Chapter 2

Statement of the Problems

In this chapter, first we introduce the history of Reliability Theory, then we state the history of Mathematical Theory of Reliability, next we introduce Definition of Reliability and Related Concepts, last we introduce Supplementary Variable Technique and put forward the problems that we will research. We mainly refer in this part to Amstadter [3], Cao and Cheng [12], Frankel [34], Gertsbakh [38], Barlow and Proschan [7], Yamada and Osaki [111], Osaki [92].

2.1 Brief Introduction to Reliability Theory

People have long been concerned with reliability of the products they use and of the friends and associates with whom they are in contact. Although the term "reliable" may not have been used specifically, its meaning was intended. The familiar complaint "things do not last as long as they used to do" is a comparison, although a subjective one, of past and present reliability. When we say that someone is reliable, we mean that the person can be depended on to complete a task satisfactorily on time. These description of reliability are qualitative, and they do not involve numerical measures.

Definition 2.1. Reliability is the probability that a device will operate adequately for a given period of time in its intended application.

Variations have been defined for single operation items such as explosive devices and for characteristics which are not time dependent, but essentially this definition applies. The definition includes the term probability, which indicates the use of a quantitative measure. Probability is the likelihood of occurrence of particular form of any event. It can be determined for any of the innumerable consumer or military equipment which are of interest. Only the methods of measuring the probability differ for the various types of equipment.

In addition to the probabilistic aspect, the reliability definition involves three other considerations: satisfactory operation, length of time, and intended applica-

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tion. There must be a definition of what constitutes satisfactory operation. Certainly, equipment does not necessarily have to be totally inoperative for it to be unsatisfactory. If the compression in two cylinders of an automotive engine is low, the performance of the engine will be less than satisfactory. On the other hand, 100 percent compliance of all desired characteristics may not be a realistic definition of satisfactory performance and something less than 100 percent may be acceptable. The United States has had many manned space flights which were considered very successful even though not every item of equipment performed perfectly. Just what constitutes satisfactory performance must be defined if a measure of reliability is to be meaningful.

The length of time of operation is more definitive. A mission is defined as covering some specific length of time. A warranty is written for a specified number of months or years. Once the criteria of satisfactory performance have been defined, the operation of the equipment can be compared with the criteria for the required time period. Even in this area, however, there may be some flexibility. The criteria of acceptability may change as a function of time so that what is considered satisfactory at the end of the operating period may be something less than what was satisfactory at the beginning. A new automobile should not use any oil between oil changes, while the addition of 1 quart of oil every 1000 miles may very well be acceptable for a 5-year old car.

The last consideration – intended application – must also be a part of the reliability definition. Equipment is designed to operate in a given manner under particular sets of conditions. These include environmental conditions and operating conditions which will be encountered in manufacturing, transportation, storage and use. If the equipment fails or degrades excessively when operated in its intended environment, it is unsatisfactory, whereas if it is subjected to stresses in excess of those for which it was designed, failures or degradation may not be reasonable measures of unreliability.

The importance of obtaining highly reliable systems and components has been recognized in recent years. From a purely economic viewpoint, high reliability is desirable to reduce overall costs. The disturbing fact that the yearly cost of maintaining some military systems in an operable state has been as high as ten times the original cost of the equipment emphasizes this need. The failure of a part or component results in the loss of the failed item but most often the old adage about the loss of a horseshoe nail is truly applicable. A leaky brake cylinder can result in a costly repair bill if it causes an accident. A space satellite may be rendered completely useless if a switch fails to operate or a telemetry system becomes inoperative.

Safety is an equally important consideration. A leaky brake cylinder could result in serious personal injury as well as creating undue expense. The collapse of a landing gear on an aircraft could result in the loss of the plane although no passengers were injured. However, the consequences could easily have been much more serious.

Also caused by reliability (or unreliability) are schedule delays, inconvenience, customer dissatisfaction, loss of prestige (possibly on a national level), and, more serious, loss national security. These conditions also involve cost and safety factors. Cost, for example, is inherent in every failure, as is inconvenience or delay. Most failures also involve at least one of the other considerations. Even the prosaic example of a defective television component involves cost, inconvenience, loss of prestige (of the manufacturer or previous serviceman), and customer dissatisfaction.

