PREFACE

Multiple criteria decision making (MCDM) is a modeling and methodological tool for dealing with complex engineering problems. Decision makers face many problems with incomplete and vague information in MCDM problems since the characteristics of these problems often require this kind of information. Fuzzy set approaches are suitable to use when the modeling of human knowledge is necessary and when human evaluations are needed. Fuzzy set theory is recognized as an important problem modeling and solution technique. Fuzzy set theory has been studied extensively over the past 40 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. Over the years there have been successful applications and implementations of fuzzy set theory in MCDM. MCDM is one of the branches in which fuzzy set theory found a wide application area. Many curriculums of undergraduate and graduate programs include many courses teaching how to use fuzzy sets when you face incomplete and vague information. One of these courses is fuzzy MCDM and its applications.

This book presents examples of applications of fuzzy sets in MCDM. It contains 22 original research and application chapters from different perspectives; and covers different areas of fuzzy MCDM. The book contains chapters on the two major areas of MCDM to which fuzzy set theory contributes. These areas are fuzzy multiple-attribute decision making (MADM) and fuzzy multiple-objective decision making (MODM). MADM approaches can be viewed as alternative methods for combining the information in a problem’s decision matrix together with additional information from the decision maker to determine a final ranking, screening, or selection from among the alternatives. MODM is a powerful tool to assist in the process of searching for decisions that best satisfy a multitude of conflicting objectives.

The classification, review and analysis of fuzzy multi-criteria decision-making methods are summarized in the first two chapters. While the first chapter classifies the multi-criteria methods in a general sense, the second chapter focuses on intelligent fuzzy multi-criteria decision making.

The rest of the book is divided into two main parts. The first part includes chapters on frequently used MADM techniques under fuzziness, e.g., fuzzy Analytic Hierarchy Process (AHP), fuzzy TOPSIS, fuzzy outranking methods, fuzzy weighting methods, and a few application chapters of these techniques. The third chapter includes the most frequently used fuzzy AHP methods and their numerical and didactic examples. The fourth chapter shows how a fuzzy AHP method can be jointly used with another technique. The fifth chapter summarizes fuzzy outranking methods, which dichotomize preferred alternatives and nonpreferred ones by establishing outranking relationships. The sixth chapter presents another commonly used multi-attribute method, fuzzy TOPSIS and its application to selection among industrial robotic systems. The seventh chapter includes many fuzzy scoring methods and their applications. The rest of this part includes the other most frequently used fuzzy MADM techniques in the literature: fuzzy information axiom approach, intelligent fuzzy MADM approaches, gray-related analysis, and neuro-fuzzy approximation.

The second part of the book includes chapters on MODM techniques under fuzziness, e.g., fuzzy multi-objective linear programming, quasi-concave and non-concave fuzzy multi-objective programming, interactive fuzzy stochastic linear programming, fuzzy multi-objective integer goal programming, gray fuzzy multi-objective optimization, fuzzy multi-objective geometric programming and some applications of these techniques. These methods are the most frequently used MODM techniques in the fuzzy literature.

The presented methods in this book have been prepared by the authors who are the developers of these techniques. I hope that this book will provide a useful resource of ideas, techniques, and methods for additional research on the applications of fuzzy sets in MCDM. I am grateful to the referees whose valuable and highly appreciated works contributed to select the high quality of chapters published in this book. I am also grateful to my research assistant, Dr. Ihsan Kaya, for his invaluable effort to edit this book.

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CONTENTS

INTELLIGENT FUZZY MULTI-CRITERIA DECISION MAKING: REVIEW AND ANALYSIS

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Abstract: This chapter highlights the implementation of artificial intelligence techniques to solve different problems of fuzzy multi-criteria decision making. The reasons behind this implementation are clarified. In additions, the role of each technique in handling such problem are studied and analyzed. Then, some of the future research work is marked up as a guide for researchers who are working in this research area.

