

Chapter 2

On Elementary Transmutations in the Interior of Stars: Paper II (1937)

2.1 Problems with the Build-Up Hypothesis

2.1.1 The Significance of the Build-Up Hypothesis

In a foregoing paper it was attempted to establish whether transmutations of atomic nuclei occur in the interior of the stars and what significance these transmutations have on stellar structure and development.^{1,2} Further investigations have shown that some of the hypothetical presumptions made there cannot be upheld. Consequently, the present paper cannot present any quantitative implementation of the theory; it confines itself to a renewed qualitative discussion of the problem under modified preconditions.

The theory is initially expected to predict, at least in certain simple cases, which nuclear reactions spontaneously occur in a piece of matter of given physical and chemical properties. Its task does not end there, however. As we cannot directly observe the physical and chemical conditions prevailing in a stellar interior, the theory must first define them itself. At this point a hypothesis is needed, as we do not know a priori whether other hitherto unknown effects alter these conditions or are not taken into account besides elemental build-up by nuclear processes whose quantitative description is the aim of the theory. In Paper I, it had been assumed that such effects were not present; this assumption was called the build-up hypothesis *Aufbauhypothese*.

This version of the hypothesis contains another uncertainty, though. The nuclear reactions exert two different influences at the same time: They change the physical state of the matter by releasing energy and its chemical composition by

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² See von Weizsäcker (1937).

transmuting the elements. The generation of energy is the unproblematic part of the theory to consider: Nuclear reactions or effects of similar energy yield are necessary to explain stellar radiation; and the build-up hypothesis is equivalent to the assumption that the nuclear processes sufficed for that on their own as well. Transmutation of the elements, however, is to a certain extent a side-effect of the nuclear reactions, yet nothing is known about its importance in the history of stellar lifetimes. The empirical frequency distribution of the chemical elements exhibits characteristic regularities apparently quite uniformly valid throughout the entire cosmos, which compel us to attempt to explain it by assuming a uniform formation process. It would suggest itself to look for this process in the element transmutations necessarily connected with the generation of energy in the stars. Yet we cannot exclude at the outset the possibility that the chemical elements were formed by another process prior to the formation of the stars as we know them today and that the present energy-generating reactions only brought on a slight change in the original frequency distribution. Hence we must distinguish between a narrower interpretation of the build-up hypothesis, which is limited to the role of energy production by the nuclear reactions taking place in a star today, and a broader interpretation that does not take into regard any other processes of element formation in the history of the cosmos besides the connected element build-up.

In Paper I, it was argued that the broader hypothesis was the simplest of possible assumptions. It became evident that it was impossible to make the same process directly responsible for the generation of energy as well as for element development because the deposit of hydrogen, which is necessary for energy production, does not lead to the build-up of heavy elements. However, a causal link was established between both processes by the assumption that the energy-generating reaction produced neutrons as a side-product, which was then supposed to take over the further element build-up. In attempting to carry it out quantitatively, this assumption now runs up against a series of problems that seem hardly surmountable. First, it is uncertain whether neutrons form in notable amounts and—if they did develop—it seems certain that helium would have to be produced at the same time in an amount that would be irreconcilable with the astrophysical data on the frequency of that element. Second, the build-up of considerable amounts of uranium and thorium via very short-lived radioactive intermediates apparently cannot be explained even by the remedial measures taken in Paper I. The build-up of neutrons ultimately does not deliver any satisfactory explanation for the empirical parallelism between binding energy and the frequency of the various sorts of nuclei.

We are therefore probably compelled to do without the broader build-up hypothesis. In fact, no empirical reason speaks against a restriction to the more limited version. It is entirely possible that the element formation occurred before the stellar formation in a state of the cosmos substantially different from the present one. The energy-generating processes necessarily lead to a change in the element distribution over the course of stellar development. Nevertheless, the stars are probably still so young that they have not had enough time yet to change their chemical composition substantially over the course of their lives. The hydrogen

reserves of a sun originally composed of pure hydrogen suffice to cover its present emission for roughly 3×10^{11} years. On the other hand, geological and astronomical data do not force us to ascribe to the Sun an age older than about 3×10^9 ; and if one may interpret the redshift in the spectra of spiral nebulas as a Doppler effect, then extrapolating backward, this explosive type of motion gives a concrete reason to ascribe to the universe a substantially different physical state from now for a point in time lying roughly 3×10^9 years back. Accordingly, the Sun would have transformed just 1 % of its mass by now. It is interesting that renouncing the broader build-up hypothesis leads to a new, independent age determination that agrees well with the mentioned figures. The radioactive elements still present today, if they are not being constantly generated in the stars, would then have to have been formed at a time lying in order of magnitude not further back than around their half-life time. Quantitative estimates³ have yielded an age for the present element distribution of approximately 5×10^9 .

The present paper treats, in the first part, the reasons speaking against the broader build-up hypothesis. The second part of this paper intends to underpin and expand upon the more limited interpretation. Finally, the third part attempts to assemble the conclusions that one might perhaps be able to draw about the state of the universe at the time the elements were formed.

2.1.2 *The Generation of Neutrons*

It has not been possible up to now to indicate with certainty which specific reactions yield the energy of the stars and therefore we do not know at all yet whether neutrons are produced in considerable amounts in these reactions. All the reaction cycles proposed in Paper I seem to be nuclear physically impossible. We dispense with a more detailed discussion of possible reactions here because we would have to go through them again further below (Sect. 2.2.2) under altered preconditions; and for the benefit of the broader build-up hypothesis, we assume that a neutron-delivering reaction has been found. Under this assumption we can determine a lower limit for the frequency of helium in the star, compared to the total frequency of the heavier elements, which seems to be in disagreement with the empirical frequency of helium.

According to Paper I, α -particles must form simultaneously with each generated neutron. Hence heavy elements cannot be generated without helium simultaneously being formed. The most efficient neutron-producing reaction is a collision between two deuterons, at which under all conditions one nucleus of mass 3 is formed that must somehow be formed into⁴ He and furthermore at which one neutron is produced in about half of all cases. Thus, on average, two helium nuclei

³ See St. Meyer (1939), Wefelmeier (1939).