The need for and importance of reliability have been reflected in the constantly increasing emphasis placed on reliability by both the government and commercial industry. Most department of Defence, NASA, and AEC contracts impose some degree of reliability requirements on the contractor. These range from the definition of system reliability gaols to requirements for actual demonstration of achievement.

The growth, recognition, and definitization of the reliability function were given much impetus during the 1960s. Reliability has become a recognized engineering discipline, with its own methods, procedures, and techniques. In arriving at this status, it encountered growing pains similar to those that quality assurance experienced in the four previous decades. Convincing corporate management that reliability was economically desirable sometimes required an effort comparable to that expended on the performance of the reliability tasks themselves. Hence, the development of reliability in the area of management and control included justification of its existence as well as application of engineering principles to the organization and direction of reliability activities.

During the growth process, three main technical areas of reliability evolved: (1) reliability engineering, covering systems reliability analysis, design review, and related tasks; (2) operations analysis, including failure investigation and corrective action; (3) reliability mathematics. Each of these areas developed its own body of knowledge and, although specific demarcations can not be drawn between one activity and another, in actual practice the reliability functions are often organized into these divisions. Some activities relate to the design organization; e.g., responsibility for design review is sometimes delegated to the design organization itself. In these instances, reliability is then usually responsible for monitoring the management or contractual directives. However, the third function – reliability mathematics – is seldom delegated outside the reliability organization. First, the methods, although not unique to the reliability function, are not familiar to most personnel in design, testing, and other organizational entities; and second, when the term "reliability" is mentioned, the mathematical aspects are the ones usually thought of. In fact, its primary definition is given in mathematical terms involving probability.

Design aspects of reliability cover such functions as system design analyses, comparison of alternate configurations, drawing and specification reviews, compilation of preferred parts and materials lists, and the preparation and analysis of test programs. Some of the specific activities include failure-modes-and-effects analyses, completion of design review checklists, and special studies and investigations.

Environmental studies are frequently included in the reliability design function, as are supplier reliability evaluations. In organizations associated with complex systems, e.g., refineries or spacecraft, the design reliability functions might be separated into systems concepts, mechanical design, and electrical design groups. A fourth group could include such functions as supplier reliability and parts evaluation. Regardless of the particular organizational structure, however, almost all the individual activities make some use of numerical procedures.

The operations reliability functions relate to manufacturing and assembly operations, test performance, failure analysis and corrective action, operating time and cycle data, field operations reports, and other activities associated with the implementation and test of the design. Personnel in this function help to ensure that the design intent is carried out and they report discrepancies in operations and procedures as well as performance anomalies. The operations reliability group provides much of the data on actual equipment reliability that is used by the other reliability groups.

The statistical group is usually the smallest but can provide equal benefits to the overall program. In addition to accomplishing the reliability numerical activities of prediction, apportionment, and assessment, this group (or individual) provides support to the reliability design and operations groups and directly to the design and test engineers. Statistical designs of experiments, goodness-of-fit tests, system prediction techniques, and other mathematical methods are developed and applied to engineering problems. The methods and procedures discussed herein are applicable to both classes of activities, and it is hoped that reliability design and operations personnel and members of engineering organizations as well as reliability statisticians find them informative and useful.

2.2 Brief Introduction to the Mathematical Theory of Reliability

The mathematical theory of reliability has grown out of the demands of modern technology and particularly out of experiences in World War II with complex military systems. One of the first areas of reliability to be approached with any mathematical sophistication was the area of machine maintenance, see Khintchine [73], Palm [94]. The techniques used to solve these problems grew out of the successful experiences of A.K. Erlang [28], C. Palm [94], and others in solving telephone trunking problems. The earliest attempts to justify the Poisson distribution as the input distribution of calls to a telephone trunk also laid the basis for using the exponential as the failure law of complex equipment.

Applications of renewal theory to replacement problems were discussed as early as 1939 by A.J. Lotka [83], who also summarized earlier work in this area. W. Feller [32, 33] is generally credited with developing renewal theory as a mathematical discipline.

In the late 1930s the subject of fatigue life in materials and the related subject of extreme value theory were being studied by Weibull [105], Gumbel [46] and Epstein [29].

