Chapter 2
General Properties of Diffraction Radiation

2.1 Diffraction Radiation as Radiation from Polarization Currents

As mentioned above, diffraction radiation can be considered as radiation generated by polarization currents induced in a medium by the field of a moving charge. The distance between the charge trajectory and medium surface is usually much larger than the mean intermolecular distance in the medium. At the same time, it is well known that the field of the charge moving in vacuum with velocity $\mathbf{v}$ and energy $E = \gamma mc^2$ decreases as $\exp(-h\omega/\gamma v)$ with distance h in the direction perpendicular to the velocity. Hence, polarization currents are located in a layer close to the surface and the properties of diffraction radiation depend strongly on the properties of this layer. In particular, radiation does not appear from a charged particle uniformly moving in parallel to the infinite plane surface of a homogeneous medium, because the conservation laws for radiation forbid the transfer of the longitudinal momentum to the medium in such a geometry. However, if the medium is inhomogeneous or its surface is not planar, the field can transfer the longitudinal momentum to the medium and radiation can appear.

As known, a medium with an electromagnetic field can be considered as homogeneous if not only the average density of the number of atoms is constant, but also the intermolecular distances are much smaller than the field wavelength. Therefore, the medium can be treated as homogeneous in the optical frequency range, but it should be considered as inhomogeneous in the high-frequency range. This means that diffraction radiation from the same surface must be considered in different ways in different frequency ranges.

Charge e whose motion in the homogeneous medium with relative permittivity $\varepsilon(\omega)$ is described by the law $x = 0$, $y = 0$, and $z = vt$ creates a field with the vector potential whose Fourier transform in space and time has the form

$$\mathbf{A}(\mathbf{q}, \omega) = \frac{e\mathbf{v}}{2\pi^2} \frac{\delta(\omega - \mathbf{qv})}{q^2 - (\omega/c)^2 \varepsilon(\omega)} \tag{2.1}$$

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In order to determine the distance dependence of the Fourier transform of the field of such a charge with frequency ω, we consider the time Fourier transform of the vector potential:

$$\mathbf{A}(\mathbf{r},\omega)=\frac{e\mathbf{v}}{2\pi^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}dq_x dq_y\frac{\exp\{iq_x x+iq_y y+i(\omega z/v)\}}{q_x^2+q_y^2+(\omega/v)^2[1-(v/c)^2(\varepsilon(\omega))]} \quad (2.2)$$

We consider only the case, where Cherenkov radiation is absent at the frequency under consideration and the following condition is satisfied:

$$(v/c)^2\varepsilon(\omega)<1 \quad (2.3)$$

In this case, the denominator of the integrand in Eq. (2.2) is positive. The leading contribution to the integral comes from the d_x^2 and d_y^2 values smaller than or about $[1-(v/c)^2(\varepsilon(\omega))]$, when the denominator in Eq. (2.3) is close to its minimum value. However, if x or y values are sufficiently large so that $q_x x \gg 1$ or $q_y y \gg 1$, the exponent in the numerator of the integrand in Eq. (2.2) oscillates rapidly and significantly reduces the integral. This does not occur if the coordinates x and y are smaller than or about the limiting values x_0 and y_0 determined from the conditions

$$x_0^2(\omega/v)^2\left[1-(v/c)^2\varepsilon(\omega)\right]=1, \qquad y_0^2(\omega/v)^2\left[1-(v/c)^2\varepsilon(\omega)\right]=1. \quad (2.4)$$

Hence, the time Fourier component of the vector potential is large in the region, where the x and y coordinates are smaller than or about values x_0 and y_0, and is small beyond this region. The calculation of the integral shows that the x dependence has the form $\exp(-x/x_0)$. The dependence of the scalar potential, electric field, and magnetic field of the charge is the same. Thus, the ω-frequency Fourier component of the field of the charge uniformly moving in the medium decreases as $\exp\left\{-\frac{h\omega}{v}\sqrt{1-(v/c)^2\varepsilon(\omega)}\right\}$ in the direction transverse to the velocity. If relative permittivity $\varepsilon(\omega)$ is not close to one, the difference $1-(v/c)^2\varepsilon(\omega)$ is not small and is on the order of one. Therefore, the exponent is comparable to $-x\omega/v$; i.e., it depends only slightly on the energy of the fast particle. However, for frequencies much higher than atomic frequencies, the relative permittivity is close to one and has the form $\varepsilon(\omega)=1-(\omega_p/\omega)^2$. For ultrarelativistic particles and high frequencies, $1-(v/c)^2\varepsilon(\omega)\approx(\omega_p/\omega)^2+\gamma^{-2}$ and the ω-frequency Fourier component of the field of the charge moving in the medium decreases as $\exp\left\{-\frac{h\omega}{v}\sqrt{(\omega_p/\omega)^2+\gamma^{-2}}\right\}$ in the direction x transverse to the velocity.

Thus, the effective range of the field of the ultrarelativistic charged particle moving in vacuum increases linearly with the particle energy at frequency ω, whereas the energy dependence of the range of the field of the ultrarelativistic charged particle moving in the medium is more complex. Under condition (2.3), the range of the field increases only for frequencies much higher than atomic frequencies. This means that the polarization currents induced by the uniformly moving charged particle are primarily concentrated in a layer with thickness $\lambda\,(v/c)$ and they are concentrated in a wider layer with a thickness of about $\lambda\,(v/c)\left((\omega_p/\omega)^2+\gamma^{-2}\right)^{-1/2}$ only for high frequencies.

The source of diffraction radiation is the polarization current that is generated by the field of the particle and whose time Fourier transform can be represented in the form

$$\mathbf{j}\,(\mathbf{r},\omega)=\frac{i\omega}{4\pi}\left\{1-\varepsilon\,(\mathbf{r},\omega)\right\}\mathbf{E}\,(\mathbf{r},\omega)\equiv\sigma\,(\mathbf{r},\omega)\,\mathbf{E}\,(\mathbf{r},\omega)\,, \tag{2.5}$$

where $\varepsilon\,(\mathbf{r},\omega)$ is the relative permittivity and $\sigma\,(\mathbf{r},\omega)$ is the conductivity of the medium.

If the layer in which polarization currents appear is sufficiently thin, the effect of the polarization currents can be considered as small perturbation and this circumstance allows one to solve the problem by the method of successive approximations. To this end, the fields are represented in the form of power series in the polarization current. In the zeroth approximation, the polarization currents can be disregarded in microscopic Maxwell's equations, so that the field in this approximation coincides with the field of the charge uniformly moving in vacuum. In the first approximation, the polarization currents in Maxwell's equations are considered to be generated by the zeroth approximation field, and exact field $\mathbf{E}\,(\mathbf{r},\omega)$ induced by the fast particle in the medium can be replaced in expression (2.5) for the polarization current by the field $\mathbf{E_0}\,(\mathbf{r},\omega)$ of this particle in vacuum.

Let us take into account that, if the homogeneous medium is bounded by the $x=0$ plane, the uniform motion of the charged particle in parallel to the z axis does not induce the radiation field. Diffraction radiation appears if the region occupied by the medium is specified by a more general condition of $x<\varsigma\,(y,z)$. Let us find the point with the minimum x coordinate on the medium surface $x=\varsigma\,(y,z)$ and chose the coordinate axes so that the $x=0$ plane pass through this point. In this case, all inhomogeneities are located inside the layer between the $x=\varsigma\,(y,z)$ and $x=0$ surfaces.

The diffraction radiation intensity depends strongly on the relation between the characteristic sizes of the problem: the range of the exponential decrease in the field of the fast particle $\lambda\,(v/c)\left[1-(v/c)^2\,\varepsilon\,(\omega)\right]^{-1/2}$, wavelength λ, and the thickness of the inhomogeneous layer $\varsigma\,(y,z)$. In the nonrelativistic case, the field of the particle decreases rapidly with the penetration depth to the medium. As a result, the thickness of the inhomogeneous layer can become much larger than the field penetration depth; i.e., $\varsigma\,(y,z)\gg\lambda\,(v/c)$. In this case, diffraction radiation is determined only by surface sections closest to the particle trajectory and information on the properties of the entire surface cannot be obtained from diffraction radiation. This information can be acquired only under the condition

$$\lambda\,(v/c) > \varsigma\,(y, z)\,. \tag{2.6}$$

The investigation of diffraction radiation began with the nonrelativistic case [1]. The results of investigations for this case were summarized in [2].

For nonrelativistic particles, $\gamma \simeq 1$ and v/c can be so small that the penetration depth of the optical-frequency field is about or smaller than intermolecular distances. In this case, optical-frequency diffraction radiation cannot be described in the framework of macroscopic electrodynamics. However, macroscopic electrodynamics at the same particle velocity and medium surface can be applicable for diffraction radiation with lower frequencies, e.g., for cm wavelength range.

Diffraction radiation generated by relativistic particles began to be investigated slightly later, but in a wider range including optical frequencies and is actively studied by many authors [3–11].