⁴ Mr. Biermann pointed this out to me. Cf. Cowling (1939).

form for one neutron. If the mean atomic weight of the heavier elements formed by the combining of neutrons onto helium is A , then a heavy nucleus contains, on average, one of the produced helium nuclei and $(A - 4)$ neutrons. Consequently, in order for a heavy nucleus to be able to form, $2(A - 4)$ α -particles must form, one of which is built into a heavy nucleus; the proportion of the number of helium atoms to heavier elements thus is $(2(A - 4) - 1)$ to 1. For $A = 10$ that already is 11:1; for the empirically approximately correct value $A = 25$, it is 41:1. The mass ratio is $4(2(A - 4) - 1):A$, i.e., 6.6:1 for $A = 25$; for $A = 10$ one obtains 4.4:1; for $A \rightarrow \infty$, 8:1.

This value is a lower limit, as all the neglected effects tend to make the helium content even higher. They are the following:

1. Neutrons are generated more rarely than assumed above. Any reaction ever drawn into consideration up to now leads to the formation of α -particles, whereas just a few lead to the formation of deuterons; likewise, a neutron can only form when one deuteron hits a second deuteron, whereas any reaction between a deuteron and a proton leads to the formation of an α -particle without leading to neutron yield. Even though the neutron-producing process is the dominant one in the star, side-reactions will surely always be occurring simultaneously that raise the relative frequency of helium.
2. Neutrons are used more rarely than assumed above for the formation of heavier nuclei. For, the build-up by neutrons from He or perhaps Be does not proceed smoothly; rather there is a certain probability that some intermediary nuclei will intervene in that decay. Thereby a part of the neutrons will always be transformed into helium.

One should therefore not be surprised if the relative frequency of helium lay substantially above the indicated limit. Experience teaches the opposite. From the new book by Unsöld⁵ we gather the following figures:

In class B stars, which one could hardly regard as particularly lacking in helium, one obtains the frequency ratio in numbers of atom type:

helium:hydrogen \sim 1:100

with an uncertainty of about factor 4 (p. 301). In solar protuberances (pp. 416, 419) this proportion results at about 1:30, in the extreme case 1:15. On the other hand,

hydrogen:metals \sim 50:1 (p. 136),
and thus one obtains for
helium:metals \sim 1:2–3:1.

⁵ See Unsöld (1938).

Even the best ratio 3:1 remains below the theoretical lower limit by a factor 10. Although the empirical data may still be very uncertain (Russell calculated for hydrogen:metals $\sim 1000:1$), the attempt to bring theory and observation into agreement can scarcely count as promising.

2.1.3 The Formation of Uranium and Thorium

In Paper I, it was shown that uranium and thorium can only be built up by neutrons if during this process a heavy nucleus captures one neutron on average at least every minute because otherwise the unstable intermediates between lead and thorium decay again prior to the further build-up. On the other hand, the rarity of all elements above iron shows that on average a combined nucleus (above helium) does not capture more than about 50 neutrons throughout its entire lifetime inside the star. Consequently, the total span of time during which a nucleus may stay in a region of the star in which it can be built-up further by neutrons is, on average, of the order of magnitude of 1 h. Now, the amount of matter that is just undergoing such kinds of transmutations relates to the total mass of the star as this time span to the time in which the total stellar mass will have undergone the build-up, hence about as 1 h to 10^{11} years or as $1:10^{16}$. Thus that fraction of the stellar material in which energy is being generated should fit inside a sphere whose radius is smaller than the 10^5 th part of the star's radius, therefore about 10 km for the Sun.

Contrary to the hope expressed in Paper I, such a concentration of the sources of energy in a centrally regular stellar model with temporally stationary generation of energy seems to be impossible. This is because the low core density of the point-source model mentioned in Paper I vanishes when one takes convection into account,⁶ and the temperature does not rise rapidly enough against the core to distinguish such a small area, given the relatively weak temperature-dependence of the energy generation (about $e \sim T^{10}$).

One way out would be the assumption that the energy is not generated uniformly at all but instead in the form of always rapidly expiring small explosions. These explosions would just have to be frequent enough not to produce any visible fluxes in the star's radius or luminous intensity. There would then be no well-defined energy-generating region in the interior of the star, just a zone of instability in the proximity of the star's centre whose magnitude would be about the same as the magnitude of the energy-generating region in the normal model. The volume of the actually exploding regions could then be very much smaller.

It is very questionable, though, whether the real reactive mechanism could lead to such a model. In any case, there must be a cause effecting the extinction of explosions of a certain magnitude, because otherwise no stable stars could exist.

⁶ Cf., e.g., Bodenstein (1937) and the presentations and discussions in *Zeitschrift für Elektrochemie*, 42 (1936): 439ff.

One must obviously invent a very special mechanism, in which an explosion is not prevented from developing by this same cause but only starts to work at a certain magnitude. Looking at the model cycle proposed in Paper I, for instance, one would in fact assume that it would either not work at all or proceed completely explosively. Physical chemistry knows of two criteria for the explosive course of a reaction⁷: a strong rise in probability of a reaction with the temperature; and a branching of the chain of reactions, that is, the generation of a product that itself can become the start of a new chain over the course of a normal chain of reactions. Both conditions are satisfied in the model cycle. On the other hand, the effect of the first condition is removed if during the reaction the released energy is too rapidly dissipated for a considerable temperature rise to be able to occur; and for the second condition, if the reaction chains abort. These two constraining factors are also active inside the star. There is available for the transport of energy, apart from radiation and convection, also the transformation of gravitation into potential energy through star expansion occurring at the speed of sound; and the β -decays may not break off the reaction chains definitively, but each time it may be for a span that ought to suffice to restore the equilibrium with the surroundings in the interim. A precise discussion of the problem is certainly very difficult and we abstain from carrying it out here; for the purpose sought here it should suffice to point out that it is not comprehensible why the restraining factors do not act much earlier to prevent the introduction of an explosion if they are able to stop an already begun explosion.

We must therefore conclude that at least uranium and thorium had already formed before the Sun existed in its present state. Because both these elements comply well with the frequency distributions of their more stable neighbours in the periodic system, one would be compelled to drop further elaboration of the build-up hypothesis for all heavy nuclei as well.