During the 1940s the major statistical effort on reliability problems was in the area of quality control, see Duncan [25].

In the early 1950s certain areas of reliability, especially life testing and electronic and missile reliability problems, started to receive a great deal of attention both from mathematical statisticians and from engineers in the military-industrial complex. Among the first groups to face up seriously to the problem of tube reliability were the commercial airlines, see Carhart [11]. Accordingly, the airlines set up an organization called Aeronautical Radio Inc. (ARINC) which collected and analyzed defective tubes and returned them to the tube manufacturer. In its years of operation with the airlines, ARINC achieved notable success in improving the reliability of a number of tube types. The ARINC program since 1950 has been focused on military reliability problems.

In December 1950 the U S Air Force formed an ad hoc Group on reliability of Electronic Equipment to study the whole question of reliability of equipment and to reduce maintenance costs. By late 1952 the Department of Defense (USA) had established the Advisory Group on Reliability of Electronic Equipment (AGREE). AGREE published its first report on reliability in June of 1957. This report included minimum acceptability limits, requirements for reliability tests, effect of storage on reliability, etc. In 1951 Epstein and Sobel [30] began to work in the field of life testing which was to result in a long stream of important and extremely influential papers. This work marked the beginning of the widespread assumption of the distribution in life-testing research.

In the missile industry Richard R. Carhart [11], Buehler [10], Steck [101], Rosenblatt [98], Madansky [85] were also active at this time in promoting interest in reliability and stating the problems of most interest to their technology.

The mathematically important paper of Moore and Shannon [89] appeared in 1956. This was concerned with relay network reliability. Moore and Shannon [89] were stimulated by von Neumann's attempt to describe certain operations of the human brain and the high reliability that has been attained by complex biological organisms.

Largely motivated by vibration problems encountered in the new commercial jet aircraft, Birnbaum and Saunders [9] in 1958 presented an ingenious statistical model of lifetimes of structures under dynamic loading. Their model made it possible to express the probability distribution of life length in terms of the load given as a function of time and of deterioration occurring in time independently of loading.

2.3 Definitions of Reliability and Related Concepts

In considering various reliability problems, we wish to analyze and calculate certain quantities of interest, designated in the literature by a variety of labels: reliability, availability, efficiency, effectiveness, etc. Even though we do not believe a comprehensive set of definitions is required at this point for understanding the models to follow, it may be of some value to present a unified treatment of the various concepts and quantities involved in the subject of mathematical reliability. Specifically, we shall define mathematically a single generalized quantity which will yield most of the fundamental quantities of reliability theory.

To this end we assume a system whose state at time t is described by $X(t) = (X_1(t), X_2(t), \ldots, X_n(t))$, a vector-valued random variable. For example, $X(t)$ may be the one-dimensional variable taking on the value 1 corresponding to the functioning state and 0 corresponding to the failed state. Alternately, $X(t)$ may be a vector of system parameter values, with each component $X_i(t)$ ranging over an interval of real numbers. $X(t)$, being a random variable, will be governed by a distribution function, $F(x_1, x_2, \ldots, x_n; t)$; explicitly, $F(x_1, x_2, \ldots, x_n; t)$ equals the probability that

$$X_1(t) \leq x_1,\ X_2(t) \leq x_2, \ldots,\ X_n(t) \leq x_n.$$

Now corresponding to any state $\mathbb{x} = (x_1, x_2, \ldots, x_n)$, there is a gain, or payoff, $g(\mathbb{x})$. Thus in the two-state example, just given accruing from being in the functioning state $(x = 1)$ might be one unit of value, so that $g(1) = 1$, and the gain from being in the field state $(x = 0)$ might be 0, so that $g(0) = 0$. The expected gain $G(t)$ at time t will be the quantity of interest; it may be calculated from

$$\begin{aligned} G(t) &= Eg(X(t)) \\ &= \int\int\cdots\int g(x_1, x_2, \ldots, x_n) dF(x_1, x_2, \ldots, x_n; t). \end{aligned} \tag{2-1}$$

Finally, we may average the expected gain $G(t)$ over an interval of time, $a \leq t \leq b$, with respect to some weight function $W(t)$ to obtain

$$H(a, b) = \int_a^b G(t) dW(t). \tag{2-2}$$

Now we are ready to specialize (2-1) and (2-2) to obtain the various basic quantities arising in reliability theory.