Note that the intensity of diffraction radiation from surfaces of certain profiles was determined in many studies through approximate numerical calculations, because this problem is rather complicated. In particular, the calculations of the energy losses of the electron moving near an inhomogeneous dielectric were reported in [12]. The numerical calculations of the energy losses of the electron beam moving near a dielectric sphere and radiation appearing in this case were presented in [13]. The characteristics of radiation appearing when the electron moves near a dielectric surface on the shape of this surface were discussed in [14] with the use of the results of the numerical calculations.

2.2 Formation Length of Diffraction Radiation

The estimate presented in Sect. 1.2 for the formation length of radiation from the fast particle refers to the case, where the charged particle itself is a source of radiation. Strictly speaking, polarization currents generated in the medium by the field of the charge uniformly moving in vacuum are directly responsible for diffraction radiation. For this reason, it is useful to estimate the formation length with the inclusion of the features of diffraction radiation. Diffraction radiation is usually considered with the use of the equations of macroscopic electrodynamics with the boundary conditions at the interface between the media. If these inhomogeneities are small, phenomenological theory can be inapplicable for describing such radiation. In this case, microscopic theory should be used. Let us estimate the formation length of diffraction radiation with the use of this theory.

From the microscopic point of view, diffraction radiation appears due to the scattering of the field of the uniformly moving charge from the atoms of the medium. Such a scattering from one atom with the formation of the radiation field was considered in Sect. 1.3.

Let us consider the scattering of one Fourier component of the self field of the fast particle from two identical medium atoms at points $\mathbf{R}_1$ and $\mathbf{R}_2$ on the z axis. Let the fast charged particle uniformly move in vacuum in parallel to the z axis according to the law $\mathbf{r} = \mathbf{b} + \mathbf{v}t$ in the $x = b$ plane. Taking the x axis along $\mathbf{b}$, we can represent the field created in such a motion of the particle in the form

$$\mathbf{E}_0(\mathbf{r}, t) = \int d^3q \int d\omega \mathbf{E}_0(\mathbf{q}, \omega) \exp(i\mathbf{qr} - i\mathbf{qv}t),$$
$$\mathbf{E}_0(\mathbf{q}, \omega) = \mathbf{E}_0(\mathbf{q})\, \delta(\omega - q_z v), \tag{2.7}$$
$$\mathbf{E}_0(\mathbf{q}) = -\frac{ie}{2\pi^2} \frac{\mathbf{v}\omega c^2 - \mathbf{q}}{q^2 - \omega^2/c^2} \exp(-iq_x b).$$

Let us consider the Fourier component of the field of the particle, $\mathbf{E}_0(\mathbf{q}, \omega)\, e^{i\mathbf{qr} - i\mathbf{qv}t}$, as the incident wave. Repeating the consideration leading to Eq. (1.47), we can obtain the following expression for the Fourier transform of the current density generated in the atoms located at points $\mathbf{R}_1$ and $\mathbf{R}_2$ by the Fourier component of the field of the particle:

$$\mathbf{j}(\mathbf{k}, \omega) = \frac{-i\omega}{(2\pi)^3} \alpha(\omega)\, \mathbf{E}_0(\mathbf{q}, \omega) \left[\exp\{i(\mathbf{q} - \mathbf{k})\mathbf{R}_1\} + \exp\{i(\mathbf{q} - \mathbf{k})\mathbf{R}_2\}\right]. \tag{2.8}$$

Spectral–angular distribution of radiation created by the Fourier component of the field of the fast particle takes the form (where $\mathbf{k} = \frac{\omega}{c}\frac{\mathbf{r}}{r}$)

$$\frac{d^2W(\mathbf{n}, \omega)}{d\omega d\Omega} = \frac{\omega^2}{c} |\alpha(\omega)|^2 \left|\left[\mathbf{k}\mathbf{E}_0(\mathbf{q}, \omega)\right]\right|^2 2\left[1 + \cos\{(\mathbf{q} - \mathbf{k})(\mathbf{R}_1 - \mathbf{R}_2)\}\right]. \tag{2.9}$$

For the case under consideration, vector $\mathbf{R}_1 - \mathbf{R}_2$ is directed along the z axis; hence, $(\mathbf{q} - \mathbf{k})(\mathbf{R}_1 - \mathbf{R}_2) = (q_z - k_z)(Z_1 - Z_2) = L(\omega/v - k_z)$. The factor $2\left[1 + \cos\{L(\omega/v - k_z)\}\right]$ takes values from zero to four in dependence on the cosine argument $(\mathbf{q} - \mathbf{k})(\mathbf{R}_1 - \mathbf{R}_2) = L(\omega/v - k_z)$. For $L(\omega/v - k_z) \ll 1$, this factor is equal to four. In this case, the energy emitted by two atoms is four times higher than the energy emitted by one atom. This means that the waves emitted by both atoms are coherent, i.e., arrive at a detector with the same phases and their amplitudes are summed. As a result, the field near the detector is doubled and the energy reaching the detector is quadrupled.

If $L(\omega/v - k_z) \gg 1$, $\cos\{L(\omega/v - k_z)\}$ is a rapidly oscillating function. The detector is detected the radiation energy arriving in finite frequency and angular ranges. The integral of the expression with the rapidly oscillating function over these ranges is equal to zero. Therefore, if $L(\omega/v - k_z) \gg 1$, the radiation energy from two atoms is twice as high as the radiation energy from one atom. In this case, the waves arriving at the detector from different atoms are incoherent, i.e., have significantly different phases; for this reason, the interference term in the energy is negligibly small and intensities, rather than amplitudes, of the waves are summed.

Thus, the condition of the coherence of radiations from two atoms can be written in the form

$$L \ll l_c, \tag{2.10}$$

where the length

$$l_c \sim \frac{2\pi}{\omega/v - k_z} = \frac{\lambda}{\beta^{-1} - n_z} \tag{2.11}$$

is called the coherence length or the length of the formation of diffraction radiation. Note that length l_c varies from $\beta\lambda$ for nonrelativistic particles or emission in the direction perpendicular to the particle velocity to $\gamma^2\lambda$ for the case of emission along the velocity of the ultrarelativistic particle.

However, according to Eq. (2.9), radiations from two atoms can also be coherent under the condition

$$L\,(\omega/v - k_z) = 2\pi n, \quad n = 1, 2, 3 \ldots \tag{2.12}$$

In terms of the variables $\beta = v/c$ and $k_z = (\omega/c)\, n_z$, this condition can be represented in the form

$$\beta^{-1} - n_z = \frac{\lambda n}{L}, \tag{2.13}$$

or, in view of Eq. (2.11),

$$L = l_c n. \tag{2.14}$$

Thus, two atoms of the medium emit coherently if distance L between them is equal to an integer number of coherence lengths l_c.

We now consider the coherence conditions for the case of the ordered arrangement of atoms, e.g., in a single crystal. Let N atoms be located on a straight line at the same distance $|\mathbf{d}| = d$ from each other so that $\mathbf{R}_g = gd$, where $g = 0, 1, 2, \ldots, N-1$. The Fourier transform of the current density generated in these atoms by the Fourier component of the field of the particle is written similarly to Eq. (2.8) as

$$\mathbf{j}\,(\mathbf{k}, \omega) = \frac{-i\omega}{(2\pi)^3}\alpha\,(\omega)\,\mathbf{E}_0\,(\mathbf{q}, \omega) \sum_{g=0}^{N-1} \exp\{i\,(\mathbf{q} - \mathbf{k})\, g\mathbf{d}\}\,. \tag{2.15}$$

When vector $\mathbf{d}$ is directed along the z axis,

$$(\mathbf{q} - \mathbf{k})\,\mathbf{d} = d\left(\frac{\omega}{v} - \frac{\omega}{c}\cos\theta\right) = d\frac{\omega}{c}\left(\beta^{-1} - \cos\theta\right), \tag{2.16}$$

where θ is the angle between the photon emission direction and z axis. The ratio of the intensity of radiation generated by N atoms to the intensity of radiation generated by one atom is given by the expression

$$\frac{d^2 W_N(\mathbf{n},\omega)}{d^2 W_1(\mathbf{n},\omega)} = \left|\sum_{g=0}^{N-1} \exp\left\{igd\frac{\omega}{c}\left(\beta^{-1}-\cos\theta\right)\right\}\right|^2 = \\ = \frac{\sin^2\left(\frac{dN}{2}\frac{\omega}{c}\left(\beta^{-1}-\cos\theta\right)\right)}{\sin^2\left(\frac{d}{2}\frac{\omega}{c}\left(\beta^{-1}-\cos\theta\right)\right)}. \tag{2.17}$$

The function on the right-hand side of Eq. (2.17) has a number of sharp peaks determined by the condition

$$d\frac{\omega}{c}\left(\beta^{-1}-\cos\theta\right) = 2\pi n, \quad n = 1, 2, 3, \ldots \tag{2.18}$$

The height and width of each peak corresponding to a given n value are proportional to N^2 and N^{-1}, respectively. Dispersion relation (2.18) describes Smith—Purcell radiation, which will be considered in detail in Chap. 6.

Note that, although we discuss radiation generated by individual atoms, the consideration is also applicable to radiation from individual inhomogeneities such as strips of a diffraction grating or target surface irregularities.