2.1.4 The Relationship Between Mass Defect and Frequency

One basic observation necessarily demanding explanation by a theory of element formation is the overwhelming frequency of energetically particularly stable nuclei. It is most clearly apparent in Harkins's rule, which states that nuclei with even numbers of particles, which are undoubtedly energetically more stable than those with odd numbers, also occur more frequently in nature than the latter, almost throughout. An attempt at an explanation of this fact was made in Paper I, within the framework of the hypothesis of neutron build-up. Meanwhile it has been demonstrated that the theoretical foundation of this explanation was not tenable. A correlation between mass defect and frequency extending beyond Harkins's rule

⁷ See Landau (1937).

now seems to have been empirically verified, which in any case does not grant the mentioned explanation.

In Paper I, it was assumed that the more energy had been gained by the combining of neutrons whose energies correspond to the star's temperature, the larger was a nucleus's mean activation cross-section. Odd-numbered nuclei would then indeed have greater cross-sections and, consequently, in stationary operation be rarer than the even ones. The cause of this postulated relationship between activation cross-section and combining energy ought to be that the number of resonance levels for neutron capture per energy interval increases very rapidly with the neutrons' combining energy.

In the meantime, Landau⁸ has made it very likely that the "reduced partial neutron width," which is proportional to the probability of capture in one level, is, for its part, inversely proportional to the number of levels per energy interval. That way, the effect of high density on a given level would be exactly compensated: The activation cross-section, averaged over many levels, would just be independent of the number of levels per energy interval.

At higher energies, at which the levels become so densely occupied and broad that they are no longer separable, the activation cross-section must in any event become equal to the geometric nuclear cross-section. Setting out from arguments of mathematical simplicity, Landau has now demanded this independence of the mean activation cross-section from the level density also for a region with disjunct levels. This extrapolation can also be supported by the following physical argument. The narrowness of a level signifies long lifetime of the nucleus in the relevant state. The empirically required very long lifetimes are explained, according to Bohr, in that owing to the interplay between the internal motions of an excited nucleus, only rarely does the necessary energy for escape concentrate onto one particle. This temporary energy concentration on a particular particle is evidently a classical concept that can only be defined at all in the limiting case of high quantum numbers, i.e., by the superpositioning of many quantum states. The further apart the states shift from each other, the further removed one is from this limiting case; and in the end, one cannot determine what the energy concentration on one particle in a single very deep state, such as, even the ground state, means anymore; consequently, this reason for the level narrowness falls away. This consideration does not prove Landau's approach, but it does show that at the current state of our knowledge there is no reason to take a different approach. Given the assumption of neutron build-up, Harkins's rule accordingly does not result as a consequence of modern nuclear physics.

Wefelmeier had put forward the general relationship between mass defect and frequency and evaluated it in nuclear theory.⁹ It is apparent above all when one compares light even-numbered elements, hence, for instance, C, O, Ne, Mg, Si and

⁸ Landau (1932, 1938), Hund (1936) Anderson: *Veröff. d. Univ.-Sternw. Dorpat*. Cf. on the following: Gamow and Teller (1938a).

⁹ See Wefelmeier (1937a, b, 1939).

S. This relationship is certainly not part of the discussed interpretation of Harkins's rule, as that interpretation yields not that a stable nucleus would be particularly prevalent but that a nucleus lighter by one unit is particularly rare; only where there is regular alternation as in the case of the even-odd rule are both the same.

The problem that we encounter here offers not only an argument against the broader build-up hypothesis but simultaneously also an indication of the demands that must be made of a correct theory of element formation. The stability of one kind of nucleus can only exert a direct influence on its frequency when during its formation not only build-up processes occur but also processes in which one building block is split off the nucleus without combining; then the unstable nucleus is most easily decomposed. For this, energies of the order of magnitude of nuclear binding energies are necessary, however. We must therefore look for conditions under which such energies can act on a large scale.

2.2 Second Part: The Mechanism of Energy Generation

2.2.1 Survey of the Known Energy Sources

We shall now treat the generation of energy independently of the question of element formation. It seems advisable to recheck the foundations of the theory against an itemization of the energy sources coming into regard according to the present state of physics. In general, four kinds of energy generation can be considered:

1. Contraction without a change in the star's chemical composition. The energy released is gravitational energy.
2. Element build-up. The energy released is nuclear energy.
3. Contraction during transmutation of part of the matter in densely packed neutrons.¹⁰ The released energy is, again, gravitational energy as well as the zero-point energy surplus of the degenerate electron gas, compared to the energy of the forming neutron gas, as a result of the transmutation; for this, nuclear energy must be expended corresponding to the mass surplus of the neutrons above the formerly present atoms. At high density this energy balance can be favourable.
4. Complete disintegration of matter into radiation. The released energy is the matter's energy at rest.

In the following we shall only take into account the second energy source and must therefore justify why the other three sources have no importance. Pure contraction is eliminated, at least for sun-type stars, owing to their low output. By

¹⁰ Landau (1932, 1938); Hund (1936); O. Anderson: *Veröff. d. Univ.-Sternw. Dorpat*. Cf. for the following: Gamow/Teller (1938).

contrast, the energy sources under 3 and 4, if they can become effective at all, are more productive than the element build-up.

Complete disintegration, for instance, by an electron uniting with a proton, has lost its likelihood in that no cause capable of bringing it about has been found in physics to date. Since the discovery of the neutron and the positron, it seems that in the cancelling out of the positive and negative charges, just the electron mass transforms into radiation energy and the proton mass is retained. The fact that this disintegration into radiation has not been observed in the laboratory until now is also a strong argument against its occurrence in a star. As the interiors of the planets and even smaller bodies do not contain this energy source, the temperature in a stellar interior must surely be an essential factor for its triggering. On the other hand, in the laboratory we can nowadays subject at least individual particles to the action of energies by orders of magnitude higher than those in all probability occurring inside a star and find no radiation-emitting disintegration. If one also takes into consideration that the assumption of radiation-emitting disintegration is not necessary to explain the empirical energy production, it does seem legitimate to drop it altogether.

