The Symmetry of the Ornament on a Jewel of the Treasure of Mycenae[1]

Introduction

It is well known that many mathematical theorems were grasped intuitively long before the systematic development of mathematics. As is known, geometry was developed systematically by the Greeks of the fourth and third centuries B.C., but long before that, in Egypt, we find ornaments that achieve complicated symmetries, that is, those based on an entirely non-trivial group.[2] There appears to be a case of an ornament inspired by a geometric theorem among the treasures found in the shaft graves of Mycenae. It appears that up to now the Mycenaean culture has barely been studied from the point of view of the history of the sciences; this small note seems therefore warranted.

The Mycenaen Jewel

The National Archaeological Museum in Athens houses the large treasure of gold objects that Heinrich Schliemann discovered in Mycenae in 1876, in the shaft graves that date from the sixteenth century B.C. (today designated as grave circle A). From tomb III, he exhumed three female skeletons covered with a large quantity of gold jewelry. Notable among these are a number of discs in gold leaf measuring about 6 cm in diameter, some of which feature geometricized figures of flowers, butterflies, octopi, etc. On others, one finds geometric figures, and one of these medallions presents a theorem that every reader learned long ago at school. This appeared as medallion 20 in Georg Karo's fine work *Die Schachtgräber von Mykenai* (fig. 1).

Karo describes the object thus:

> 20. Pl. XXVIII. Small gold discs with star pattern. Diam. 6.2 [cm] …
> 60 exemplars, 16 with rather large, roughly punched holes. A six-pointed framework is produced from flat arcs, which is filled by a 6-pointed star, in its turn formed by 6 overlapping arcs. Between the leaves small circles are situated, with recessed insides. In 3 exemplars, the pointed leaves are filled with an engraved fishbone pattern …; one of these and another one are made out of lighter, thinner sheet metal.[3]

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[1] Originally published as "La symétrie sur un bijou du trésor de Mycènes," *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, Anno I, Fascicolo 20, 1976.

[2] On this subject, see Andreas Speiser, *Theorie der Gruppen von endlicher Ordnung*, 4th ed. (Basel und Stuttgart: Birkhäuser Verlag, 1956) and Hermann Weyl, *Symmetry* (Princeton: Princeton University Press, 1952).

[3] "*20. Taf. XXVIII. Goldplättchen mit Sternmuster. Dm. 6,2 ….*
60 Exemplare, 16 mit ziemlich grossem, roh eingeschlagenem Loch. Aus flachen Kreisbögen ist ein sechsspitziger Rahmen hergestellt, der, wiederum durch 6 sich überschneidende Kreisbögen, mit einem Stern aus 6 spitzen. Blättern gefüllt ist. Zwischen den Blättern sitzen

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Fig. 1. Mycenaean medallion.
From George Karo, *Die Schachtgräber von Mykenai*, vol. II, pl. XXVIII

This medallion was also reproduced by Giovanni Becatti in *Oreficerie Antiche*, where he describes it thus:

> Circular disc in gold stamped with a rosette with six petals inside a hexagon with curved sides, and small circles between the petals. From tomb III of Mycenae. Diam. 6.2 cm. Second half of the 16th century B.C.[4]

The essential characteristic of this figure is the fact that *all six* interior arcs have been drawn *with the same compass opening* that determined the big circle, i.e., the perimeter of the medallion (fig. 2).

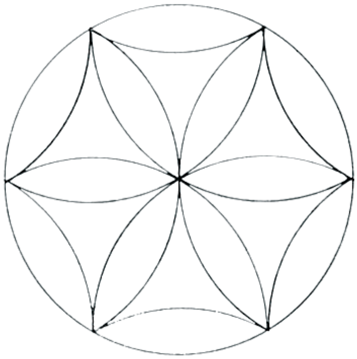

Fig. 2.

kleine Kreisen mit eingetieftem Innern. Bei 3 Exemplaren sind die spitzen Blättern mit eingeritztem Fischgräternmuster gefüllt ...; eines von diesen und ein anderes besteben aus hellerem, dünnerem Blech (Georg Karo, *Die Schachtgräber von Mykenai*, vol. l (text) and vol. II (plates), Munich: F. Bruckman AC, 1930, p. 47, fig. 20).