Formula (2.18) can be represented in the form of the requirement that the period of the structure is equal to an integer number of coherence lengths:

$$d = nl_c. \tag{2.19}$$

It is convenient to use radiation coherence condition (2.19) for periodic structures, whereas condition (2.10) is more convenient for qualitative analysis of phenomena caused by one irregularity or irregularities are located chaotically.

2.3 Radiation from Relativistic Particle Near a Screen

It is useful to consider the manifestations of the features of diffraction radiation generated by ultrarelativistic particles for a simple case of radiation generated by a particle moving near a flat screen. Let the screen of a homogeneous medium occupy a spatial region specified by the inequalities $-a < z < 0$, $x < 0$ and the particle with the charge e move in vacuum near the screen according to the law $z = vt$, $y = 0$, and $x = b > 0$. We consider diffraction radiation with optical and lower frequencies, i.e., when the wavelength is much larger than intermolecular distances and the medium of the screen can be treated as homogeneous. In this case, the Fourier transform of the polarization current given by Eq. (2.5) is represented as

$$\mathbf{j}(\mathbf{r},\omega) = \frac{i\omega}{4\pi}\{1-\varepsilon(\mathbf{r},\omega)\}\mathbf{E}(\mathbf{r},\omega) = \sigma(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega), \\ \sigma(\mathbf{r},\omega) = \sigma(\omega)\theta(z+a)\theta(-z)\theta(-x); \quad \theta(x) = \frac{x+|x|}{2|x|}. \tag{2.20}$$

As pointed out above, diffraction radiation is formed not on the entire screen, but only in a small screen part that is the screen layer with a thickness of about of the $\lambda\,(v/c)\left[1-(v/c)^2\,\varepsilon\,(\omega)\right]^{-1/2}$ closest to the particle trajectory. The total field $\mathbf{E}\,(\mathbf{r},\omega)$ in Eq. (2.20) is the sum of the self field of the fast particle, $\mathbf{E}_0\,(\mathbf{r},\omega)$, and radiation field, $\mathbf{E}_1\,(\mathbf{r},\omega)$. Since the volume of the screen part in which the radiation field is formed is small, we can assume that $\mathbf{E}_1 \ll \mathbf{E}_0$ and use the method of successive approximations to solve the problem. In the first approximation, the polarization current can be treated as being generated only by the self field of the fast particle and the effect of the radiation field can be neglected. In this case, the polarization current density is known and the problem reduces to the calculation of radiation generated by the given current. In order to use expression (1.11) for the spectral–angular distribution of the emitted energy, it is necessary to determine the Fourier transform of the polarization current density given by Eq. (2.20) in space and time. We take into account that the field of the charged particle whose motion is specified by the law $z=vt$, $y=0$, $x=b$ has the form of Eq. (2.7). With the use of Eqs. (2.20) and (2.7), it is easy to obtain the Fourier transform of the polarization current in the coordinates and time in the form (where $Q\equiv\omega/v-q_z$)

$$\mathbf{j}\,(\mathbf{q},\omega)=\frac{\sigma\,(\omega)}{4\pi^2 v}\int_{-a}^{0}dz\int_{-\infty}^{0}dx\int_{-\infty}^{\infty}ds_x\,\exp\left(is_x x+iQz\right)\mathbf{E}\left(q_x+s_x,q_y,\omega/v\right). \tag{2.21}$$

First, we integrate with respect to s_x with the use of the relation [15]

$$\int_{-\infty}^{\infty}ds\frac{(1;\,s)}{s^2+G^2}\exp\left(isp\right)=(1;\;iG)\,\frac{\pi}{G}\exp\left(-pG\right). \tag{2.22}$$

Introducing the notation

$$G\left(q_y\right)=\left[q_y^2+\gamma^{-2}\,(\omega/c)^2\right]^{1/2} \tag{2.23}$$

and disregarding the γ^{-2} corrections, we can reduce Eq. (2.21) for the ultrarelativistic case to the form

$$\begin{aligned}\mathbf{j}\,(\mathbf{q},\omega)=&\frac{\sigma\,(\omega)}{4\pi^2 v}\left\{i\mathbf{e_x}+\frac{q_y\mathbf{e_y}}{G\left(q_y\right)}\right\}\times\\&\times\int_{-a}^{0}dz\int_{-\infty}^{0}dx\,\exp\left\{iQz-(b-x)\,G\left(q_y\right)-iq_x x\right\},\end{aligned} \tag{2.24}$$

where $\mathbf{e_x}$ and $\mathbf{e_y}$ are the orts of the x and y axes, respectively. Integration with respect to x and z yields

$$\mathbf{j}(\mathbf{q},\omega) = \frac{e\sigma(\omega)}{8\pi^3 v} \frac{\left[1-\exp(iQa)\right]\exp\left\{-bG\left(q_y\right)\right\}}{G\left(q_y\right)Q\left\{G\left(q_y\right)+iq_x\right\}} \left\{\mathbf{e_x}G\left(q_y\right) - i\mathbf{e_y}q_y\right\}. \quad (2.25)$$

The energy emitted by polarization current (2.25) in vacuum for the total observation time in frequency range $d\omega$ in solid angle element $d\Omega$ in the direction of vector $\mathbf{k} = \frac{\omega}{c}\frac{\mathbf{r}}{r}$ is given by the expression

$$d^2W(\mathbf{n},\omega) = \frac{1}{c}(2\pi)^6 \left|\left[\mathbf{kj}(\mathbf{k},\omega)\right]\right|^2 d\omega d\Omega. \quad (2.26)$$

The substitution of Eq. (2.25) into Eq. (2.26) yields the spectral–angular distribution of diffraction radiation generated by the ultrarelativistic particle

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{e^2}{v}|\sigma(\omega)|^2 \frac{\left[\mathbf{ke_y}\right]^2 k_y^2 + \left[\mathbf{ke_x}\right]^2 G^2\left(k_y\right)}{G^2\left(k_y\right)\left\{G^2\left(k_y\right)+k_x^2\right\}} \times$$
$$\times \frac{4\sin^2\left(\frac{a}{2}(\omega/v-k_z)\right)}{(\omega/v-k_z)^2}\exp\left\{-2bG\left(k_y\right)\right\}. \quad (2.27)$$

Radiation in the ultrarelativistic case is concentrated in the region of small angles θ with respect to the particle velocity: $k_z \approx k\left(1-\theta^2/2\right)$, $k_y \approx k\theta\sin\varphi$, and $k_x \approx k\theta\cos\varphi$. Therefore, $(\omega/v)-k_z \approx (\omega/2)\left(\theta^2+\gamma^{-2}\right)$. The function $G(k)$ is on the order of k at $\theta \sim 1$ and $\sin^2\varphi \sim 1$, so that $\exp(-2bG) \sim \exp(-2bk)$, whereas $G(k) \sim k/y$ for small θ and $\sin^2\varphi$ values, so that $\exp(-2bG) \sim \exp(-2bk/y)$. Thus, the contribution to radiation from the angular range $\theta \sim 1$ and $\sin^2\varphi \sim 1$ is exponentially small and the main contribution to radiation comes from the region of small angles $\theta \leq \gamma^{-1}$ and angles φ close to zero or π. This dependence on angle φ means that the radiation emission directions are primarily concentrated near the xz plane, i.e., near the symmetry plane of the problem.

The radiation intensity depends strongly on the ratio of screen thickness a to coherence length $\lambda\gamma^2$. If the screen thickness is much smaller than the coherence length, i.e., $a\left[(\omega/v)-k_z\right]/2 \ll 1$, the radiation distribution given by Eq. (2.27) takes the form

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{e^2}{v}4a^2|\sigma(\omega)|^2 \frac{\left[\mathbf{ke_y}\right]^2 k_y^2 + \left[\mathbf{ke_x}\right]^2 G^2\left(k_y\right)}{G^2\left(k_y\right)\left\{G^2\left(k_y\right)+k_x^2\right\}}\exp\left\{-2b\,G\left(k_y\right)\right\}. \quad (2.28)$$

For optical frequencies and $\gamma \sim 10^3$, the coherence length can be equal to several centimeters, so that the condition $a \ll \lambda\gamma^2$ is easily satisfied. If $a\left[(\omega/v)-k_z\right]/2 = \pi/2$, Eq. (2.27) is modified to the form

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{e^2}{v}4|\sigma(\omega)|^2 \frac{\left[\mathbf{ke_y}\right]^2 k_y^2 + [\mathbf{ke_x}]^2 G^2(k_y)}{G^2(k_y)\left\{G^2(k_y)+k_x^2\right\}(\omega/v-k_z)^2} \exp\left\{-2b\,G(k_y)\right\}. \tag{2.29}$$

However, if the frequency and direction of radiation satisfy the condition $a(\omega/v - k_z) = 2K\pi$, where K is an integer number, the radiation intensity is equal to zero. This means that angles corresponding to the maxima and minima of the intensity exist at a given radiation frequency. The appearance of these maxima and minima is attributed to the coherence of radiation formed in various sections of the screen. We emphasize that the coherence length for diffraction radiation generated by the nonrelativistic particle is on the order of $\lambda(v/c)$ and the problem of creating the screen with the thickness comparable with the coherence length for optical and higher frequencies is impracticable.