Definition 2.2. Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered.

Ordinarily "the period of time intended" is $[0, t]$. Let $X(t) = 1$ if the device is performing adequately at time t, 0 otherwise; we assume that adequate performance at time t implies adequate performance during $[0, t]$. Then from (2-1),

$$G(t) = Eg(X(t)) = P\{X(t) = 1\} = \text{probability that the device performs adequately over } [0, t]. \tag{2-3}$$

Thus $G(t)$ is the reliability of the device as defined above. In general, we shall assume that, unless repair or replacement occurs, adequate performance at time t implies adequate performance during $[0, t]$.

Definition 2.3. Pointwise availability is the probability that the system will be able to operate within the tolerances at a given instant of time.

As before we let $X(t) = 1$ if the system is operating within tolerances at time t, 0 otherwise. Also as before $g(1) = 1,\ g(0) = 0$. We do not exclude the possibility of repair or replacement before time t. Then

$$G(t) = Eg(X(t)) = P\{X(t) = 1\} \text{ is the probability that the system is operating within tolerances at time } t. \tag{2-4}$$

Thus $G(t)$ now yields pointwise availability at the time t.

Definition 2.4. Interval availability is the expected fraction of a given interval of time that the system will be able to operate within the tolerances (Repair and /or replacement is permitted).

Suppose the given interval of time is $[a, b]$. Then with X, g defined as above and $W(t) = \frac{(t-a)}{b-a}$, we compute from (2-2),

$$H(a, b) = \frac{1}{b-a}\int_a^b G(t)dt = \frac{1}{b-a}\int_a^b Eg(X(t))dt, \tag{2-5}$$

so that under suitable regularity conditions

$$H(a, b) = E\frac{\int_a^b g(X(t))dx}{b-a} \tag{2-6}$$

which is the expected fraction of the time interval $[a, b]$ that the system is operating within tolerances. Thus $H(a, b)$ is the interval availability for the interval $[a.b]$.

Definition 2.5. Limiting interval availability is the expected fraction of time in the long run that the system operates satisfactorily.

To obtain limiting interval availability simply compute

$$\lim_{T\to\infty} H(0, T) \tag{2-7}$$

in (2-5) or (2-6), which in some papers is called "limiting efficiency".

Definition 2.6. Interval reliability is the probability that at a specified time, the system is operating and will continue to operate for an interval of duration, say x (Repair and / or replacement is permitted).

To obtain this quantity, let $X(t) = 1$ if the system is operating at time t, 0 otherwise. Then the interval reliability $R(x, T)$ for an interval of duration x starting at time T is given by

$$R(x, T) = P\{X(t) = 1,\ T \leq t \leq T + x\}. \tag{2-8}$$

Limiting interval reliability is simply the limit of $R(x, T)$ as $T \to \infty$, some researchers call it "strategic reliability".

2.4 Supplementary Variable Technique

Throughout the history of reliability theory, large numbers of reliability problems were solved by using reliability models. There are several methods to establish such models. Among them the supplementary variable technique plays an important role. In 1955, D.R. Cox [18] first put forward the "supplementary variable technique" and established the M/G/1 queueing model. After that, the supplementary variable technique was used by many authors to solve a good number of queueing problems (see Chaudhry and Templeton [13]). In the steady-state case, many problems are more readily treated by the supplementary variable technique than by the imbedded Markov chain. In 1963, Gaver [36] first used the supplementary variable technique to study a reliability model. After that, other researchers widely applied this idea to study many reliability problems, see Linton [81], Subramanian and Ravichandran [100], Ohashi and Nishida [93], Goel et al. [43], Chung [15], Dhillon [19], Garg and Goel [35], Gupta and Sharma [52], Kumar et al. [77], Dhillon and Anude [20], Dhillon and Yang [23], Itoi et al. [71], Adachi et al. [1], Yamashiro [113], Murari and Maruthachalam [90], Yamashiro [112], Kodama and Sawa [75], Dhillon and Natesan [22], Goel et al. [44], Goel and Gupta [39], Goel and Gupta [41], Goel and Gupta [40], Goel et al. [42], Gupta and Agarwal [48], Kodama and Sawa [76], Chung [16], Gupta and Agarwal [47], Gupta and Kumar [49], Kodama et al. [74], Gupta and Sharma [52], Gupta et al. [51], Gupta et al. [50], Mokhles and Abo El-Fotouh [88], Wu et al. [106], Liu and Cao [82], Dhillon and Fashandi [21].