[4] "*Bratta aurea circolare stampigliata con rosette a sei petali entro un esagono dai lati curvi, e cerchietti fra i petali. Dalla tomba III di Micene. Diam. cm. 6,2. Seconda metà deI XVI sec. a.C.*" (Giovanni Becatti, *Oreficerie Antiche*, Rome: Istituto Poligrafico dello Stato, Libreria dello Stato, 1955, p. 154, tav. XVIII, fig. 62).

This construction is based on the following theorem:

> The radius *r* of the regular hexagon is equal to each of its sides.

Or, inversely,

> If one starts at any given point on a circle, and determines a second point on the circle as far from the first as it is from the center, and then repeats this process from the last point determined and continues in this manner, one will return, after the sixth point, *exactly* to the initial point (fig. 3).

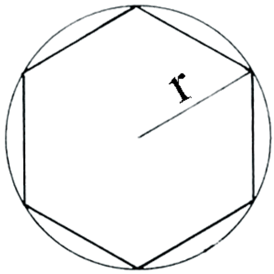

Fig. 3.

Many of the readers of these lines will remember the pleasure that they experienced when, for the first time, they understood this theorem, when they drew and redrew these circles with their first schoolboy's compass, and when the figures took shape in front of their eyes. There is no reason to believe that it was any different in ancient Mycenae, and we think that the *intuitive discovery of this theorem prompted the ornamentation of these medallions.*

Fig. 4. Six Mycenaean medallions. From Karo, *op. cit.*, pl. XXIV

Karo had already suspected the possibility that the goldsmith who made these discs knew this theorem. Yet, he expresses this hypothesis about another ornament, shown in the upper right hand corner, numbered 10, in Karo's plate XXIV (fig. 4).

Like all the other disks reproduced in our fig. 4, this one too is the fruit of the discovery of this theorem; in our opinion, however, it does not provide the proof of it. On the other hand, the description of medallion 20, "Admittedly, the star consisting of six large arcs drawn from the rim with connecting flat arcs between the points remains an isolated case"[5] (note that ornament 20, XXVIII = ornament 81, XXXIV), in suggesting a distinction between "large arcs" and "flat arcs," probably does not do justice to the ornament. Note that we say: probably. Some of the exterior arcs, *but not all*, are indeed flatter; however, we believe that these inequalities are due to an imprecision in the execution rather than to the artist's intention. The fact that the twelve arcs of the ornament can be drawn with the same opening of the compass reinforces our thesis.

This procedure can be continued in all directions. From every pair of neighboring points on the circle, one can determine the exterior point equidistant from each of them. This point will then be the center of the concave arc (exterior) of the ornament, which is the starting point of another circle, and so on. Thus the medallion is only one part of an infinite ornament (fig. 5).

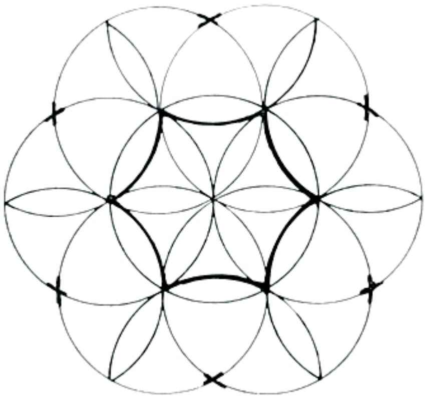

Fig. 5.