Key words: Intelligent optimization, fuzzy multi-criteria decision making, research directions

1. INTRODUCTION

1.1 Mathematical Model of Fuzzy Multi-Criteria Decision Making

Multi-criteria decision making (MCDM) represents an interest area of research since most real-life problems have a set of conflict objectives. MCDM has its roots in late-nineteenth-century welfare economics, in the works of Edgeworth and Pareto. A mathematical model of the MCDM can be written as follows:

$$\underset{s}{\text{Min}} \quad Z = [z_1(x), z_2(x), \ldots, z_K(x)]^T \tag{1}$$

C. Kahraman (ed.), Fuzzy Multi-Criteria Decision Making.

where

$$S = \{x \in X \,|\, Ax \le b, x \in R^n, x \ge 0\}$$

where:

Z(x) = C x is the *K*-dimensional vector of objective functions and *C* is the vector of cost corresponding to each objective function,

S is the feasible region that is bounded by the given set of constraints,

A is the matrix of technical coefficients of the left-hand side of constraints,

b is the right-hand side of constraints (i.e., the available resources),

x is the *n*-dimensional vector of decision variables.

When the objective functions and constraints are linear, then the model is a linear multi-objective optimization problem (LMOOP). But, if any objective function and/or constraints are nonlinear, then the problem is described as a nonlinear multi-objective optimization problem (NLMOOP). Since problem (1) is deterministic, it can be solved by using different approaches such as follows:

1. Utility function approach,
2. Interactive programming,
3. Goal programming, and
4. Fuzzy programming.

But, in the real world, the input information to model (1) may be vague, for example, the technical coefficient matrix (*A*) and/or the available resource values (*b*) and/or the coefficients of objective functions (*C*). Also, in other situations, the vagueness may exist, such as the aspiration levels of goals ($z_i(x)$) and the preference information during the interactive process. All of these cases lead to a fuzzy multi-criteria model that can be written as follows:

$$\underset{S}{\text{Min}} \quad Z \cong [z_1(x), z_2(x), ..., z_K(x)]^T \tag{2}$$

where

$$S = \{x \in X \,|\, \tilde{A}x \,\tilde{\le}\, \tilde{b}, \quad x \in R^n, \quad x \ge 0\}\,.$$

This fuzzy model is transformed into crisp (deterministic) by implementing an appropriate membership function. So, the model can be classified into two classes. If any of the objective functions, constraints, and membership functions are linear, then the model will be LFMOOP. But, if any of the objective functions and/or constraints and/or membership functions are nonlinear, then the model is described as NLFMOOP.

Different approaches can handle the solution of problem (2). All of these approaches depend on transforming problem (2) from fuzzy model to crisp model via determining an appropriate membership function that is the backbone of fuzzy programming.

Definition 1.1: Fuzzy set

Let X denote a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function:

$$\mu \ddot{A} : X \to [0,1] \tag{3}$$

That assigns to each element $x \in X$ a real number in the interval [0, 1] and $\mu_{\tilde{A}}(x)$ represents the grade of membership function of x in A.

The main strategy for solving model (2) can be handled according to the following scheme:

Step 1. Examine the type of preference information needed.

Step 2. If a priori articulation of preference information is available use, one of the following programming schemes:

2.1 Fuzzy goal programming,

2.2 Fuzzy global criterion, or

2.3 Another appropriate fuzzy programming technique.

Otherwise, go to step (3).

Step 3. If progressive articulation of preference information is available, use the following programming scheme:

3.1 Fuzzy interactive programming,

3.2 Interactive fuzzy goal programming, or

3.3 Another appropriate fuzzy interactive programming technique.

Step 4. End strategy.

Each programming scheme involved different solution methodologies that will be indicated in Section 1.3.

1.2 Historical Background of Fuzzy MCDM

In 1970, Bellman and Zadah highlighted the main pillar of fuzzy decision making that can be summarized as follows:

$$D = G \cap C \tag{4}$$

where G is the fuzzy goal, C is the fuzzy constraints, and D is the fuzzy decision that is characterized by a suitable membership function as follows:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)). \tag{5}$$

The maximizing decision is then defined as follows:

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min\,(\mu_G(x), \mu_C(x)). \tag{6}$$

For k fuzzy goals and m fuzzy constraints, the fuzzy decision is defined as follows:

$$D = G_1 \cap G_2 \cap \ldots \cap G_k \cap C_1 \cap C_2 \cap \ldots \cap C_m \tag{7}$$

and the corresponding maximizing decision is written as follows:

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min\,(\mu_{G_1}(x), \ldots, \mu_{G_k}, \mu_{C_1}(x) \ldots, \mu_{Cm}(x)). \tag{8}$$

For more details about this point, see Sakawa (1993). Since this date, many research works have been developed. In this section, the light will be focused on a sample of research works on FMCDM from the last 25 years to extract the main shortcomings that argue for us to direct attention toward the intelligent techniques as an alternative methodology for overcoming these drawbacks.