The possibility of the third energy source mentioned above does follow directly out of modern physics, though. Hence we can only eliminate it if we show that the conditions under which it becomes effective do not occur in the known stars. As a matter of fact, the usual estimates for the interior of a star do yield a density far below the critical density at which this energy source starts to operate. However, we can perhaps make ourselves independent of these estimates as well by a genetic consideration. If a star begins its lifetime as a gaseous ball of low density, as the density increases, first energy source 1 will become accessible to it, then 2, and finally 3. If under contraction the star undergoes a series of equilibrium states, it will not increase its density and temperature more rapidly than the energy that is released at this increase can be radiated. Energy source 2 becomes active at a quite precisely defined temperature, and therefore the star should remain in the vicinity of this temperature (10^7) for a long while. It fits here that empirically temperatures of this order of magnitude have to be assigned to all main-sequence stars. Since this energy source can last for a time span that is about a hundred times longer than the presumed age of the Sun, one should assume that the Sun (and likewise the entire main sequence) is not old enough yet to have attained the density necessary for energy source 3.

The sole possibility to achieve this density more rapidly would accordingly be in a stellar development that does not undergo a series of equilibrium states. It does appear, though, that such a development must always lead to the star exploding. This is because an increase in density is connected to an increase in temperature which leads to an accelerated release of nuclear energy. Thereby the star is at least returned to the state of equilibrium; if the deviation from equilibrium was already too large, it can only explode, either immediately or along the route of 'overstable' pulsation. This argument can also be stated this way: At the high density of matter required before a "neutron clump" can form and at the corresponding temperature, the thermodynamic equilibrium of the nuclear reactions must set in promptly

(cf. also the third part of this paper). At equilibrium the proportionate mix of elements is defined by the physical conditions. Every star that does not have the right ratio mix at the outset must set it during contraction and if this process is completed in a period shorter than about 10^{11} years, the energy emitted in the process destroys the star's cohesion.

2.2.2 The Course of Energy-Producing Nuclear Reactions

Which special nuclear reactions are in fact responsible for the generation of energy could not be decided in Paper I. Advances in nuclear physics since then still do not allow a sure answer. The assumption that the elements had essentially already been formed before the development of the present state of the stars casts this problem in a new light, however. For, now all the known stable nuclei are available as initial substances of chain reactions, and yet, the properties of neutron generation and autocatalysis do not have to be demanded of those chains anymore; hence it is less significant for the basic assertions of the theory which reaction should ultimately prove to be the most important.

On the question regarding which of the hitherto proposed reactions are possible, nuclear physics today provides the following information:

The model cycle of Paper I can only run if the nuclei of mass 5 are able to persist. According to experimental research published in the interim, this does not seem to be the case. They have not been found as stable nuclei and on the basis of nuclear reactions a mass has been attributed to ${}^5\text{He}$ that is larger than the sum of the masses of a α -particle and a neutron.¹¹

Most apparently possible ways for a build-up, circumventing mass 5, lead via the nucleus ${}^8\text{Be}$. But this nucleus seems not to be able to survive either. At least, one would have to assign it a lifetime that with considerable likelihood is too short, compared to α -decay, for another charged particle to be able to add itself on within that period under the prevailing conditions inside the star.¹²

Reaction chains starting out with helium, which lead to the build-up of higher nuclei or to a multiplication of the amount of helium, solely through the addition of hydrogen or of helium itself, accordingly seem not to be possible by two-body collisions; and three-body collisions ought not to play a part in stars of normal density.

One must, however, reckon with the possibility of a direct reaction between two protons. Albeit, according to present-day knowledge about the forces between two protons, the thereby initially formable nucleus. He would not be any more stable.

¹¹ Williams et al. (1937). As Mr. Gamow has informed me by letter shortly before completion of this paper, according to new findings by Joliot, ${}^5\text{He}$ and ${}^5\text{Li}$ are stable, after all. Then the model cycle would have to be regarded as relevant and only heavy hydrogen must be assigned the energy source of giant stars (cf. Sect. 7).

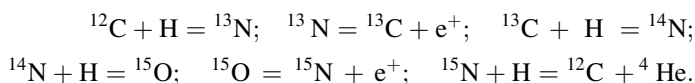
¹² Cf. Livingston/Bethe (1937).

During the brief period of its existence, this unstable nucleus can emit a positron, though; so the process $H + H = D + e^+$ does take place, overall. This process was proposed as an energy source by Atkinson¹³ and has been more recently examined quantitatively by Bethe.¹⁴ Nevertheless, no more can probably be said than that our knowledge about the nuclear physics is not adequate to exclude it as an energy source. The very low a priori probability of β -decay occurring in it is balanced out by the frequency of collisions between two protons inside a star; it is very difficult to find a reliable estimate for the probability of the β -decay, though. The assumption of this energy source would probably present astrophysical difficulties because its temperature dependence is weak, owing to the faint Coulomb field between two protons; within the main sequence the approximate constancy in the core temperatures, despite the very different requirements that stars of different masses set for their energy sources, would be difficult to explain by it.¹⁵

At this point another proposal by Döpel¹⁶ should be mentioned: that reactions particles are unable to undergo by thermal energy could be triggered by particles accelerated by the electric fields within the star. To this the reply must be that electric fields are surely only maintainable in the outermost atmospheric layers of the stars, because the stellar interior has to be an ideal conductor of electricity due to the great density of free electrons.

If we now assume that all the elements in the star had been there at the outset, then we are no longer limited to reactions beginning with hydrogen or helium. On the basis of laboratory experiments, the following is predictable about the behaviour of the immediately higher elements within a star.

All known stable isotopes of lithium, beryllium and borium are decomposed by proton additions and thus ultimately lead to the formation of helium. However, added to this must be a cycle of reactions setting in on the ^{12}C nucleus, during the course of which helium is also produced but the initial nucleus remains unchanged and hence just has the effect of a catalyst.¹⁷ This is the cycle:



The energy source of the star would accordingly first constitute a decay of the elements below carbon and thereafter the indicated cycle. If owing to secondary reactions the frequency of carbon should also ultimately diminish, an analogous cycle beginning from oxygen is available.

¹³ Atkinson (1936). Cf. also Döpel (see note 5 [16] below).

¹⁴ Pursuant to an oral note by Mr. Gamow.

¹⁵ Mr. Biermann pointed this out to me.

¹⁶ See Döpel (1937).

¹⁷ Mr. Gamow informed me that Bethe has recently examined this cycle quantitatively.