Still, one might object that it is not necessary to read the ornament in this way, and maintain that it merely depicts a geometricized star or a six-petaled flower, which are quite often found in Mediterranean art of this period. In response to this criticism, we put forward the following three observations:

- This interpretation doesn't do justice to the concave exterior circles;
- If the figure were a flower, we would expect to find similar flowers with 4, 5, 7, 8 … "petals." It is true that a few examples of these are found in the Mycenaean treasure ; for example, on the large diadems found in shaft grave III, one finds some figures with "petals," but the great majority of the

[5] *Zwar bleibt Stern der aus sechs vom Rande aus geschlagenen grossen Bögen, mit verbindenden flachen Bögen zwischen den Spitzen, ein vereinzelter Fall* (Karo, *op. cit.*, p. 269).

ornaments have six or even twelve petals. Of the discs, the ornament of the figure 20 seems to be unique in its genre;

- Karo says of Mycenaean art that "the basis of the whole ornamentation is built on circles, wavy lines and spirals, which gives this art its stylistic character."[6] He tried to have the artist K. Grundmann reconstruct the genesis of some of these ornaments,[7] in order to show how the goldsmiths had arrived at these refined drawings, which he calls, rightly so, "quite complicated."

We have underlined the geometric aspect of medallion 20. This doesn't exclude the attribution of a symbolic significance, or even one that is religious or magical. In this regard, it is interesting to note that two of these medallions, also found in tomb III, were used as the two pans of a scale (fig. 6).[8]

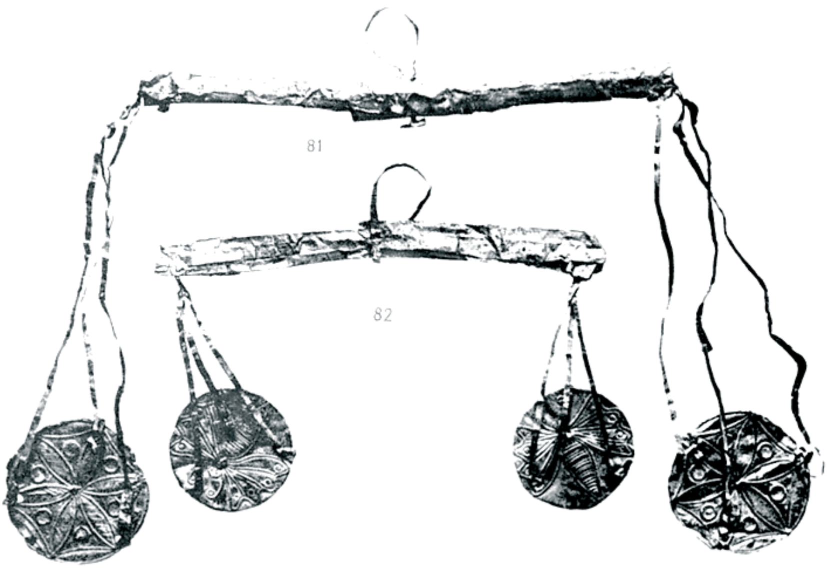

Fig. 6.

As this ornament is based on a precise theorem, it is a perfect expression of the idea of exact distribution and payment. Was it a symbol for this? Yves Duhoux communicated to me that it has been hypothesized that these scales may indicate a belief in the weighing of souls. Our remark may therefore provide support for this hypothesis.

Lessons for the history of the sciences

What lessons does this ornament hold for the history of the sciences?

First of all, that the use of the compass was well known in sixteenth-century B.C. Mycenae. Prof. B. L. van der Waerden drew my attention to the fact that the use of

[6] *Die Grundlage der ganzen Verzierung bilden Kreis, Wellenlinie, Spirale, sie geben dieser Kunst ihr stilistisches Gepräge* (Karo, *op. cit.*, p. 259).

[7] Cf. Karo, *op. cit.*, p. 265.

[8] See Oscar Montelius, *La Grèce Préclassique* (Stockholm, var Haeggströms Boktryckeri A.B., 1924), 1re partie, fasc. I, pp. 18–218. See also G. Karo*, op. cit*, vol. II, pl. XXXIV.

the compass is attested to in ancient Babylonia of the time of King Hammurabi (eighteenth century B.C.), by showing three circles drawn *with precision.*[9]

This comparison puts the discovery of the Mycenaeans in relief. For the Babylonians, the ratio between the perimeter and the diameter of the circle was 3:1,[10] whereas, in the Mycenaean figure, it is sufficient to draw the six arcs to see at once that this value is too small (fig. 7): the circumference of the circle is obviously larger than that of the hexagon, which, according to this figure, is equal to three times the diameter.