In FMCDM problems, the membership function depends on where the fuzziness existed. If the fuzziness in the objective functions coefficients, the membership function may be represented by

$$\mu_k(Z^k(x)) = \begin{cases} 1 & \text{if} \quad Z^k(x) \leq L_k, \\ \dfrac{U_k - Z^k(x)}{U_k - L_k} & \text{if} \quad L_k < Z^k(x) < U_k \\ 0 & \text{if} \quad Z^k(x) \geq U_k \end{cases} \tag{9}$$

where U_k is the worst upper bound and L_k is the best lower bound of the objective function k, respectively. They are calculated as follows:

$$\begin{aligned} U_k &= (Z^k)^{\max} = \max_{x \in X} Z^k(x) \\ L_k &= (Z^k)^{\min} = \min_{x \in X} Z^k(x), \; k = 1, 2, \ldots, K \end{aligned} \tag{10}$$

If the fuzziness is existed in the right-hand side of the constraints, the constraints are transformed into equalities and then the following membership function is applied (Lai and Hwang, 1996):

$$\mu_k(Z^k(x)) = \begin{cases} [(Ax)_i - (b_i - d_i)]/d_i & \text{if} \quad (b_i - d_i) \leq (Ax)_i < b_i, \\ [(b_i - d_i) - (Ax)_i]/d_i & \text{if} \quad b_i \leq (Ax)_i \leq (b_i - d_i) \\ 0 & \text{if} \quad (b_i - d_i) \leq (Ax)_i or (Ax)_i < (b_i - d_i) \end{cases} \tag{11}$$

where the membership function is assumed to be symmetrically triangular functions. The problem solver may assume any other membership function based on his/her experience. Besides, some mathematical and statistical methods develop a specific membership function. On the other side, the intelligent techniques provide the problem solver with a powerful techniques to create or estimate these functions as will be indicated later. If we assumed that the FMCDM problem has fuzzy objective functions, then the deterministic model of the FMCDM is written as follows:

$$\begin{aligned} &\max \; \beta \\ &\text{subject to} \\ &\beta \leq \mu_k(Z^k(x)), \; k = 1,2,\ldots,K \\ &\sum_{j=1}^{n} a_{ij} x_j = b_i, \; i = 1,2,\ldots,m \\ &x_j \geq 0, \; i = 1,2,\ldots,m; \; j = 1,2,\ldots,n, \; k = 1,2,\ldots,K \\ &0 \leq \beta \leq 1 \end{aligned} \tag{12}$$

where β is an auxiliary variable and can be worked at a satisfaction level. Model (7) can be solved as a single objective linear/nonlinear programming problem.

After the Bellman and Zadah paper, several research studies were adopted, such as Hannan (1983) and Zimmerman (1987) who handled fuzzy linear programming with multiple objectives by assuming a special form of the membership function. Hannan assumed discrete membership function, and Zimmerman used a continuous membership function. Boender (1989), Sakawa (1993), and Baptistella and Ollero (1980) implemented the fuzzy set theory in interactive multi-criteria decision making. For more historical information, see Sakawa (1993) and Lai and Hwang (1996). Also, see Biswal (1992), Bhattacharya et al. (1992), Bit (1992), Boender et al. (1989), Buckley (1987), Lothar and Markstrom (1990) for more solution methodologies.

Many real-life problems have been formulated as FMCDM and have been solved by using an appropriate technique. Some of these applications involved production, manufacturing, location−allocation problems, environmental management, business, marketing, agriculture economics, machine control, engineering applications and regression modeling. A good classification with details can be found in Lai and Hwang (1996). A new literature review (Zopounidis and Doumpos, 2002) assures the same field of applications.

1.3 Shortcomings of the FMCDM Solution Approaches

The problems that meet either the solution space construction or the model development can be classified into three categories as follows: 1) ill-structured, 2) semi-well structured or, 3) well structured.