2.2.3 Consequences for the History of the Formation of Stars

One can estimate the alteration to the element distribution in the Sun that has taken place by the proposed processes up to now. In order to have simple figures, let us assume the Sun were originally composed of equal mass proportions of hydrogen and heavy elements and the latter were evenly distributed over all the atomic weights from 1 to 50, so that for 25 hydrogen atoms there is one atom that is more massive. In its lifetime up to now the Sun has transmuted about 1 % of its mass from hydrogen into higher elements, therefore, it has lost a 50th part of its hydrogen. On average, two hydrogen atoms react with the same heavy nucleus or, resp., its daughter products before it has transformed completely into helium (e.g., for ${}^6\text{Li}$, one needs 2 protons; for ${}^7\text{Li}$, 1; for ${}^9\text{Be}$, 3, etc.); thus a fourth part of all heavy nuclei should be affected by the transformation if each heavy nucleus reacts just once (hence disregarding cyclic reactions). The lightest nuclei are affected first, and consequently over the elapsed history of the Sun's development the element distribution should be decomposed just about up to carbon. At carbon, at the onset of the cycle, decomposition stops anyway. It is known that Li, Be and B are particularly rare in the Sun,¹⁸ not only compared to other elements but apparently also compared to the Earth,¹⁹ in which element transmutation ceased a very long time ago, of course.

The frequency in helium occurrence now also results in satisfactory agreement with experience. About as many helium nuclei should form as protons disappear. Thus results a ratio in numbers of atoms of about H:He = 50:1.

These considerations can be translated onto the other stars of the main sequence, with appropriate modifications. A problem is posed by giant stars, however, which combine high luminosity with a core temperature that is about ten times lower than in the main sequence. It is certain that the energy sources for nuclear reactions in the giant branch and the main sequence cannot be the same. Yet, given the assumption that giant stars still drew their energy from pure contraction, it is known that such a short time span is available that the pulsation periods of the Cepheids ought to have already changed as a consequence of the contraction within historical times—in contradiction to a series of observations.

One must thus surely seek two different nuclear reactions for giant stars and for main sequence stars. The above considerations offer two different possibilities. As Bethe has noted,²⁰ one can assume that giants are still decomposing Li, Be and B, whereas the main sequence is already engaged in the carbon cycle. One can also assume that large amounts of heavy hydrogen isotopes are still present in giant stars, and they are yielding energy whereas the decomposition of lithium and its neighbours is either limited to a rapidly progressing intermediary state or else

¹⁸ The significance of the rarity of these elements was probably first emphasized by Goldschmidt (1926)

¹⁹ At least Li and Be. Cf. Goldschmidt (1938). Mr. Wefelmeier pointed out this matter to me.

²⁰ Pursuant to an oral note by Mr. Gamow.

already belongs to a stellar type that is in very close proximity to the main sequence. As the energy gain is essentially a function of Z^2/T , the first assumption would reduce the required core temperature of giant stars, compared to the main sequence, by about factor 4, the second by about a factor that is in any case greater than 10. The first assumption may seem less forced; the second perhaps lends clearer expression to the characteristically wide separation between giant and dwarf stars. Which of the two is right depends on the as yet unknown original frequency distribution of the lightest elements. Independent of this individual issue, the build-up hypothesis suggests, in any event, that one must assign to the main sequence a stationary reaction cycle and to giant stars the decomposition of nuclei lighter than the nuclei participating in the cycle; they therefore already have to have been converted before the temperature could rise high enough to stimulate the cycle.

This assumption has a few consequences that can perhaps be tested empirically. According to it, giant stars must be very young. Based on the above estimates, the decomposition within the Sun of a single sort of nucleus of average frequency lasts around 10^8 years. Since giants possess a higher specific luminosity than the Sun, for them this period is lowered to about 10^7 years or less. That star clusters do not simultaneously contain B-stars and yellow or red giants points in the direction that a uniform age (within certain limits) must be assigned to giant stars. But perhaps more precise tests exist. Furthermore, giants would have to contain light elements that are already very rare in the main sequence, in particular, either lithium and the next-heavier elements or heavy hydrogen (the latter obviously in an amount perhaps no longer spectroscopically detectable, besides light hydrogen).

Against the assumption that giants also had normal nuclear reactions as one source of energy, Gamow²¹ has raised the objection that they would then have to be ordered in a line approximately parallel to the main sequence, whereas in reality the giant branch is positioned about perpendicular to the main sequence. One has to note, though, that the giant branch is very diffuse and in reality fills an extended level range above the main sequence. According to our assumption, the energy source of the main sequence is temporally roughly constant, whereas that of giant stars is spent over time. Accordingly, a giant's luminosity can change over the course of its development, such that giants observed today as differing in mass and age should, in fact, fill one level in the Russell diagram. The line of the largest number of stars in the diagram, normally called a giant branch, then does not need to be a line of constant age along which the stars would be distributed according to mass; they could, conversely, depict the developmental path of the mass occurring most frequently among giants. Perhaps, owing to the mainly convective construction of giant stars, another mass-luminosity relation is valid than for dwarfs.

Gamow²² has pointed out another problem for the entire build-up hypothesis. He has shown that the mass-luminosity relation of a star depends on its hydrogen

²¹ Gamow (1938a).

²² See Gamow (1938b).

content and if that changes during stellar development, the empirical continuance of a universal mass-luminosity relation²³ (at least for the main sequence) is not comprehensible. The assumption of a resonance in the energy-generating process introduced by Gamow himself to remove this difficulty has a low a priori probability and seems, in addition, only to reduce the problem but not remove it.²⁴ Perhaps, however, the fact recently put forward by Gamow,²⁵ that the strong deviation from the normal relation is limited to a short part of the star's lifetime, suffices for that. The whole problem disappears, though, if one abandons the broader build-up hypothesis and assumes that during the lifetimes of the stars to date, their original chemical compositions have not changed essentially at all. This remark suffices for stars with luminosities smaller per gram than ten to a hundred times the Sun's luminosity. Accordingly, the cosmos would still be so young that stellar evolution had not taken place yet, apart from going through the giant stage. Just the existence of white dwarfs poses a problem for this interpretation; almost the only explanation that the build-up hypothesis can offer for their low luminosity is the assumption that their entire hydrogen content has already been depleted.

These considerations on the historical development are still at a speculative stage. But the concepts they use stem throughout from a part of physics whose foundations today can already be regarded as elucidated. One may therefore hope that some experimental advances and a closer collaboration between astrophysics and nuclear physics will permit a well-rounded theory to be completed in the near future.