Disregarding the γ^{-2} corrections, we can represent Eq. (2.28) for angles $\theta \leq \gamma^{-1}$ in the form

$$\begin{aligned}\frac{d^2W(\omega,\theta,\varphi)}{\theta d\theta d\varphi d\omega} &= \frac{e^2}{v}|\sigma(\omega)|^2 \frac{4a^2}{k^2}\frac{2\theta^2\sin^2\varphi+\gamma^{-2}}{\left(\theta^2\sin^2\varphi+\gamma^{-2}\right)\left(\theta^2+\gamma^{-2}\right)}\times \\ &\times \exp\left\{-2bk\sqrt{\theta^2\sin^2\varphi+\gamma^{-2}}\right\}.\end{aligned} \tag{2.30}$$

According to this expression, the radiation intensity is low if the distance between the trajectory of the ultrarelativistic particle and the dielectric screen, b (impact parameter), is much larger than $\gamma\lambda$.

This result is obtained under the assumption that the radiation field is much lower than the self field of the particle. Although this assumption is always valid for the frequencies exceeding optical frequencies, it will be shown in Chap. 4 that a more accurate approach similar to that developed in Sect. 1.4 for calculating the characteristics of transition radiation at the frequencies exceeding optical frequencies is required in ultraviolet and soft X-ray frequency ranges.

The imaginary part of the relative permittivity should be taken into account for optical and lower frequencies. This is most substantial for conducting media when $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ and $\varepsilon'(\omega) \ll \varepsilon''(\omega)$. In this case, the field of the particle varies strongly in the medium at the skin-layer thickness $\delta \sim \frac{c}{\omega}\sqrt{\varepsilon''(\omega)}$; therefore, the method of successive approximations is inapplicable. Hence, Eqs. (2.27), (2.28), (2.29), (2.30) are applicable in a frequency region, where $\varepsilon''(\omega) \ll \varepsilon'(\omega) < 1$.

2.4 Diffraction Radiation from Ultrarelativistic Particles

As known, radiation in the ultrarelativistic case is concentrated in the region of small θ angles near the particle velocity direction. In this case, the thickness of the surface layer with polarization currents for the ultrarelativistic particle can be on the order of or much smaller than the wavelength, whereas the formation length of diffraction radiation, $l_c \sim \lambda \left(\beta^{-1} - \cos\theta\right)^{-1}$, for the characteristic emission angles about $\theta \ll 1$ is much larger than the field wavelength λ. The diffraction radiation intensity is low when the linear sizes of inhomogeneities either much smaller or much larger than the radiation formation length. This means that diffraction radiation is most intense when the sizes of the surface inhomogeneities are on the order of the radiation formation length. Therefore, diffraction radiation generated by the ultrarelativistic particle moving near the medium whose surface irregularities have a linear size of b is most intense for the frequencies on the order of $4\pi\,(c/b)\left(\gamma^{-2} + \theta^2\right)^{-1}$. This is much different from diffraction radiation generated by the nonrelativistic particle, where the intensity maximum is in a frequency range of about $4\pi\,(v/b)$.

Let us consider diffraction radiation in the X-ray frequency range when the radiation frequency is much higher than atomic frequencies and the wavelength is smaller than or about the atomic size. Note that the consideration of X-ray diffraction radiation is meaningful only for the ultrarelativistic particles, because such a radiation for the nonrelativistic particles is negligibly small due to the fast decrease in the self field in the direction perpendicular to the velocity.

In the x-ray frequency range, macroscopic electrodynamics is inapplicable, averaging over the volume is not performed, but the electron number density is averaged over the quantum-mechanical states and thermal motion of the atoms. The polarization current density depending on the atomic coordinates is involved in microscopic Maxwell's equations in order to include the reverse effect of the polarization current on the field. This effect in the x-ray frequency range is small and can be considered as small perturbation. Since atomic frequencies are much lower than the field frequency, the coupling forces of the atomic electrons interacting with the field can be disregarded and the electrons can be considered as free particles when calculating the polarization current. In this approximation, microscopic Maxwell's equations can be represented in a form similar to the equations of the macroscopic electrodynamics of an inhomogeneous medium by introducing the following analogue of the relative permittivity:

$$\varepsilon\,(\mathbf{r},\omega) = 1 - \chi\,(\mathbf{r},\omega) \equiv 1 - \frac{4\pi e^2}{m\omega^2}\sum_a f\,(\mathbf{r} - \mathbf{R}_a)\,, \tag{2.31}$$

where the summation is performed over all the molecules of the medium, $\mathbf{R}_a$ is the radius vector of the center-of-mass of the molecule, and $f\,(\mathbf{r})$ is the electron number density in the molecule averaged over the quantum-mechanical electronic states and thermal motion. The microscopic electron number density in the medium can be represented in the form

$$n_e = \sum_a f\,(\mathbf{r} - \mathbf{R}_a)\,. \tag{2.32}$$

The electric displacement can be formally introduced by the expression

$$\mathbf{D}\,(\mathbf{r},\omega) = \varepsilon\,(\mathbf{r},\omega)\,\mathbf{E}\,(\mathbf{r},\omega)\,. \tag{2.33}$$

However, this quantity is a microscopic quantity and depends on the coordinates of the atoms of the medium. The formal coincidence of Eq. (2.33) with the relation between the displacement and field of the inhomogeneous medium in macroscopic electrodynamics is natural, because the medium cannot be treated as homogeneous for a field with the wavelength of about atomic sizes. When the charged particle moves in parallel to the plane surface of the inhomogeneous medium, the field can transfer momentum to the irregularities of the medium and thereby generate diffraction radiation. This phenomenon is the simplest example of X-ray diffraction radiation.

In this region, the relative permittivity is close to one and, hence, perturbation theory in the small quantity $\chi(\mathbf{r}, \omega) = 1 - \varepsilon(\mathbf{r}, \omega)$ can be used. In the first approximation, only the expansion terms linear in $\chi(r, \omega)$ can be retained. In the zeroth approximation, the quantity $\chi(\mathbf{r}, \omega)$, i.e., polarization currents in the medium, can be neglected. For the problem concerning diffraction radiation, this means that the zeroth approximation field is the field of the charged particle uniformly moving in infinite vacuum. The field of the charged particle whose motion is described by the law $x = b,\ \ y = 0,\ z = vt$ has the form of Eq. (2.7).

A source of diffraction radiation is the polarization current that is induced by the self field $\mathbf{E}$ in the medium and whose time Fourier transform can be represented in the form

$$\mathbf{j}(\mathbf{r}, \omega) = \frac{i\omega}{4\pi}\chi(\mathbf{r}, \omega)\,\mathbf{E}_0(\mathbf{r}, \omega) = \frac{ie^2}{m\omega}\mathbf{E}_0(\mathbf{r}, \omega)\sum_a f(\mathbf{r} - \mathbf{R}_a). \tag{2.34}$$

Passing to the space Fourier transforms in Eq. (2.34) with the use of Eq. (2.7) and the relation

$$f(\mathbf{r}) = \int d^3q f(\mathbf{q})\exp(i\mathbf{q}\mathbf{r}), \tag{2.35}$$

we easily obtain

$$\mathbf{j}(\mathbf{k}, \omega) = \frac{ie^2}{m\omega}\int d^3p\mathbf{E}_0(\mathbf{k} - \mathbf{p})\,\delta(\omega - k_z v + p_z v)\, f(\mathbf{p})\sum_a \exp\left(-i\mathbf{p}\mathbf{R}_a\right). \tag{2.36}$$

For a crystal, $\mathbf{R}_a = \mathbf{e_x}a_x l + \mathbf{e_y}a_y m + \mathbf{e_z}a_z s$, where $l = 1, 2, \ldots L$, $m = 1, 2, \ldots M$, $s = 1, 2, \ldots S$, so that

$$\sum_a \exp\left(-i\mathbf{p}\mathbf{R}_a\right) = \frac{\exp(-ip_x a_x L) - 1}{\exp(-ip_x a_x) - 1} \times \frac{\exp\left(-ip_y a_y M\right) - 1}{\exp\left(-ip_y a_y\right) - 1} \times \frac{\exp(-ip_z a_z S) - 1}{\exp(-ip_z a_z) - 1}. \tag{2.37}$$

Each factor in Eq. (2.37) is a rapidly oscillating function of vector **p** and almost does not contribute to the integral except for the **p** magnitudes for which the arguments of the exponentials are small, $p_x a_x L \ll 1$, $p_y a_y M \ll 1$, and $p_z a_z S \ll 1$. Thus, the right-hand side of Eq. (2.37) is equal to the product $L \cdot M \cdot S$, i.e., to the number of molecules, N, in the entire crystal volume. If $L \gg 1$, $M \gg 1$, and $S \gg 1$, the **p** vector magnitudes for which sum (2.37) is not small coincide with the reciprocal lattice vectors

$$\mathbf{g}\left(\frac{2\pi}{a_x}n_1, \frac{2\pi}{a_y}n_2, \frac{2\pi}{a_z}n_3\right), \tag{2.38}$$

where n_1, n_2, n_3 are arbitrary integers. Thus,

$$\sum_a \exp\left(-i\mathbf{p}\mathbf{R}_a\right) = N\delta\left(\mathbf{p} - \mathbf{g}\right). \tag{2.39}$$