Each category has been characterized by specific criteria to indicate its class. Some of these indicator criteria of ill-structured problems are as follows:

1. There is no available solution technique to solve the model.
2. There is no standard mathematical model to represent the problem.
3. There is no ability to involve the qualitative factors in the model.
4. There is no available solution space to pick up the optimal solution.
5. There is a difficulty in measuring the quality of the result solution(s).
6. There is kind of vagueness of the available information that leads to complexity in considering it into the model account.

If some of these criteria exist, then the problem will belong to the second category, which is called *semi-ill-structured* problems. But, if all of these criteria and others do not exist, then the problem will belong to the third category, which is called *well-structured* problems. It is clear that there is no problem regarding the third category. Fortunately, the first and second categories represent a rich area for investigation, especially in the era of information technology where all the sciences are interchanged in a complex manner to a degree that one can find difficulty in separating between sciences. In other words, biological sciences, sociology, insects' science, and so on attracted researchers to simulate them by using computer technology that consequently reflects its positive progress on the optimization research work.

Let us now apply these criteria of ill-structured problems on FMCDM problems. For FMCDM model structure, the following problems are represented as an optical stone to more progress in this area. Some of these problems are as follows:

1. Incorporating fuzzy preferences in the model still needs new methodologies to take the model into account without increasing the model complexity.
2. Right now, the FMCDM models are transformed into crisp models to solve it by using the available traditional techniques. This transformation reduces both the efficiency and the effectiveness of the fuzzy solution methodologies. So, we need to look for a new representation methodology to increase or at least keep the efficiency of the fuzzy methodology.
3. As mentioned above, the membership function is the cornerstone of fuzzy programming, and right now, the problem solvers assumed it according the experience. As a result, the solution will be different according to the selected membership function. This will lead to another problem, which is which solution is better or qualifies more for the problem under study. In this case, there is an invitation to implement the progress in information technology to discover an appropriate membership function.
4. Large-scale FMCDM models still need more research especially when incorporating large preference information.

Regarding the solution methodologies, there are some difficulties in enhancing them. Some of them are:

1. Some of the existing ranking approaches that have been used to solve the FMCDM problem are not perfect.

2. Fuzzy integer programming with multi-criteria can be considered a combinatorial optimization problem, and as a result, it needs an exponential time algorithm to go with it.
3. In 0-1 FMCDM problems (whatever small scale or large scale), the testing process of the Pareto-optimal solution is considered the NP-hard problem.
4. In FMCDM problems, a class of problems exist that are known as the global convex problems, where the good solutions in the objective space are similar to those in the decision space. So, we need a new methodology to perform well with them.
5. In fuzzy and nonfuzzy MCDM problems, there is a difficulty in constructing an initial solution that should be close to the Pareto-optimal solution to reduce the solution time. So, we need powerful methodology–based information technology to deal with this problem.

Because of these shortcomings and others, FMCDM attracts the attentions of researchers to enhance the field of FMCDM by developing more powerful links (bridges) between it and other sciences. In this chapter, we will highlight the link between artificial intelligence and FMCDM to overcome all or some of the mentioned problems. This link leads to a new and interesting area of research called “intelligent optimization.” The general strategy for the integration between artificial intelligence (AI) techniques and FMCDM problems may be done according the following flowchart seen in Figure 2. In the next subsection, some of the intelligent techniques will be introduced briefly.

1.4 Some Intelligent Techniques

AI is the branch of computer technology that simulates the human behavior via intelligent machines to perform well and better than humans. Computer science researchers are wondering how to extract their ideas from the biological systems of human beings such as thinking strategies, the nervous system, and genetics. AI also extends to the kingdom of insects such as the ant colony. The tree that summarizes the different commercial forms of AI techniques is shown in Figure 1. Each AI technique can perform well in specific situations more so than in others. For example, expert systems (ESs) can handle the qualitative factors or preferences that can not be included in the mathematical model. Artificial neural networks (ANNs) are successfully applied in prediction, classification, pattern and voice recognition, and so on. Simulated annealing

(SA), genetic algorithms (GA), and particle swarm optimization (PSO) are used as stochastic search methods to deal with multi-criteria combinatorial optimization problems.

The implementation of AI techniques to handle different problems in FMCDM depends on the following conditions:

1. The nature of the problem that FMCDM suffers from,
2. The availability of the solution techniques and its performance,
3. The environmental factors that affect the problem under study.