Did the Mycenaeans grasp this?

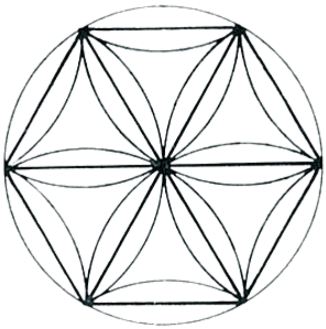

Fig. 7.

Further, the ornament shows that the Mycenaeans not only had a clear idea of geometric drawings and studied them systematically, but also that a theorem of the geometry of the circle was familiar to them, as well as a construction with the aid of a compass.

Questions for further investigation

We would very much like to have answers to both of the following questions:

1. Where did these goldsmiths get their geometric knowledge: did they discover this ornament for themselves, or did they learn it from somewhere else?

One is tempted to look for the source of this ornament in Minoan art. Indeed, some ornaments found in Crete – for example, the geometric ornament on the scepter in the shape of leopard[11] (somewhat earlier) or the ornament in spiral on the pithos[12] of the same period – would seem to lend weight to this hypothesis.

But Minoan art doesn't seem to have shared the "circular obsession" of Mycenaean art. In any case, a superficial examination didn't provide any tangible evidence, and the particular character of this period of Mycenaean art has often been underlined.

[9] See B. L. Van Der Waerden, *Erwachende Wissenschaft* (Basel und Stuttgart: Birkhaüser Verlag, 1956), fig. 20, p. 109.
[10] See B. L. Van Der Waerden, *op. cit*., p. 120.
[11] Spyridon Marinatos and Max Hirmer, *Kreta und das Mykenische Hellas* (Munich: Hirmer Verlag, 1959), figs. 68, 80.
[12] Ibid.

2. Can we find subsequent traces coming from this ornament that show us that its mathematical content had been conserved and transmitted?

All that we were able to find is a clasp, found in Boeotia, reproduced in Erwin Bielefeld,[13] which, according to a friendly communication from Yves Duhoux, dates from the first third of the last millennium B.C (Fig. 8). The ornament is an extension of the ornament of Mycenae (compare to our fig. 5).

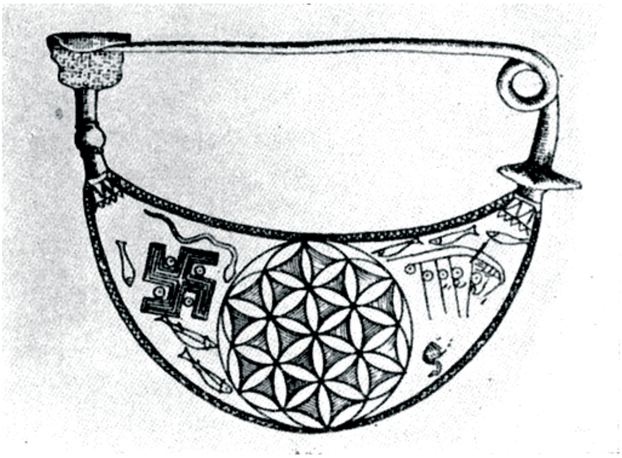

Fig. 8.

Fig. 9.

[13] Erwin Bielefeld, "Schmuck", *Archaeologia Homerica*, Band 1, Kap. C (Göttingen: Vandenhoeck Ruprecht, 1968), p. C51, fig. 6j.

This must be compared to two others found in Thebes in Boeotia, dating from the same time and reproduced by Oscar Montélius (fig. 9).[14] In one point, this represents a real progress. The figure shows the circle divided not only in six but in twelve equal angles.

These two questions deserve to be examined in greater depth.

Translated from the French by Kim Williams

[14] Oscar Montelius, *op. cit.*, 1ère partie, pl. 24.

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