Foreword

This book is about applications of the theory of pseudoanalytic functions. The Swedish mathematician T. Carleman was the first to characterize this class of functions via a generalized Cauchy-Riemann system which is sometimes called a Carleman-Vekua-Bers system, as the investigation of the solutions of these systems reached fruition many years later through work of I.N. Vekua and L. Bers. In the 1940s, L. Bers accepted an invitation to participate in the program Advanced Research and Instruction in Applied Mathematics at Brown University. There, as part of work relevant to the war effort, he studied two-dimensional subsonic fluid flow problems, which led him to pseudoanalytic functions. Independently, I.N. Vekua from Tiflis called them "generalized analytic functions" and described applications in elasticity and fluid dynamics. Under the influence of his famous book, *Generalized Analytic Functions* (1959), operator theoretical aspects dominated for a time, while Bers' ideas of a theory similar to classical complex analysis fell into relative oblivion.

The current renaissance of Bers' theory is primarily due to recent research by V.V. Kravchenko, who independently and later in collaboration with other colleagues uncovered striking new relations and applications of pseudoanalytic function theory. These developments have now been very carefully prepared and presented in a style accessible to a wide audience. Through several interesting examples from physics it is shown how concepts of Bers' theory give new results. The book is an interplay between pseudoanalytic theory and a collection of partial differential equations of mathematical physics which are quite important in applications: inter alia, the Schrödinger equation, the Klein-Gordon equation, Maxwell's equations and the Dirac equation. One basic idea which can be found throughout the book is that, starting from a special solution, it is often possible to construct large classes of solutions and even complete systems of solutions to an important equation. Mathematical topics are motivated by physical problems, each leading to a corresponding Carleman-Vekua-Bers system that is the main subject of this book.

The reader will find in this book suprising relations among equations from different genres. To fully appreciate them, some knowledge is required of complex analysis, ordinary differential equations including Sturm-Liouville problems, and second-order elliptic partial differential equations. Researchers may take note that

the theory presented in this book is not yet complete; new applications can easily be visualized, and the author has formulated a number of open problems which may be tackled in the future. A comprehensive list of papers and books is provided at the end of the volume, suited for readers who wish to deepen their research studies. Kravchenko's book is to be recommended to higher undergraduates, graduates and postdoctoral researchers.

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