AI techniques can be classified according to their functions as follows:

1. Symbolic processing, where the knowledge is treated symbolically not numerically. In other words, the process is not algorithmical. These techniques are ES, fuzzy expert system (FES), and decision support system (DSS).
2. Search methods that are implemented to search and scan the large solution space of combinatorial optimization problem. These techniques are able to pick an acceptable or preferred solution in less time compared with the traditional solution procedures. Examples of these search methods are GA, SA, ant colony optimization (ACO), PSO, DNA computing, and any hybrid of them.
3. Learning process that is responsible for doing forecasting, classifications, and function estimating based on enough historical data about the problem under study. These techniques are ANN and neuro-fuzzy systems.

Now, we shall classify the intelligent FMCDM problems based upon the implemented technique.

1.4.1 Expert System and FMCDM

ES is an intelligent computer program that consists of three modules: 1) inference engine module, 2) knowledge-base module, and 3) user-interface module. This system can produce one of the following functions: 1) conclusion, 2) recommendation, and 3) advice. The main feature of the ES is its ability to treat the problems symbolically not algorithmically. So, it can perform a good job regarding both the decision maker's preferences and the qualitative factors that cannot be included in the mathematical model because of its increase in the degree of model complexity.

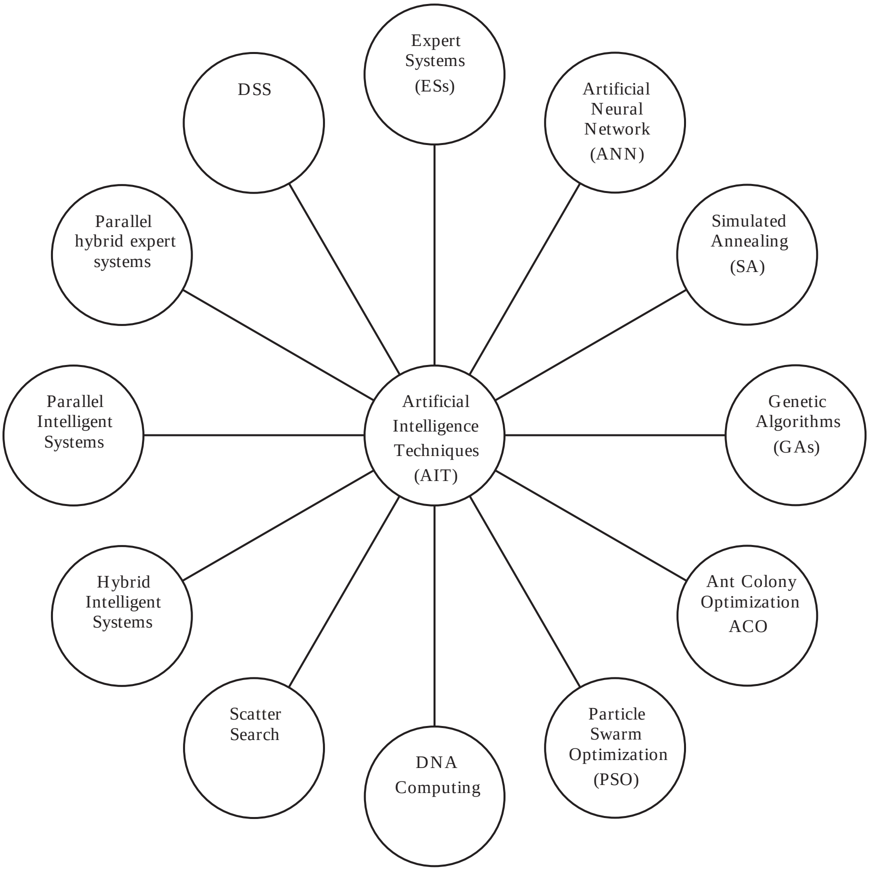

Figure 1. The tree diagram of artificial intelligence techniques

Generally speaking, ES has been applied to solve different applications that can be modeled in MCDM. For example, Lothar and Markstrom (1990) developed an expert system for a regional planning system to optimize the industrial structure of an area. In this system, AI paradigms and numeric multi-criteria optimization techniques are combined to arrive at a hybrid approach to discrete alternative selection. These techniques include 1) qualitative analysis, 2) various statistical checks and recommendations, 3) robustness and sensitivity analysis, and 4) help for defining acceptable regions for analysis. Jones et al. (1998) developed an intelligent system called "GPSYS" to deal with linear and integer goal programming. The intelligent goal programming system is one that is designed to allow a nonspecialist access to, and clear understanding of a goal programming solution and analysis techniques. GPSYS has an analysis tool such as Pareto