The substitution of Eq. (2.39) into Eq. (2.36) yields

$$\mathbf{j}\left(\mathbf{k}, \omega\right) = \frac{ie^2}{m\omega} N \sum_{\mathbf{g}} \mathbf{E}_0\left(\mathbf{k} - \mathbf{g}\right)\delta\left(\omega - k_z v + g_z v\right) f\left(\mathbf{g}\right). \tag{2.40}$$

Here, the Dirac delta function means that the Fourier transform of the polarization current is nonzero only at certain values of angle θ between vector **k** and particle velocity **v**:

$$\cos\theta = \frac{c}{v}\left(1 + \frac{vg_z}{\omega}\right). \tag{2.41}$$

The energy emitted by a certain current in vacuum for the total observation time in the frequency range $d\omega$ to the solid angle element $d\Omega$ in the direction of vector **k** is given by the expression

$$d^2 W\left(\mathbf{n}, \omega\right) = \frac{(2\pi)^6}{c}\left|\left[\mathbf{kj}\left(\mathbf{k}, \omega\right)\right]\right|^2 d\omega d\Omega. \tag{2.42}$$

The substitution of Eq. (2.40) into Eq. (2.42) gives rise to the appearance of the Dirac delta function squared

$$\delta^2\left(\omega - k_z v + g_z v\right) = \frac{T}{2\pi}\delta\left(\omega - k_z v + g_z v\right), \tag{2.43}$$

where T is the total observation time. This means that the angular distribution of diffraction radiation from the crystal consists of a set of narrow peaks near the angles satisfying inequality (2.41) for various g_z values. For this reason, each peak can be analyzed independently. X-ray diffraction radiation generated by the ultrarelativistic particle moving near a single crystal is discussed in detail in Sect. 4.5.

The properties of the electromagnetic waves with the frequencies between optical frequencies and x-ray frequencies differ from the properties of the optical and X-ray waves. This frequency range is the range of ultraviolet and soft x-ray radiations. In this range, the field wavelength is larger than atomic sizes:

$$\lambda \gg \frac{\hbar^2}{me^2}, \quad \text{i.e.} \quad \hbar\omega \ll \alpha mc^2 \tag{2.44}$$

(where $\alpha = e^2/\hbar c \simeq 1/137$ is the fine structure constant) and the frequencies are higher than atomic frequencies:

$$\hbar\omega \gg \alpha^2 mc^2. \tag{2.45}$$

In the frequency range, where both conditions (2.44) and (2.45) are satisfied,

$$\alpha mc^2 \gg \hbar\omega \gg \alpha^2 mc^2, \tag{2.46}$$

macroscopic electrodynamics is applicable; i.e., the properties of the medium are described by the usual relative permittivity. Since the wavelength is much larger than atomic sizes, the homogeneous medium approximation is applicable. Owing to inequality (2.45), the binding forces of the electrons in the atom can be disregarded. Hence, the relative permittivity of the medium in the frequency range specified by inequalities (2.46) can be represented as

$$\varepsilon(\omega) = 1 - \omega_p^2/\omega^2, \quad \omega_p = \sqrt{4\pi NZe^2/m}, \tag{2.47}$$

where N is the number of atoms per unit medium value and Z is the nuclear charge number. The thickness of the medium layer, where the polarization currents induced by an ultrarelativistic particle with the energy $E = \gamma Mc^2$ in the frequency range specified by inequalities (2.46) are located, is given by the expression (where $\beta = v/c$)

$$a \sim \frac{\lambda\beta}{\sqrt{(\omega_p/\omega)^2 + \gamma^{-2}}}. \tag{2.48}$$

The thickness of this layer is much larger than the radiation wavelength.

Let an ultrarelativistic charged particle move near the nonplanar surface of a homogeneous medium occupying the spatial region specified by the inequality $\varsigma(y,z) > x$ and changes in the surface profile occur in the region $x > 0$. In this case, the coordinate dependence of the relative permittivity is determined only by the medium surface profile and, hence,

$$1 - \varepsilon(\mathbf{r},\omega) = (\omega_p/\omega)^2 \theta[\varsigma(y,z) - x], \qquad \theta(u) = \frac{u + |u|}{2|u|}. \tag{2.49}$$

As shown in Sect. 2.1, diffraction radiation in this case is generated by polarization currents in the layer between the $x = 0$ plane and $x = \varsigma(y, z)$ surface. The Fourier transform of the polarization current density can be represented as

$$\mathbf{j}(\mathbf{r}, \omega) = \frac{i\omega_p^2}{4\pi\omega}\mathbf{E}(\mathbf{r}, \omega)\,\theta(x)\,\theta\left[\varsigma(y, z) - x\right], \tag{2.50}$$

where $\mathbf{E}(\mathbf{r}, \omega)$ is the Fourier transform of the field generated by the moving particle in the medium. In order to determine this field by usual methods of macroscopic electrodynamics, it is necessary to find the general solution of Maxwell's equations in the medium and in vacuum and to match these solutions on the surface of the homogeneous medium, $x = \varsigma(y, z)$. The exact solution can be obtained for the surfaces of the simplest profile, whereas approximate methods should be used for other cases. When $\omega_p^2/\omega^2 \ll 1$, this ratio can be used as a small parameter in the method of successive approximations. This problem is analyzed in detail in Chap. 4.

2.5 Effect of the Excitation of the Medium on Diffraction Radiation

Diffraction radiation appearing when the charged particle moves near the surface of the stationary medium is considered above. The properties of such a medium are time independent and its energy is conserved. If the relative permittivity of the medium is $\varepsilon(\omega) < (c/v)^2$, Cherenkov radiation is impossible, the energy and momentum conservation laws in the emission process are satisfied due to the transfer of momentum to the medium, but the transfer of momentum is possible only in the inhomogeneous medium. If the properties of the medium vary in time, the energy of the medium is not conserved. The properties of the nonstationary medium in macroscopic electrodynamics were considered in [16, 17]. In such a medium, the energy exchange between the medium and field is possible; in particular, the energy and momentum conservation laws for radiation can be satisfied without the transfer of momentum to the medium due only to the energy transfer from the medium to the field. This means that diffraction radiation appears when the charged particle uniformly moves in parallel to the planar surface of the homogeneous nonstationary medium [18]. If the medium is nonstationary and inhomogeneous, momentum transfer $\Delta\mathbf{p}$ to the medium and energy transfer ΔE from the medium can exist in the process of emission. In this case, the energy conservation law for the emission of the transverse wave with frequency ω and wave vector $\mathbf{k}$ has the form of Eq. (1.54):

$$\hbar\omega = \hbar\mathbf{k}\mathbf{v} + \mathbf{v}\Delta\mathbf{p} + \Delta E. \tag{2.51}$$

If $\mathbf{v}\Delta\mathbf{p} \ll \Delta E$, the transfer of the energy from the medium to the field obviously plays the leading role in the emission process and the momentum transfer can be disregarded in the first approximation. In this approximation, the problem reduces to the calculation of diffraction radiation from the homogeneous nonstationary medium.

As an example, we consider the medium excited by a certain interaction. Our consideration is limited to the case, where the ionization of the medium in the process of excitation can be disregarded; i.e., excitation energy per atom is lower than the ionization potential. This means that excitation is weak. After the termination of such an interaction, the electronic excitation energy migrates in the medium in the form of long-lived elementary excitations of the medium, i.e., electromagnetic longitudinal plane waves.

As known, the relation between frequency ω and wave vector $\mathbf{q}$ of the longitudinal plane electromagnetic wave is determined by the condition of vanishing the relative permittivity as a function of the frequency and wave vector: $\varepsilon(\mathbf{q}, \omega) = 0$. For small $\mathbf{q}$ magnitudes, the solution of this equation has the form

$$\omega(\mathbf{q}) = \omega_p + (\alpha/2)\, q^2, \tag{2.52}$$

where α is a constant and ω_p is the frequency at which the relative permittivity is equal to zero at $q = 0$ [19] : $\varepsilon\left(\omega_p, q = 0\right) = 0$. Hence, the electron field of the longitudinal plane wave for small q values has the form

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E} \exp\left\{i\mathbf{qr} - i\omega_p t - i(\alpha/2)\, q^2 t\right\}. \tag{2.53}$$

The propagation velocity of such a wave is $\mathbf{u} = \alpha\mathbf{q}$.

Let us consider diffraction radiation appearing when the ultrarelativistic charged particle moves in parallel to the planar surface of the homogeneous medium whose excitation is described by longitudinal wave (2.53). In such a geometry of the problem, diffraction radiation is impossible in the absence of the longitudinal wave. In the presence of the longitudinal wave, the medium becomes nonstationary and inhomogeneous. The energy conservation law in such a medium has the form of Eq. (2.51). The typical momentum transferred to the medium is on the order of $\hbar\mathbf{q}$ and the typical energy acquired from the medium is on the order of $\hbar\omega(q)$. The inequality $\mathbf{v}\Delta\mathbf{p} \ll \Delta E$ takes the form $qv \ll \omega_p + (\alpha/2)\, q^2$ and, under this inequality, radiation in the first approximation can be considered as diffraction radiation from the homogeneous nonstationary medium by changing the field of the longitudinal waves to $\mathbf{E}_p \exp\left(-i\omega_p t\right)$. However, since the momentum transfer from the field to the medium is disregarded, $\mathbf{q} = \mathbf{k}$ and the equality $\mathbf{kv} \ll \omega_p + \alpha k^2/2$ should be satisfied. This equality is always satisfied for the nonrelativistic particles and is satisfied for the ultrarelativistic particles only at the frequencies $\omega \ll \omega_p$. This condition determines the region of applicability of the approximate solution method.