2.3 Third Part: The Formation of the Elements

2.3.1 *The Necessary Physical Conditions*

If we do not want to abstain entirely from understanding the development of the elements, after having given up the broader build-up hypothesis, we must draw out of the frequency distribution of the elements conclusions about the former state of the cosmos in which this distribution could arise. It is self-evident that such conclusions rest on a very much more uncertain basis than the theory of energy production, as they, unlike the latter, presume not only the spatial permanence but also the temporal permanence of our natural laws. That the laws of nature retain their form in the transition from the terrestrial laboratory to planetary space and from there into galactic space has been confirmed by experience at least for a series of individual cases. How far we can describe an essentially different earlier state of the cosmos by the laws of nature as we know it is, a priori, wholly

²³ The German original reads 'Reaktion', apparently a typo. (*ed.MD*).

²⁴ See Gamow/Teller (1938b).

²⁵ See footnote 20.

uncertain; and the essence of time dictates that as soon as the past is no longer accessible to our memory, we cannot possess any direct experiences, but must instead rely on conclusions drawn from documents. There is no other choice than initially to presuppose hypothetically the temporal permanence of our natural laws and to be prepared that an error may be revealed upon comparison of this theory against documents from the past still available today.

The relationship between mass defect and frequency of occurrence compels one to assume that while the elements were forming kinetic energies were available to the coreactants of the order of magnitude of nuclear binding energies. At such high energies thermodynamic equilibrium must set in very rapidly in the nuclear reactions. For, first, Coulomb repulsion no longer plays a part anymore then, at least for lighter nuclei; and second, the transmutation of free protons into neutrons then becomes a very frequent process, so an arbitrary amount of neutrons is available for the combining and decomposition of heavy nuclei.

The first impression that the frequency distribution of the elements leaves contradicts this assumption, of course. It should be noted, however, that nuclear processes were still taking place in the cosmos even after the basic features of the element distribution had come about. The theory has thus already achieved what one can expect of it, if it shows that the discrepancy with the present-day distribution can be explained by equilibrium in nuclear processes that we must anyway require as having occurred for physical or astronomical reasons also after the first act of formation. Thus the abnormal rarity of the lightest elements is a consequence of energy-generating reactions in present-day stars. We shall discuss another discrepancy at the end of [Sect. 2.3.2](#).

Hence, if one sets the original distribution at thermal equilibrium, one can attempt to calculate out of the empirical frequency of the elements what temperature and what density must have dominated at the time of their formation. The frequency relation between two neighbouring nuclei must be set according to the Saha equation.²⁶ We shall look at a neutron being taken up. Let n_A denote the number of nuclei of atomic weight A per cm^3 , hence specifically n_1 , the number of neutrons, and E_A the energy that must be expended in order for one neutron to be torn away from the nucleus of mass A , then

$$\frac{n_{A-1} \cdot n_1}{n_A} = g_A \cdot e^{-\frac{E_A}{kT}} \quad (2.1)$$

holds, with

$$g_A = \frac{G_{A-1}}{G_A} \cdot \frac{2(2\pi M k T)^{3/2}}{h^3} \quad (2.2)$$

²⁶ This attempt has been undertaken often already. The following publications are known to me: Farkas/Harteck (1931), Sterne (1933), Guggenheimer (1934).

G_A is the statistical weight of the nucleus of mass A , hence a number of order of magnitude 1. The second factor in (2.2) contains the statistical weight of the free neutrons; it has the dimension of a reciprocal volume, namely, it signifies the number of neutrons per unit volume for which phase cells are available at the given temperature.

One can now apply Eq. (2.1) to two neutrons being successively taken up. One obtains from this the following equations for the temperature and neutron density:

$$kT = \frac{E_A - E_{A-1}}{\ln \frac{n_{A-2} n_A}{n_{A-1}^2} \cdot \frac{G_{A-1}^2}{G_{A-2} G_A}} \quad (2.3)$$

and

$$\frac{n_1}{g_A} = \frac{n_A}{n_{A-1}} e^{-\frac{E_A}{kT}}. \quad (2.4)$$

We apply these formulas to the reactions



If one sets $n_{16} = 10^4$, then empirically, $n_{17} = 4$ and $n_{18} = 20$. Furthermore, $E_{17} = 4.5 \text{ TME}$ and $E_{18} = 9.8 \text{ TME}$. G_{16} and G_{18} may be set equal to one. The spin of ${}^{17}\text{O}$ is unknown; if it is set equal to $1/2$, then follows $G_{17} = 2$. It follows that

$$kT = 0.44 \text{ TME} = 0.41 \text{ MeV}$$

or

$$T = 4.7 \times 10^9 \text{ degrees.}$$

Furthermore $n_1/g_{18} = 1.2 \times 10^{-9}$.

With the just calculated temperature,

$$1/g_{18} = (5 \times 10^{-12} \text{ cm})^3,$$

hence $n_1/g_{18} = 1$ would be a neutron density already comparable to the density inside the atomic nucleus. The real figure yields a mean distance between the two neutrons that is about a thousand times larger. There results $n_1 = 10^{25}$, that is, about tenfold the density of water just for the neutrons.

These figures are still very imprecise. Above all, the density determination can still be wrong by a number of powers of ten because of the exponential dependence on the energy. Yet given the inaccuracy of the indicated mass defects and occurrence frequencies, a factor of order of magnitude 2 must also be left open in

determining the temperature. The next task of theory would therefore be initially to check many other reactions by an analogous consideration for whether somewhat uniform values for temperature and density are generally determinable out of the entire frequency distribution of the elements and, if this hope is confirmed, to define the numerical values as precisely as possible. Perhaps by this route one can come far enough along ultimately to use the empirical frequencies of the individual nuclei directly as quantitative information about their mass defects.

Unfortunately, we possess accurate knowledge about mass defects at the same time as their frequencies required for this examination only for the nuclei below oxygen, whose frequency is completely changed by secondary processes. A couple of data on heavier nuclei yield figures of the same order of magnitude as those calculated here. We shall abstain from continuing on toward a quantitative analysis in this paper and shall only regard more exactly a qualitative feature of the empirical distribution.