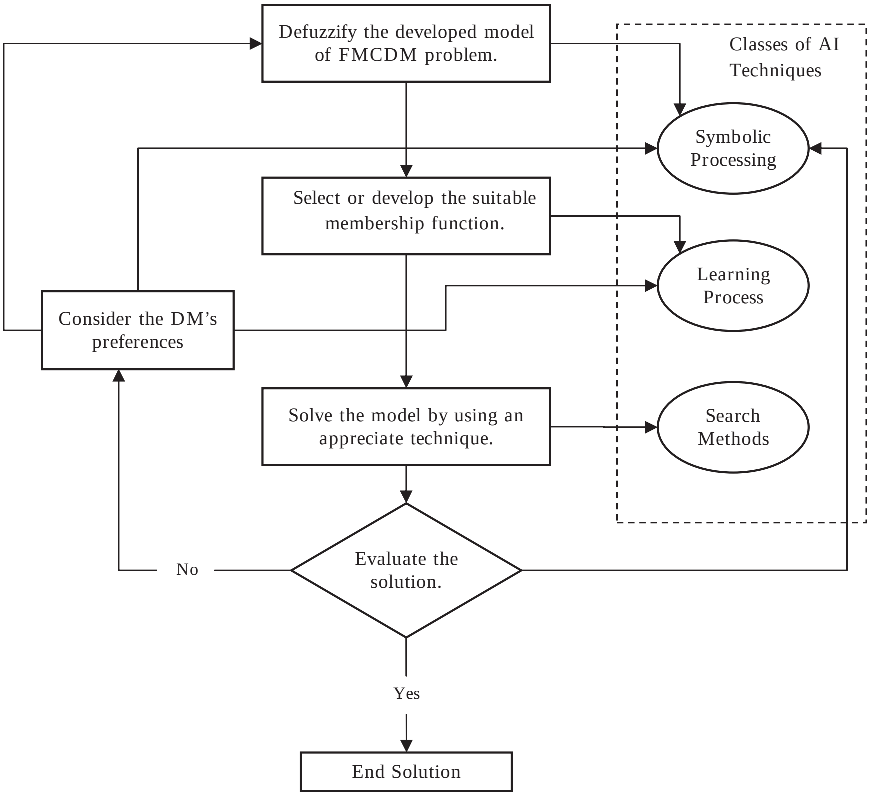

Figure 2. The integration between AI techniques and FMCDM phases

detection and restoration, normalization, automated lexicographic redundancy checking, and an interactive facility. Abd El-Wahed (1993) developed a decision support system with a goal programming based ES to solve engineering problems. In this research, the statistical analysis and the decision maker's preferences are combined in an ES to assign the differential weights of the sub-goals in goal programming problems. Also, Rasmy et al. (2001) presented a fuzzy ES to include the qualitative factors that could not be involved in the mathematical model of the multi-criteria assignment problem in the field of bank processing. The approach depends on evaluating the model solution by using the developed fuzzy ES. If the solution is coincided with the evaluation criteria, the approach is terminated. Otherwise, some modification on the preferences is done in the feedback to resolve the model again and so on until getting a solution coincides with the evaluation criteria. Little research work regarding FMCDM has been done. For example, Rasmy et al. (2002) presented an interactive approach for solving the MCDM problem with fuzzy preferences in both aspiration level determination and priority structure by using the framework of the fuzzy expert system. The main idea of this approach is to convert the MCDM problem into its equivalent goal programming model by setting the aspiration levels and priority of each objective function based on fuzzy linguistic variables. This conversion makes the implementation of ES easy and effective.

Liu and Chen (1995) present an integrated machine troubleshooting expert system (IMTES) that enhances the efficiency of the diagnostic process. The role of fuzzy multi-attribute decision-making in ES is determined to be the most efficient diagnostic process, and it creates a “meta knowledge base” to control the diagnosis process.