In order to avoid insignificant complications, we assume that the distance from the particle trajectory to the medium surface is not small and the self field in the medium is low, so that the layer in which the polarization current exists is sufficiently thin. In this case, the effect of the polarization current can be treated by the method of successive approximations in the powers of the polarization current. In the zeroth approximation, the polarization currents can be disregarded in microscopic Maxwell's equations, so that the field in this approximation coincides with the field of the charged particle uniformly moving in vacuum. In the first approximation, the terms linear in the polarization current are taken into account. In this approximation, it can be assumed that the polarization current in the equations for the first approximation field is induced by the zeroth approximation field, i.e., by the field $\mathbf{E}_0(\mathbf{r}, t)$ of the charged particle uniformly moving in vacuum. Thus, the calculation of the first approximation field reduces to the problem of the field generated by a given current in vacuum.

Let us find the polarization current density in the medium for the case of the sufficiently thin layer with the polarization current. Assuming that the fields acting on a bound atomic electron are much lower than atomic fields, the electron can be treated as quasielastically bound. The equations of motion of the electron quasielastically bound in an atom under the action of the field of the fast particle,

$$\mathbf{E}_0(\mathbf{r}, t) = \int d^3q \int d\omega \mathbf{E}_0(\mathbf{q}, \omega) \exp(i\mathbf{q}\mathbf{r} - i\omega t), \tag{2.54}$$

and field $\mathbf{E}_p \exp\left(-i\omega_p t\right)$ have the form

$$\frac{d^2\mathbf{r}(t)}{dt^2} + \omega_\mu^2 \mathbf{r}(t) = \frac{e}{m}\left\{\mathbf{E}_p \exp\left(-i\omega_p t\right) + \int d^3q \int d\omega \mathbf{E}_0(\mathbf{q}, \omega)\, e^{i\mathbf{q}\mathbf{r}(t)}\right\}, \tag{2.55}$$

where e is the elementary charge and m, and ω_μ are the mass, and oscillation frequency of the bound atomic electron, respectively. Taking into account that the field wavelength is much larger than the oscillation amplitude, we can solve this equation by the method of successive approximations in the field powers:

$$\mathbf{r}(t) = \mathbf{r}_0(t) + \mathbf{r}_1(t) + \mathbf{r}_2(t) + \ldots, \tag{2.56}$$

where $\mathbf{r}_0(t)$ is field independent, $\mathbf{r}_1(t)$ is linear in the field, $\mathbf{r}_2(t)$ is square in the field, etc. In the zeroth approximation in the field, this equation is the equation of the free oscillations of the electron. In the first approximation, only terms linear in field hold in Eq. (2.55). In this case, since the field wavelength is much larger than atomic sizes, $\mathbf{r}_0(t)$ can be replaced by the radius vector of the atomic nucleus, $\mathbf{R}_a$:

$$\frac{d^2\mathbf{r}_1(t)}{dt^2} + \omega_\mu^2 \mathbf{r}_1(t) = \frac{e}{m}\left\{\mathbf{E}_p \exp\left(-i\omega_p t\right) + \int d^3q \int d\omega \mathbf{E}_0(\mathbf{q}, \omega)\, e^{i\mathbf{q}\mathbf{R}_a - i\omega t}\right\}. \tag{2.57}$$

The solution of this equation makes it possible to determine the polarization current density $\mathbf{J}(\mathbf{r}, t)$ appearing in the case of the homogeneous stationary medium. However, as shown above, diffraction radiation does not appear in the problem geometry under consideration for the case of the homogeneous stationary medium.

In this approximation, when the solution linear in the polarization current is sought, current $\mathbf{J}(\mathbf{r}, t)$ can be disregarded and the additional polarization current $\mathbf{j}(\mathbf{r}, t)$ appearing due to the excitation of the medium can be calculated. To this end, it is necessary to solve the second-approximation equation for motion of the electron quasielastically bound in the atom in the presence of the field of the fast particle, $\mathbf{E}_0(\mathbf{r}, t)$, in the excited medium. This equation contains the field-squared terms:

$$\frac{d^2\mathbf{r}_2(t)}{dt^2} + \omega_\mu^2 \mathbf{r}_2(t) = \frac{ie}{m}\int d^3p\left(\mathbf{p}\mathbf{r}_1(t)\right)\int d\omega\,\mathbf{E}_0(\mathbf{p}, \omega)\exp\left(i\mathbf{p}\mathbf{R}_a - i\omega t\right). \tag{2.58}$$

The transition to the time Fourier transforms of the coordinates in Eq. (2.57) provides

$$\left(\omega_\mu^2 - \omega\right)\mathbf{r}_1(\omega) = \frac{e}{m}\left\{\mathbf{E}_p\delta\left(\omega - \omega_p\right) + \int d^3p\,\mathbf{E}_0(\mathbf{p}, \omega)\exp\left(i\mathbf{p}\mathbf{R}_a\right)\right\}, \tag{2.59}$$

so that

$$\mathbf{r}_1(t) = \mathbf{r}_{1p}(t) + \mathbf{r}_{10}(t); \qquad \mathbf{r}_{1p}(t) = \frac{e}{m}\mathbf{E}_p\frac{\exp\left(-i\omega_p t\right)}{\omega_\mu^2 - \omega_p^2}, \tag{2.60}$$

$$\mathbf{r}_{10}(t) = \frac{e}{m}\int d^3p\int d\omega\frac{\mathbf{E}_0(\mathbf{p}, \omega)}{\omega_\mu^2 - \omega^2}\exp\left(i\mathbf{p}\mathbf{R}_a - i\omega t\right). \tag{2.61}$$

The substitution of the expression for $\mathbf{r}_1(t)$ into Eq. (2.58) results in the appearance of two terms on the right-hand side of Eq. (2.58). The first term is squared in $\mathbf{E}_0$ and the second term is proportional to both self field $\mathbf{E}_0$ and longitudinal wave field $\mathbf{E}_p$. Correspondingly, the solution of Eq. (2.58) consists of two terms, $\mathbf{r}_2(t) = \mathbf{r}_{2p}(t) + \mathbf{r}_{20}(t)$. Radiation of interest is associated with the solution $\mathbf{r}_{2p}(t)$ of the equation following from Eq. (2.58):

$$\frac{d^2\mathbf{r}_{2p}(t)}{dt^2} + \omega_\mu^2\mathbf{r}_{2p}(t) = \frac{ie}{m}\int d^3p\left(\mathbf{p}\mathbf{r}_{1p}(t)\right)\int d\omega\mathbf{E}_0(\mathbf{p}, \omega)\,e^{i\mathbf{p}\mathbf{R}_a - i\omega t}. \tag{2.62}$$

The substitution of Eq. (2.60) into Eq. (2.62) provides the expression

$$\mathbf{r}_{2p}(t) = i\left(\frac{e}{m}\right)^2\left(\omega_\mu^2 - \omega_p^2\right)\int d^3q\left(\mathbf{p}\mathbf{E}_p\right)\int d\omega\frac{\mathbf{E}_0\left(\mathbf{p}, \omega - \omega_p\right)}{\omega_\mu^2 - \omega^2}e^{i\mathbf{p}\mathbf{R}_a - i\omega t}. \tag{2.63}$$

From this expression, the current density in the atom for such a motion of the electrons is easily obtained in the dipole approximation (summation is performed over all the atomic electrons):

$$\mathbf{j}_a(\mathbf{r},t) = e\sum_{\mu}\frac{d\mathbf{r}_{2p}(t)}{dt}\delta(\mathbf{r}-\mathbf{R}_a). \tag{2.64}$$