2.3.2 The Conditions for the Formation of Every Kind of Nucleus in Comparable Amounts

In the bigger picture, the empirical frequency distribution of nuclei exhibits conspicuous uniformity, despite strong individual fluctuations. It often happens that a nucleus occurs 10^3 times more frequently than its neighbour; nonetheless, the frequency of any kind of nucleus, from oxygen to lead, varies only by about factor 10^6 . If starting out from oxygen one wants to calculate the frequency of lead, formula (1) must be applied almost 200 times (where the build-ups applied are partly neutrons, partly protons); in order for the resulting relation to become $n_{\text{O}}/n_{\text{Pb}} \sim 10^6$, the factor $g_A e^{E_A/kt} / n_1$, which indicates the relevant frequency ratio between two successive nuclei, must have a mean value that is almost exactly equal to one, despite the very large individual fluctuations. If we call the mean value f , then $f^{200} \sim 10^6$ must be valid and hence $f = 10^{0.03} = 1.07$. Such a noticeable relation between temperature and neutron density cannot be chance; we must rather expect the theory to explain this matter physically. In fact, this relation proves to be an almost direct consequence of the saturation of the nuclear forces.

Let us have the density of the matter vary at constant temperature. At low density, f is large against one (the statistical weight of the free protons is so great that its influence outweighs the energetic preference for the bound state); that is why the lightest nuclei are present practically on their own. At increasing density the balance shifts in favour of composite nuclei. If, however, the binding energy is proportional to the number of particles, that is, if in the mean E_A is independent of A , this shift never leads to favouring one sort of nucleus over the next lighter one but instead, in the limiting case of very large density, yields a distribution in which nuclei of every size are exactly equally frequent (hence $f = 1$). The only precondition for this is that the density stay small against the density of the matter in

the interior of the nuclei, i.e., that the formation of separated nuclei is generally still possible.

We ground this assertion on a simplified model. We first just consider the combining of a single sort of particle of concentration n_1 and mass M ; that is, we disregard the difference between neutrons and protons. This is unproblematic because neutrons and protons become practically equally frequent particularly at high temperature and density. We assume furthermore exact saturation of the nuclear forces and thereby substitute E_A by the constant E . We continue to neglect for now the upper end of the periodic system (which is, of course, determined only by the deviations from saturation in the heaviest nuclei because of the Coulomb force), thus we assume that A could run from one to infinity. Finally, we set all weighting factors $G_A = 1$; g_A then takes on a constant value g . (2.1) then becomes

$$\frac{n_{A-1}}{n_A} = \frac{g}{n_1} e^{-\frac{E}{kT}} = f; \quad (2.6)$$

f is independent of A , and there results

$$n_A = n_1 f^{-(A-1)}. \quad (2.7)$$

If the total mass density ρ of the matter is defined, then n_1 (and hence f) is determined out of (2.6) and the additional condition:

$$\rho = \sum_A n_A \cdot AM = n_1 M f \sum_A A f^{-A} = \frac{n_1 M}{(1 - 1/f)^2}. \quad (2.8)$$

One notes that the density becomes infinite even for $f = 1$ as there are then infinitely many sorts of nuclei with the same occurrence frequency. $f < 1$ therefore cannot occur at all.

This result can also be expressed physically like this: We examine the condensation of a neutron gas. So that $n_{A-1} = n_A$, that is, in order for “drops” of every size to form with the same probability,

$$n_1 = g e^{-\frac{E}{kT}} \quad (2.9)$$

must be valid. This is precisely the vapour-pressure equation of the neutron liquid, which indicates the concentration of a neutron gas at equilibrium with its condensate. This condensate, however, which by its presence provides for the validity of Eq. (2.9), is the forming nuclei themselves.

The question now is whether the presumptions made apply to the real nuclei. Exact saturation is the only doubtful assumption. The finiteness of the periodic system presents no difficulty as, although the elements above lead are radioactive, they are viable and their frequency diminishes according to (2.7) exponentially with the mass; if they decay subsequently, the frequency of lead, compared against

its neighbouring elements, is merely raised by factor 10–100, which fits well with experience.

The temperature must be high enough to cancel out the empirical deviation of the binding energies from saturation (that is, not the fluctuations but the systematic course). The outcome is a substantially higher value than in the foregoing paragraphs. If the combining energy of a neutron varies over the 200 mass numbers from oxygen to lead by ϵ , and if this variation is set linearly, then even for very great ρ , oxygen must occur more frequently than lead by the factor $e^{100 \cdot \epsilon kT}$. With $\epsilon = 3$ MeV and $n_O/n_{Pb} = 10^6$, there results

$$kT = 20 \text{ MeV}; \quad T = 2.3 \times 10^{11} \text{ degrees.}$$

This very rough estimate yields, in any event, that a temperature must be assumed of an order of magnitude as would arise from a complete conversion of the nuclear binding energy into heat. The associated density is likewise already in the neighbourhood of the density inside the nucleus.

It is not surprising that the comparison between immediately neighbouring nuclei yielded a very much lower temperature when one considers the strong fluctuations in the frequency distribution on the small scale. In the physics, it must be taken into account that the temperature must have steadily dropped after the formation of the elements, even if perhaps very rapidly. Nuclear reactions must have still been going on in the process. Above all, if the density was diminishing simultaneously and thereby the lighter nuclei were then being favoured in the equilibrium distribution, it is possible that the energy of the gas simply did not suffice anymore to produce the distribution corresponding to the new temperature, but instead just to allow a couple of more reactions to run their course in the vicinity around each nucleus and thereby conform the fine structure of the distribution to the lower temperatures. Perhaps one ought to be able to read off from the present distribution on the large scale the highest temperature attained, and from the distribution on the small scale the lowest temperature attained at which reactions were still occurring. The latter temperature could be, according to its value calculated above, about the temperature at which the formation of neutrons out of free protons and electrons stops.

How can these notions be tested further, now? On the practical side, above all, more precise knowledge about mass defects of many series of isotopes in the manner of the oxygen isotope applied above would be desirable, which would make possible further quantitative applications of Eq. (2.1). From theory one can ask for verifiable postulates about when and where in the history of the cosmos the required temperatures and densities could have been realized.