The results of an update search in some available database sites regarding the combination of both ES and FMCDM can be summarized as follows:

1. The mutual integration between ES and MCDM/FMCDM is a rich area for more research,
2. The implementation of ES for dealing with the problems of FMCDM still needs more research,
3. The combination of ES and other AI techniques needs more research to gain the advantages of both of them in solving the problems of FMCDM problems.

The researchers are invited to investigate the following points where they are not covered right now:

1. Applying ES to guide the determination process of the aspiration levels of fuzzy goal programming.
2. Applying ES to handle the DM’s preferences in solving interactive FMCDM to reduce the solution time and the solution efforts.
3. Implementing the ES in ranking approaches that have been used to solve FMCDM problems to include the environmental qualitative factors.
4. Handling ES in solving large-scale FMCDM problems.
5. Combining ES with both parametric analysis and sensitivity analysis to pick a more practical solution.

1.4.2 ANN and FMCDM Problems

ANN is a simulation of a human nervous system. The ANN simulator depends on the Third Law of Newton: “For any action there is an equal reaction with negative direction.” A new branch of computer science is opened for research called “neural computing.” Neural computing has been viewed as a promising tool to solve problems that involve large date/preferences or what is called in optimization large-scale optimization problems. Also, the transformation of FMCDM into crisp model needs an appropriate membership function. In other situations, ANN is implemented to solve the FMCDM problems without the need to defuzzify the mathematical model of FMCDM problems. ANN offers an excellent methodology for estimating continuous or discrete membership functions/ values. To do that, an enormous amount of historical data is needed to train and test the ANN as well as to get the right parameters and topology of it to solve such a problem. On the other side, the complex combinatorial FMCDM problems (NP hard problems) may be not represented in a standard mathematical form. As a result, ANN can be used to simulate the problem for the purpose of getting an approximate solution based on a simulator. The main problem facing those who are working in this area is the development of the energy (activation) function, which is the central process unit of any ANN. This function should have the inherited characteristics of both the objective function and the constraints to train and test the network. There are many standard forms of it such as the sigmoid function and the hyperbolic function. The problem solver must elect a suitable one from them such that can be fitted with the nature of the problem under study. For the FMCDM with fuzzy objective functions [model (7)], the energy function can be established by using the Lagrange multiplier method as follows:

$$E(x,\beta,\lambda,\eta) = \beta + \lambda^t(-\mu_k(Z^k(x)) + \beta + \chi) + \eta^t(\sum_{j=1}^{n} a_{ij}x_j - b_i) \tag{11}$$

where λ and η are the Lagrange multipliers. χ is the vector of slack variables. By taking the partial derivative of an equation with respect to x, λ, and η, we obtain the following differential equations:

$$\begin{aligned} \partial E / \partial x &= \rho \nabla_x E(x, \beta, \lambda, \eta) \\ \partial E / \partial \lambda &= \rho \nabla_\lambda E(x, \beta, \lambda, \eta) \\ \partial E / \partial \eta &= \rho \nabla_\eta E(x, \beta, \lambda, \eta) \end{aligned} \tag{12}$$

where ρ is called a learning parameter. By setting the penalty parameters λ and η, the adaptive learning parameters ρ, and initial solution $x_j(0)$, then we can solve the system (9) to obtain β.

Previous research works use ANN to solve some optimization problems as well as FMCDM specifically. These works can be classified according to the type of treating method of the FMCDM model as follows:

1.4.2.1 Treating the Fuzzy Preferences in MCDM Problems

For example, Wang (1993) presented a feed-forward ANN approach with a dynamic training procedure to solve multi-criteria cutting parameter optimization in the presence of fuzzy preferences. In this approach, the decision maker's preferences are modeled by using fuzzy preference information based on ANN. Wang and Archer (1994) modeled the uncertainty of multi-criteria, multi-persons decision making by using fuzzy characteristics. They implemented the back propagation learning algorithm under monotonic function constraints. Stam et al. (1996) presented two approaches of ANNs to process the preference ratings, which resulted from analytical, hierarchy process, pair-wise comparison matrices. The first approach, implements ANN to determine the eigenvectors of the pair-wise comparison matrices. This approach is not capable of generalizing the preference information. So, it is not appropriate for approximating the preference ratings if the decision maker's judgments are imprecise. The second approach uses the feed-forward ANN to approximate accurately the preference ratings. The results show that this approach is working well with respect to imprecise pair-wise judgments. Chen and Lin (2003) developed the decision neural network (DNN) to use in capturing and representing the decision maker's preferences. Then, with DNN, an optimization problem is solved to look for the most desirable solution.