Summing this expression over all the atoms of the medium, we can arrive at the following expression for the polarization current density, which is responsible for diffraction radiation:

$$\mathbf{j}(\mathbf{r},t) = \sum_a \delta(\mathbf{r}-\mathbf{R}_a)\int d^3p\mathbf{E}_0(\mathbf{p})\left(\mathbf{p}\mathbf{E}_p\right)\int d\omega U(\omega)\,\delta\left(\omega-\omega_p-p_z v\right)e^{i\mathbf{p}\mathbf{R}_a-i\omega t}, \tag{2.65}$$

where

$$U(\omega) = \sum_{\mu}\frac{e^3\omega}{m^2\left(\omega_\mu^2-\omega_p^2\right)\left(\omega_\mu^2-\omega^2\right)} \tag{2.66}$$

and it is taken into account that the field of the charged particle whose motion is described by the law $y = 0$, $x = b$, and $z = vt$ is given by Eq. (2.7). Assuming that the field wavelength is much larger than interatomic distances, we can average the current density over the coordinates of the medium atoms and arrive at the expression

$$\mathbf{j}(\mathbf{r},t) = n_0\int d^3R\,\theta(-X)\int d^3p\left(\mathbf{p}\mathbf{E}_p\right)\int d\omega\,U(\omega)\,\delta\left(\omega-\omega_p-p_z v\right)\mathbf{E}_0(\mathbf{p})\,e^{i\mathbf{p}\mathbf{R}-i\omega t}, \tag{2.67}$$

where n_0 is the average number of atoms per unit volume and $\theta(x)$ is the Heaviside step function (see, Eq. (2.49)). The Fourier transform of polarization current (2.67) in space and time is easily obtained in the form

$$\mathbf{j}(\mathbf{k},\omega) = U(\omega)\;\delta\left(\omega-\omega_p-k_z v\right)\int_0^\infty dX\int_{-\infty}^\infty dp_x\left\{\mathbf{k}\mathbf{E}_p+(p_x-k_x)\,\mathbf{e_x}\mathbf{E}_p\right\}\times$$
$$\times\,\mathbf{E}_0\left(p_x,k_y,k_z\right)\exp\{i\,(k_x-p_x)\,X\}, \tag{2.68}$$

where $\mathbf{e_x}$ is the ort of the x axis. The substitution of Eq. (2.7) into Eq. (2.68) yields (where $\gamma = 1/\sqrt{1-v^2/c^2}$)

$$\mathbf{j}(\mathbf{k},\omega) = \beta(\omega)\;\delta\left(\omega-\omega_p-k_z v\right)\int_0^\infty dX\exp(ik_xX)\int_{-\infty}^\infty dp_x\exp\{-ip_x(X+b)\}\times$$
$$\times\left\{\mathbf{k}\mathbf{E}_p+(p_x-k_x)\,\mathbf{e_x}\mathbf{E}_p\right\}\frac{ie}{2\pi^2c^2}\,\frac{\mathbf{e_x}(p_x-k_x)+\mathbf{e_y}k_y+\mathbf{e_z}\left(\omega-\omega_p\right)/v\gamma^2}{p_x^2+k_y^2+\left[(\omega-\omega_p)/v\gamma\right]^2}. \tag{2.69}$$

Introducing the notation

$$G\left(k_y\right)=\sqrt{k_y^2+\left(\frac{\omega-\omega_p}{v\gamma}\right)^2} \tag{2.70}$$

and integrating with respect to p_x with the use of the known relation [15]

$$\int_{-\infty}^{\infty} du\,\left(1;u;u^2\right)\frac{\exp\left(iua\right)}{u^2+G^2}=\left(1;\ iG\mathrm{sign}(a);\ -G^2\right)\frac{\pi}{G}\exp\left(-\left|a\right|G\right), \tag{2.71}$$

we reduce Eq. (2.69) to the form

$$\begin{aligned}\mathbf{j}\left(\mathbf{k},\omega\right)=U\left(\omega\right)\,\delta\left(\omega-\omega_p-k_z v\right)\int_0^{\infty} dX\exp\left\{ik_xX-(X+b)G\right\}\times\\ \times\left\{\mathbf{kE}_p+\left(iG-k_x\right)\mathbf{eE}_p\right\}\frac{ie}{2\pi G}\left\{\mathbf{e_x}\left(iG-k_x\right)+\mathbf{e_y}k_y+\mathbf{e_z}\frac{\omega-\omega_p}{v\gamma^2}\right\}.\end{aligned} \tag{2.72}$$

The integration with respect to X yields

$$\begin{aligned}\mathbf{j}\left(\mathbf{k},\omega\right)=U\left(\omega\right)\,\delta\left(\omega-\omega_p-k_z v\right)\frac{\exp\left(-bG\right)}{ik_x-G}\times\\ \times\left\{\mathbf{kE}_p+\left(iG-k_x\right)\mathbf{eE}_p\right\}\frac{ie}{2\pi G}\left\{\mathbf{e_x}\left(iG-k_x\right)+\mathbf{e_y}k_y+\mathbf{e_z}\frac{\omega-\omega_p}{v\gamma^2}\right\}.\end{aligned} \tag{2.73}$$

The spectral–angular distribution of the energy emitted by this current at long distances has the form

$$\frac{d^2W}{d\omega\,d\Omega}=(2\pi)^6\,\frac{1}{c}\,\left|\left[\mathbf{kj}\left(\mathbf{k},\omega\right)\right]\right|^2. \tag{2.74}$$

The substitution of Eq. (2.73) into Eq. (2.74) provides the distribution of the diffraction radiation energy from the homogeneous medium excited by one longitudinal wave (where T is the total observation time):

$$\begin{aligned}\frac{d^2W}{d\omega\,d\Omega}&=(2\pi)^3\,T\,e^2\left|U\left(\omega\right)\right|^2\exp\left(-2bG\right)\delta\left(\omega-\omega_p-k_z v\right)\times\\ &\times\frac{\left\{\left(\left(\mathbf{k}-k_x\mathbf{e_x}\right)\mathbf{E}_p\right)^2+\left(G\mathbf{eE}_p\right)^2\right\}\left\{\left[\mathbf{ke_x}\right]^2G^2+\left[\mathbf{k}\left(\mathbf{e_y}k_y-\mathbf{e_x}k_x+\mathbf{e_z}(\omega-\omega_p)/v\gamma^2\right)\right]^2\right\}}{G^2\left(k_y^2+G^2\right)}.\end{aligned} \tag{2.75}$$

The excitation of the medium is often distributed homogeneously and isotropically. This corresponds to the isotropic distribution of the longitudinal waves in the medium. In this case, the distribution of the energy of diffraction radiation can be obtained from Eq. (2.75) by integrating over the directions of the field of the longitudinal wave $\mathbf{E}_p$ and the result has the form

$$\frac{d^2W}{d\omega\, d\Omega} = (2\pi)^3\, Te^2\, |U(\omega)|^2 \exp(-2bG)\, \delta\left(\omega - \omega_p - k_z v\right) \times$$
$$\times\, \frac{1}{2}\mathbf{E}_p^2 \left\{G^2 + k_y^2 + k_z^2\right\} \frac{\left\{[\mathbf{ke}]^2 G^2 + \left[\mathbf{k}\left(\mathbf{e_y}k_y - \mathbf{e_x}k_x + \mathbf{e_z}(\omega - \omega_p)/v\gamma^2\right)\right]^2\right\}}{G^2\left(k_x^2 + G^2\right)}. \tag{2.76}$$

The medium can be excited by an acoustic wave. In this case, the homogeneous stationary initial medium becomes both inhomogeneous and nonstationary. If a surface acoustic wave is generated in the medium, the surface profile of the medium also changes. Radiation generated by the charged particle moving near the medium surface along which the surface acoustic wave propagates was investigated in [21, 22]. Considering the effect of the acoustic wave as small perturbation, the authors of those works obtained the spectral–angular distribution of the diffraction-radiation energy and showed that such a radiation can be observed in the range of millimeter and submillimeter waves.

2.6 Diffraction Radiation from a Charged Particle Reflected from a Single Crystal

The fast charged particle incident on the surface of a single crystal at small grazing angle ς (angle between the particle velocity and surface) undergoes mirror reflection from the surface if $\varsigma < \theta_L = (U/E)^{1/2}$ [23] (here, θ_L is the Lindhard angle, U is the potential barrier of the surface, and E is the particle energy).

Change in the velocity in the process of reflection leads to bremsstrahlung. At the same time, the polarization of the surface by the charged particle results in the appearance of polarization currents also leading to radiation. Therefore, such a radiation appears due to the join action of the mechanisms of bremsstrahlung and diffraction radiation [24].

Estimation of the intensity of such a radiation is of interest, because the effective surface of the single crystal from which electrons are reflected can differ from the effective surface from which the electromagnetic field is reflected. Indeed, the electrons are reflected from the surface atomic layer, whereas the electromagnetic field is reflected from the electrons of the medium. Meanwhile, the conduction electron density near the surface in metals undergoes small Fridel oscillations and vanishes outside the surface ion layer at distance $z \sim \hbar/p_F$, where $\hbar$ is Planck's constant, p_F is the Fermi momentum, and the $z = 0$ plane coincides with the surface ion layer. The effective crystal surfaces determined from the reflection of particles and light are generally spaced by a certain distance b from each other. The intensity of the radiation under consideration is a function of this distance and can provide information on it.

It should be taken into account that radiation is formed in a finite time of about $\tau = 1/(\omega - \mathbf{kv})$ (where ω and $\mathbf{k}$ are the frequency and wave vector of radiation, respectively, and v is the particle velocity). The distance from the particle to the surface changes by $\tau \varsigma v$ in time τ.

When $b \ll \tau \varsigma v$, the distance between the effective reflection surface for particles and light does not affect the radiation intensity. We emphasize that the case, where $\varsigma > \theta_L$ and the charged particle penetrates into the crystal rather than is reflected from it, is not considered below.