2.3.3 Materializing the Conditions in the History of the Cosmos

Inasmuch as we know the present state of the cosmos, it does not contain any areas at the temperature required. Neither can we imagine an earlier state of the cosmos out of which the stars could have steadily emerged, that is, by going through a sequence of equilibrium states and during which such temperatures would have prevailed. For, according to Sect. 5, a star can only attain very high temperature without subsequently exploding if it had already practically depleted its hydrogen. So, even if heavy elements should be forming somewhere in the cosmos along this steady path of development, this at least cannot be in the presently existing stars, as these stars still contain their hydrogen (the hydrogen content that is today being attributed to the stars lies far above the equilibrium amount corresponding to the distribution of heavy elements and can only be interpreted as an admixture to the heavy elements by matter that had not participated in the thermal equilibrium). We must therefore look for a possible state of matter prior to the formation of the present stars.

This state must have been a stellar-type cluster of matter, at any rate, as we could not understand the high density required without the participation of gravitation. Now, all empirically known star types are stable and hence unsuitable for attaining those high temperatures; we must therefore look for an empirically unknown but possible type of star. It suggests itself to think of those stars with a mass lying above the empirical upper limit of stellar masses. In Paper I, it was supposed that these stars would be unstable against pulsations of augmenting amplitude. The details of the arguments given there were probably wrong because they postulated a too low real stability for the stars, even without any delay in the generation of energy.²⁷ Nevertheless, the assumption that these stars are unstable is legitimate because, on one hand, they should otherwise be empirically locatable at least in a few exemplars; and on the other hand, the slower a star reacts to changes in the state of its interior, the more defenceless it becomes against the nuclear reactions taking an explosive course, irrespective of how they specifically occur; and as the connection between pulsation period and luminosity already shows, at any event, the sluggishness of a star to react increases with growing mass.

One can therefore assume major original conglomerations of matter that perhaps were composed of pure hydrogen. By contracting under the influence of gravitation and thereby raising their central temperature, they finally reached a state at which nuclear reactions occurred in their interiors. If their mass was small enough, they could remain stable as stars; if the mass was too large, the nuclear reactions proceeded explosively and blasted the star apart which then either got lost in space as a diffuse cloud or clustered together into new smaller stars. With

²⁷ Cf. Cowling (footnote 4).

these the same game repeated itself until only stable stars were left behind. These explosions, if they proceed completely at a certain volume, can attain temperatures of the order of magnitude required above, because then the total energy contents of the nuclei are temporarily converted into heat.

How large, now, may one imagine the first conglomeration to have been? Theory does not set any upper limit on its mass, and our imagination has the liberty to conceive not just the Milky Way but the whole cosmos known to us as united within it. One can even draw an empirical fact into the field in support of this speculation. The energy released by nuclear reactions is about 1 % of the energy of matter at rest and conveys to the nuclei on average a velocity of order of magnitude of a tenth of the speed of light. A star's debris should have flown apart at about that velocity. In answer to the question where such velocities of this scale are still being observed, they are found only in the escaping motions of spiral nebulae. That is why one should at least reckon with the possibility that this motion has its origin in a starting catastrophe of the kind considered. Milne²⁸ pointed out a few years ago that any 'gas' escaping into empty space from spiral nebulae must exhibit the Hubble relation between distance and radial velocity as soon as its expansion has become large against its original volume. Our proposal fits into this picture. However, it distinguishes itself from the present theory by Milne in that it indicates a concrete cause for the expansion and instead dispenses with an infinite and uniform distribution of spiral nebulae. From the standpoint of cosmological speculation, Milne's theory or one of the older ones about the "expanding universe" may be more elegant; it does seem to us, though, rather an advantage of this new proposal that it makes dispensable the difficult-to-prove assumption that the part of the universe known to us had the same structural quality as the whole.

An empirical test of this proposal would be possible if a drop in the frequency of spiral nebulae, which ought to start above a certain critical radial velocity, lay within the observational range of modern telescopes. The rough estimate for this critical velocity, of about a tenth of the speed of light, is surely wrong by a factor in order of magnitude 2. To be able to obtain a more accurate prediction, one would have to be able to describe the explosion process quantitatively.

2.4 Conclusion

2.4.1 First Part

1. The assumption that all known chemical elements were formed and are still forming in the presently existing stars is abandoned; for the following reasons:
2. The heavier elements, according to Paper I, would have to be built up by neutrons whose generation is necessarily connected with the formation of

²⁸ Milne (1933).

helium. A quantitative analysis of this mechanism leads to the setting of a lower limit in the frequency of helium occurring in a star that is incompatible with experience.

3. Uranium and thorium must be built up by rapidly decaying intermediate nuclei. Contrary to the assumption in Paper I, the spatial concentration of the energy sources does not suffice to give the build-up process the necessary velocity.
4. Nuclear physics cannot justify the explanation for Harkins's rule offered in Paper I. The general relationship between mass defect and distribution frequency compels the assumption of element formation at a temperature permitting equilibrium to set in between decomposition and build-up processes.

2.4.2 Second Part

5. The generation of energy by stars is probably uniquely based on the reactions of lighter nuclei.
6. Which reactions are the most important cannot be decided yet. If composed elements don't just form in present-day stars, no autocatalytic reaction cycle is called for. The model cycle of Paper I is probably nuclear physically impossible. A cycle in which carbon acts as a catalyst in the formation of helium is the most likely.
7. The frequency distribution of the lightest elements following out of these cycles is in conformity with experience. One should probably assume two different types of reactions for giant and dwarf stars, namely, a stable cycle for the dwarfs, and the decomposition of nuclei lighter than those participating in the cycle for giant stars.

2.4.3 Third Part

8. It is assumed that the elements were formed in nuclear reactions at thermodynamic equilibrium. For some elements above oxygen, precise knowledge about the mass defects of many successive isotopes would be necessary for exact verification of the posited formulas. A confirmation of these formulas would legitimate drawing quantitative conclusions about mass defects from the frequency of types of nuclei.
9. From the—in the mean—smooth frequency distribution of the elements, an initial formation temperature of around 2×10^{11} degrees follows that just corresponds to the energy released on the whole by nuclear reactions. The fine structure of the distribution seems to have been ultimately defined by a lower temperature of about 5×10^9 .

10. In a 'star' of very large mass, the required temperatures could probably temporarily arise but would lead to its explosion. The connection between this notion and the escape motion of spiral nebulae is discussed.

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