Chapter 1

Overview

Power generation based on renewable energy sources plays an important role in the development of a sustainable and environmentally friendly generation of energy, motivated by the finite nature of fossil energy sources and environmental pollution. In particular, wind energy is considered to be most promising to provide a substantial part of the electrical energy supply. But due to the fluctuating behavior of power production from renewable energies, especially caused by wind power production, new challenges are posed to the structure of power generation systems. In this context, we approach the question of how energy storages and flexible generation units may contribute to decouple fluctuating supply and demand, yielding a sustainable and cost efficient energy production. To this end, the problem is formulated as an optimization model including combinatorial, nonlinear, and stochastic aspects. By approximating the nonlinearities, we receive a stochastic multistage mixed-integer program. The aim of this thesis is the development of a solution algorithm which is capable to solve test instances sufficiently large to provide reliable results. This is accomplished by developing a decomposition approach based on splitting the corresponding scenario tree, enhanced by mixed integer programming techniques, such as primal methods and cutting plane generation.

In detail, the thesis is structured as follows. In Chapter 2, we give an introduction to the power generation problem, which arises when large amounts of fluctuating energy are fed into the public supply network. In this context, the focus is on the potential of energy storages in order to decouple supply and demand. Next to a description of the basic technical characteristics of the facilities considered in the generation system, a survey on related literature is given regarding modeling and solution approaches.

Chapter 3 contains the mathematical modeling of the power generation problem. In the first part, a deterministic model is presented assuming all data to be known in advance. With the aim of a realistic description, partial load efficiencies of the facilities are taken into account leading to the integration of nonlinear functions into the model. In order to handle the resulting mixed integer nonlinear problem, an approximation of the nonlinearities by piecewise linear functions is described. In the second part of this chapter, we extend the model towards the inclusion of uncertainty concerning the amount of wind power available and the market prices for electricity. Using a scenario tree approach to describe the evolution of the uncertain data, we formulate a multistage stochastic mixed-integer problem.

Chapter 4 addresses the investigation of polyhedral substructures of the problem. In particular, we investigate the facial structure of the polytope arising from the description of switching processes with minimum running time and minimum down time restrictions in a scenario tree formulation. Based on the results for the deterministic case, we derive a complete linear description of the polytope occurring within the stochastic formulation. Using these inequalities as cutting planes we incorporate them in the solution process of the problem described above.

The focus of Chapter 5 is on the generation of good feasible solutions based on the idea of relax-and-fix. With regard to the deterministic problem formulation, an adapted rolling horizon algorithm is presented, where the relaxation of the integrality conditions is enhanced by problem specific approximation schemes. Assuming the original problem to be feasible, we investigate the possibility of running into infeasible subproblems. In this context, we show that the algorithm terminates with a feasible solution of the entire problem, imposing certain conditions on the input data and the approximation scheme. Subsequently, we provide an adaptation to the stochastic problem by extending the generation of the subproblems, approximation schemes, and feasibility results yielding an approximate-and-fix heuristic.

One crucial point of this thesis concerns the development of a novel solution approach to the stochastic power generation problem from above which is presented in Chapter 6. We reformulate the original problem by decomposing it into several subproblems coupled by few coupling constraints which is based on the splitting of the scenario tree into subtrees. In order to determine global optimal solutions of the problem, we integrate this approach into a branch-and-bound framework called *SD-BB* (scenario tree based decomposition combined with branch-and-bound). Furthermore, we extend this method by applying Lagrangian relaxation in order to generate tighter lower bounds of the optimal solution value.

In Chapter 7, we describe the implementation of the *SD-BB* algorithm mentioned above. First, we focus on its initialization phase whose core comprises the splitting of the scenario tree into several subtrees where we present a polynomial time algorithm. Furthermore, we discuss and specify suitable branching techniques for *SD-BB* focusing on variable selection rules and the determination of branching points in case of continuous variables. Finally, we address the computation of dual bounds as well as the determination of feasible solutions, exploiting the special structure of the problem at hand.

In order to evaluate the performance of the developed methods various test runs are performed which are summarized in Chapter 8. Besides the presentation of the numerical effects applying the developed separation algorithm and the primal heuristics, we focus on the computational investigation of the *SD-BB* algorithm. By applying the algorithm to various instances scaled with respect to the basic properties of the *S-OPGen* problem and comparing the results with the commercial solver CPLEX the performance of the algorithm is investigated. Finally, we complete this thesis in Chapter 9 with a conclusion and suggestions for further improvements and investigations.