Let us consider radiation appearing when the particle with charge e is reflected from the planar surface of the semi-infinite ($z < 0$) cubic crystal with relative permittivity ε_1. The particle moves in the homogeneous isotropic medium with relative permittivity ε_2. The Fourier transform of the current density of the charge undergoing mirror reflection from the $z = 0$ surface can be represented in the form

$$\begin{aligned} \mathbf{j}\left(q_x, q_y, z, \omega\right) &= (2\pi)^{-3} \iiint dxdydt\mathbf{j}\left(\mathbf{r}, t\right) \exp\left(-iq_y y + i\omega t\right) = \\ &= \frac{e}{4\pi^3 u} \left\{\mathbf{v}\cos\left[(\omega - \mathbf{qv})\, zu\right] + i\mathbf{u}\sin\left[(\omega - \mathbf{qv})\, z/u\right]\right\}. \end{aligned} \tag{2.77}$$

Here, $\mathbf{v}$ and $\mathbf{u}$ are the velocity components tangential and normal to the surface, respectively.

The partial solution of Maxwell's equations for the self field of the particle has the form

$$E_{0z}\left(\mathbf{q}, z, \omega\right) = \frac{e}{\pi^2 \varepsilon_2 c^2} \frac{\omega \varepsilon_2 u^2 - c^2 (\omega - \mathbf{qv})}{u^2 k_2^2 - (\omega - \mathbf{qv})^2} \sin\left[(\omega - \mathbf{qv})\, z/u\right], \tag{2.78}$$

$$H_{0z}\left(\mathbf{q}, z, \omega\right) = \frac{ieu}{\pi^2 c} \times \frac{[\mathbf{qv}]}{u^2 k_2^2 - (\omega - \mathbf{qv})^2} \cos\left[(\omega - \mathbf{qv})\, z/u\right], \tag{2.79}$$

$$k_{1(2)} = \sqrt{(\omega/c)^2 \varepsilon_{1(2)} - q^2}. \tag{2.80}$$

The total field outside the crystal consists of the self field of the particle and the field $\mathbf{E}_2$ of the transverse waves leaving the surface. The field inside the crystal consists only of the field $\mathbf{E}_1$ of the transverse waves leaving the surface:

$$\mathbf{E}_{1(2)}\left(\mathbf{q}, z, \omega\right) = E_{1(2)}\left(q, \omega\right) \exp\left\{-(+)\, ik_{1(2)} z\right\}. \tag{2.81}$$

From the condition that fields $\mathbf{E}_1$ and $\mathbf{E}_2$ are transverse and the boundary conditions on the crystal surface $z = 0$, it is easy to derive the relations determining the normal components of the radiation fields:

$$(\varepsilon_1 k_2 + \varepsilon_2 k_1)\, E_{1(2)z}\,(\mathbf{q},\omega) = +(-)\varepsilon_{2(1)} k_{2(1)} E_{0z}\,(\mathbf{q},0,\omega) + \varepsilon_{2(1)}\,(\mathbf{q}\mathbf{E}_0\,(\mathbf{q},0,\omega))\,, \tag{2.82}$$

$$(k_2 + k_1)\, H_{1(2)z}\,(\mathbf{q},\omega) = +(-)k_{2(1)} H_{0z}\,(\mathbf{q},0,\omega) + \left(\mathbf{q}\mathbf{H}_0\,(\mathbf{q},0,\omega)\right). \tag{2.83}$$

The other components are expressed in terms of the normal components with the use of Maxwell's equations. The spectral–angular distribution in the medium, where the particle moves, can be obtained in the form

$$\frac{d^2W\,(\mathbf{n},\omega)}{d\Omega\, d\omega} = \frac{4\pi^2\omega^4\varepsilon_2^{3/2}}{q^2c^3}\left(\varepsilon_2\,|E_{2z}|^2 + |H_{2z}|^2\right)\cos^2\theta. \tag{2.84}$$

The substitution of the explicit expressions of the field provides the distribution of the emitted energy in the form

$$\frac{d^2W}{d\omega\, d\Omega} = \frac{4e^2\varepsilon_2^{3/2}u^2\cos^2\vartheta}{\pi^2c^3\left|\left(1-(v/c)\,\varepsilon_2^{1/2}\sin\vartheta\cos\varphi\right)^2 - (u/c)^2\,\varepsilon_2\cos^2\vartheta\right|^2}\times$$
$$\times\left\{\frac{\left|\varepsilon_1 - \varepsilon_2\sin^2\vartheta\right|(v/c)^2\sin^2\varphi}{\left|\varepsilon_2^{1/2}\cos\vartheta + \left[\varepsilon_1 - \varepsilon_2\sin^2\vartheta\right]^{1/2}\right|^2} + \frac{|\varepsilon_1|^2\left|\sin\vartheta - (v/c)\,\varepsilon_2\cos^2\varphi\right|^2}{\left|\varepsilon_1\varepsilon_2^{1/2}\cos\vartheta + \varepsilon_2\left[\varepsilon_1 - \varepsilon_2\sin^2\vartheta\right]^{1/2}\right|^2}\right\}. \tag{2.85}$$

The radiation distribution in the crystal from which the particle is reflected differs noticeably from that given by Eq. (2.85). The calculations are similar and at $\mathrm{Im}\varepsilon_1 = 0$ yield the expression

$$\frac{d^2W}{d\omega\, d\Omega} = \frac{4e^2\varepsilon_2^{3/2}u^2\cos^2\vartheta}{\pi^2c^3\left|\left(1-(v/c)\,\varepsilon_2^{1/2}\sin\vartheta\cos\varphi\right)^2 - (u/c)^2\left(\varepsilon_2 - \varepsilon_1\sin^2\vartheta\right)\right|^2}\times$$
$$\times\left\{\frac{\left|\varepsilon_2 - \varepsilon_1\sin^2\vartheta\right|(v/c)^2\sin^2\varphi}{\left|\varepsilon_1^{1/2}\cos\vartheta + \left[\varepsilon_2 - \varepsilon_1\sin^2\vartheta\right]^{1/2}\right|^2} + \frac{\left|\varepsilon_1\sin\vartheta - (v/c)\,\varepsilon_1^{1/2}\varepsilon_2\cos^2\varphi\right|^2}{\left|\varepsilon_2\varepsilon_1^{1/2}\cos\vartheta + \varepsilon_1\left[\varepsilon_2 - \varepsilon_1\sin^2\vartheta\right]^{1/2}\right|^2}\right\}. \tag{2.86}$$

The problem of the generation of the surface waves in the process of the mirror reflection of the fast charged particle from the surface of the single crystal is considered similarly [25].

The dependence of the radiation intensity on the difference between the surfaces of the effective reflection of particles and field can be illustrated by an example of reflection from a metal when it is convenient to use the image method. Let the electromagnetic field be reflected from the $z = 0$ plane. If the reflection planes of particles and field coincide, the spectral—angular distribution of the emitted energy has the form

$$\frac{d^2W}{d\omega\, d\Omega} = \frac{4}{\pi^2}\frac{e^2}{c}\frac{u^2}{c^2} \times \frac{(v/c)^2\cos^2\vartheta\,\sin^2\varphi + \left[\sin\vartheta - (v/c)\cos\varphi\right]^2}{\left(\left[1-(v/c)^2\sin\vartheta\,\cos\varphi\right]^2 - \left[(u/c)\cos\vartheta\right]^2\right)^2}. \quad (2.87)$$

If the particle is reflected not reaching the effective field reflection planes at the distance b from it, the laws of motion of the actual charge and charge image have the form $\left(\theta\,(t) = \dfrac{t+|t|}{2\,|t|}\right)$

$$\begin{aligned} \mathbf{r}_+\,(t) &= \mathbf{b} + (\mathbf{v}+\mathbf{u})\,t\theta\,(t) + (\mathbf{v}-\mathbf{u})\;t\theta\,(-t)\,; \\ \mathbf{r}_-\,(t) &= -\mathbf{b} + (\mathbf{v}-\mathbf{u})\,t\theta\,(t) + (\mathbf{v}+\mathbf{u})\;t\theta\,(-t)\,. \end{aligned} \quad (2.88)$$

The ratio of the spectral–angular distributions of the emitted energy at a finite positive b value and $b = 0$ can be obtained in the form

$$\frac{d^2W\,(b)}{d^2W\,(b=0)} = \cos^2\left(b\frac{\omega}{c}\cos\vartheta\right). \quad (2.89)$$

When the particle is reflected from “inner” layers of the medium, i.e., at negative b values, the laws of the motion of the charge and charge image have the form

$$\begin{aligned} \mathbf{r}_+\,(t) &= (\mathbf{v}+\mathbf{u})\,t\theta\,(t-|b|\,/u) + (\mathbf{v}-\mathbf{u})\;t\theta\,(-t-|b|\,/u)\,; \\ \mathbf{r}_-\,(t) &= (\mathbf{v}-\mathbf{u})\,t\theta\,(t-|b|\,/u) + (\mathbf{v}+\mathbf{u})\;t\theta\,(-t-|b|\,/u)\,. \end{aligned} \quad (2.90)$$

The ratio of the distributions of the emitted energy is written as

$$\frac{d^2W\,(-|b|)}{d^2W\,(b=0)} = \cos^2\left\{(\omega - \mathbf{kv})\,|b|\,/u\right\}. \quad (2.91)$$

Thus, the measurement of the angular distribution of the radiation under consideration provides the possibility of measuring the magnitude and sign of the displacement of the effective reflection planes of light and particles from the surface.

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