In the supplementary variable technique a non-Markovian process in continuous time is made Markovian by inclusion of one or more supplementary variables. But the above mathematical models established by the supplementary variable technique were described by partial differential equations with integral boundary conditions. So, it is necessary to study their well-posedness. In addition, many of the researchers who established the above reliability models were interested in steady-state reliability indices and therefore they researched steady-state reliability indices under the following hypothesis (see Chung [15], Dhillon [19], Garg and

Goel [35], Gupta and Sharma [52], Kumar et al. [77], Dhillon and Anude [20], Dhillon and Yang [23]):

$$\lim_{t\to\infty} p(x,t) = p(x),$$

here

$$p(x,t) = (p_0(t), p_1(x,t), p_2(x,t), \ldots, p_n(x,t)),$$
$$n \text{ is finite or infinite},$$
$$p(x) = (p_0, p_1(x), p_2(x), p_3(x), \ldots, p_n(x)),$$
$$n \text{ is finite or infinite}.$$

The above hypothesis means that the time-dependent solutions of the models converge to their steady-state solutions.

There were many researchers who tried to study the time-dependent solutions of the reliability models. Almost all of them first established mathematical models to describe corresponding reliability problems, then studied the time-dependent solution by using the Laplace transform and probability generating functions, next at most determined the expression of the probability generating function's Laplace transform or expression of their time-dependent solutions' Laplace transform if the boundary conditions are simple. Last, they studied steady-state reliability indices such as pointwise availability, operational behavior, and cost analysis (see Itoi et al. [71], Adachi et al. [1], Yamashiro [113], Murari and Maruthachalam [90], Yamashiro [112], Kodama and Sawa [75], Dhillon and Natesan [22], Goel et al. [44], Goel and Gupta [39], Goel and Gupta [41], Goel and Gupta [40], Goel et al. [42], Gupta and Agarwal [48], Kodama and Sawa [76], Chung [16], Gupta and Agarwal [47], Gupta and Kumar [49], Kodama et al. [74], Gupta and Sharma [52], Gupta et al. [51], Gupta et al. [50], Mokhles and Abo El-Fotouh [88], Wu et al. [106], Liu and Cao [82], Dhillon and Fashandi [21]). Roughly speaking, the above researchers obtained existence of the time-dependent solution of the above models and their steady-state reliability indices but have not answered whether the above hypothesis holds. As we know, steady-state solutions of the systems depend on their time-dependent solutions and time-dependent solutions reflect clearly tendency of the systems. Hence, we should study the existence of the time-dependent solutions of the above models and their asymptotic behavior, time-dependent reliability indices and their asymptotic behavior.

In reliability theory literature, solutions to reliability models have been mostly obtained in terms of probability generating functions or Laplace-Stieltjes transform of the probability distributions of interest.This book is an effort to study time-dependent solutions of reliability models, their asymptotic behavior and asymptotic behavior of the reliability indices.

In Chapter 3, we do dynamic analysis for a system which was described by a finite number of partial differential equations with integral boundary conditions. Firstly, we establish the mathematical model by using the supplementary variable technique to describe the system, next by using the knowledge in Chapter 1 we prove that the model has a unique positive time-dependent solution. Thirdly, by using the knowledge in Chapter 1, we prove that the time-dependent solution of the model exponentially converges to its steady-state solution. Lastly, we obtain asymptotic behavior of reliability indices for the system. In Chapter 4, we study a system which was described by an infinite number of partial differential equations with integral boundary conditions. First of all, by using the supplementary variable technique we establish the mathematical model to describe this system, next by using the knowledge in Chapter 1 we obtain the well-posedness of the model. In addition, we prove that the time-dependent solution of this model strongly converges to its steady-state solution and show that our result about convergence is best. Moreover, by using the cone theory we deduce the asymptotic behavior of the reliability indices for the system.

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