Renn, J., Ritter, J., Sauer, T. (eds.) The Expanding Worlds of General Relativity, pp. 161–206. Birkhauser, Boston-Basel-Berlin (1999)
3. Frank, P.: Einstein, his Life and Times. Alfred A. Knopf, New York (1947)
4. Pais, A.: ‘Subtle is the Lord..’ The Science and the Life of Albert Einstein. Clarendon Press, Oxford (1982)
5. Bičák, J.: Einstein a Praha (in Czech and German). Prometeus, Prague (1979)
6. Bičák, J.: Einstein’s route to the general theory of relativity (in Czech). Československý Časopis pro Fyziku **A29**, 222 (1979)
7. Bičák, J.: Einstein’s Prague articles on gravitation. In: Blair, D.G., Buckingham, M.J. (eds.) Proceedings of the 5th M. Grossmann Meeting on General Relativity, pp. 1325–1334. World Scientific, Singapore (1989)

8. Bičák, J.: Einstein's Prague ideas on gravitation: the history and the present. In: Janta, J., Pantoflíček, J. (eds.) Trends in Physics, pp. 65–75. Union of Czechoslovak Mathematics and Physics, Prometheus, Prague (1985)
9. Těšínská, E.: Theoretical physics at the Prague University and its connections with Prague stay of A. Einstein 100 years ago (in Czech), Pokroky Matematiky, Fyziky a Astronomie **57**, 146–168 (2012)
10. Semerák, O., Podolský, J., Žofka, M. (eds.): Gravitation: Following the Prague Inspiration. World Scientific, New Jersey (2002)
11. Bičák, J., Ledvinka, T. (eds.): Relativity and Gravitation: 100 Years After Einstein in Prague. Springer, Switzerland (2014)
12. Einstein, A.: Die Relativitäts-Theorie. Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich **56**, 1 (1911)
13. Einstein, A.: Zum Ehrenfestschen Paradoxon. Bemerkung zu V. Varičaks Aufsatz. Phys. Z. **12**, 509 (1911)
14. Einstein, A.: Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes. Ann. d. Phys. **35**, 898 (1911)
15. Einstein, A.: Lichtgeschwindigkeit und Statik des Gravitationsfeldes. Ann. d. Phys. **38**, 355 (1912)
16. Einstein, A.: Zur theorie des statischen Gravitationsfeldes und Nachtrag zur Korrektur. Ann. d. Phys. **38**, 443 (1912)
17. Einstein, A.: Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist? Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen **44**, 37 (1912)
18. Einstein, A.: Relativität und Gravitation: Erwiderung auf eine Bemerkung von M. Abraham. Ann. d. Phys. **38**, 1059 (1912)
19. Will, C.M.: The confrontation between general relativity and experiment. Living Rev. Relativ. **9**(3), (2006). http://www.livingreviews.org/lrr-2006-3
20. Sumner, T.J., et al.: Satellite test of equivalence principle. Adv. Space Res. **39**, 254 (2007). doi:10.1016/j.asr.2006.09.019
21. Müller, H., Peters, A., Chu, S.: A precision measurement of the gravitational redshift by the interference of matter waves. Nature **463**, 926 (2010). doi:10.1038/nature08776
22. Wolf, P., et al.: Does an atom intorferometer test the gravitational redshift at the Compton frequency? Class. Quantum Grav. **28**, 145017 (2011). doi:10.1088/0264-9381/28/14/145017
23. Hohensee, M.A., Estey, B., Hamilton, P., Zeilinger, A., Müller, H.: Force-free gravitational redshift: proposed gravitational Aharonov-Bohm experiment. Phys. Rev. Lett. **108**, 230404 (5 pages) (2012). doi:10.1103/PhysRevLett.108.230404
24. LoPresto, J.C., Schrader, C., Pierce, A.K.: Solar gravitational redshift from the infrared oxygen triplet. Astrophys. J. **376**, 757 (1991). doi:10.1086/170323
25. Cacciani, A., Briguglio, R., Massa, F., Rapex, P.: Precise measurement of the solar gravitational redshift. Celest. Mech. and Dyn. Astrometry **95**, 425 (2006). doi:10.1007/s10569-006-9014-0
26. Wojtak, R., Hansen, S.H., Hjorth, J.: Gravitational redshift of galaxies in clusters as predicted by general relativity. Nature **477**, 567 (2011). doi:10.1038/nature10445
27. Hentschel, K.: Erwin Finlay Freundlich and testing Einstein's theory of relativity. Arch. Hist. Exact Sci. **47**, 143 (1994)
28. Will, C.M.: Resource letter PTG-1: precision tests of gravity. Amer. J. Phys. **78**, 1240 (2010). doi:10.1119/1.3481700
29. Clowe, D., et al.: A direct empirical proof of the existence of dark matter. Astrophys. J. Lett. **648**, L109 (2006)
30. Renn, J., Sauer, T.: Eclipses of the stars-Mandl, Einstein, and the early history of gravitational lensing. In: Ashtekar, A., Cohen, R., Howard, D., et al. (eds.) Revisiting the Foundations of Relativistic Physics—Festschrift in Honour of John Stachel, pp. 69–92. Kluwer Academic Publishers, Dordrecht (2003)

31. Link, F.: Eclipse Phenomena in Astronomy. Springer, New York (1969)
32. Renn, J., et al.: Einstein's Zurich notebook: gravitation in the twilight of classical physics. In: Renn J. et al. (eds.) The Genesis of General Relativity, Boston Studies in the Philosophy of Science, vol. 1–4. Springer, Dordrecht (2007)

Springer
http://www.springer.com/978-3-319-06348-5

General Relativity, Cosmology and Astrophysics
Perspectives 100 years after Einstein's stay in Prague
Bičák, J.; Ledvinka, T. (Eds.)
2014, XV, 535 p. 117 illus., 80 illus. in color., Hardcover
ISBN: 978-3-319-06348-5