1.4.2.2 Handling Fuzziness in FMCDM Models

It is clear that ANN is capable of solving the constrained optimization problems, especially the applications that require on-line optimization. Gen et al. (1998) discussed a two-phase approach to solve MCDM problems with fuzziness in both objectives and constraints. The main proposed steps to solve the FMCDM model (2) can be summarized as follows:

1. Construct the membership function based on positive ideal and negative ideal (worst values) solutions.
2. Apply the concept of α-level cut, where $\alpha \in [0,1]$ to transform the model into a crisp model.
3. Develop the crisp linear programming model based on steps (1) and (2).
4. According to the augmented Lagrange multiplier method, we can create the Lagrangian function to transform the result model in step (3) into an unconstrained optimization problem. The Lagrangian function is implemented as an energy (activation) function to activate the developed ANN.
5. If the DM accepts the solution, stop. Otherwise, change α and go to the step (1).

The results show that the result solution is close to the best compromise solution that has been calculated from the two-phase approach. The method has an advantage; if the decision maker is not satisfied with the obtained solutions, he/she can get the best solutions by changing the α-level cut.

1.4.2.3 Determining the Membership Functions

Ostermark (1999) proposed a fuzzy ANN to generate the membership functions to new data. The learning process is reflected in the shape of the membership functions, which allows the dynamic adjustment of the functions during the training process. The adopted fuzzy ANN is applied successfully to multi-group classification-based multi-criteria analysis in the economical field.

1.4.2.4 Searching the Solution Space of Ill-Structured FMCDM Problems

Gholamian et al. (2005) studied the application of hybrid intelligent system based on both fuzzy rule and ANN to:

- Guide the decision maker toward the noninferior solutions.
- Support the decision maker in the selection phase after finishing the search process to analyze different noninferior points and to select the best ones based on the desired goal levels.

The idea behind developing this system is the ill-structured real-world problem in marketing problems where the objective can not be expressed in a mathematical form but in the form of a set of historical data. This means that ANN can do well with respect to any other approach. From the above analysis, we can deduce that many research points are still uncovered. It means that the integration area between ANN and FMCDM is very rich for more research. These points are summarized as follows:

1. Applying the ANN to solve FMCDM problems in its fuzzy environment without transforming it into a crisp model to obtain more accurate, efficient, and realistic solution(s).
2. Developing more approaches to enhance the process of generating real membership functions.
3. Studying the effect of using different membership functions on the solution quality and performance.
4. Implementing the ANN to solve more large-scale FMCDM problems that represented the real-life case.
5. Combining both ES and ANN to develop more powerful approaches to consider the preference information (whatever quantitative/qualitative) in FMCDM problems.
6. Applying the ANN to do both parametric and sensitivity analysis of the real-life problems that can be represented by the FMCDM model.

1.4.3 Tabu Search

A tabu search (TS) was initiated by Glover as an iterative intelligent search technique capable of overcoming the local optimality when solving the CO problems. The search process is based on a neighborhood mechanism. The neighborhood of a solution is defined as a set of all formations that can be obtained by a move that is a process for transforming the search from the current solution to its neighboring solution. If the move is not listed on the TS, the move is called an “admissible move.” If the produced solution at any move is better than all enumerated solutions in prior iterations, then this solution is saved as the best one. The candidate solutions, at each iteration, are checked by using the following tabu conditions:

1. Frequency memory that is responsible for keeping the knowledge of how the same solutions have been determined in the past.
2. Recency memory that prevents cycles of length less than or equal to a predetermined number of iterations.

TS has an important property that enables it to avoid removing the powerful solutions from consideration. This property depends on an element called an aspiration mechanism. This element means that if the TS list captured a solution with a value strictly better than the best obtained so far, the TS can stop.

TS is applied to solve some FMCDM problems. For example, Bagis (2003) proposed a new approach based on TS to determine the membership functions of a fuzzy logic controller. The simulation results indicated that the given approach is performed well, and as a result it is effective in determining such a membership function. Li et al. (2004) presented a TS method as a stochastic global optimization method for solving very large combinatorial optimization tasks and for extending a continuous-valued function for the fuzzy optimization problems. They approved the performance of the proposed method by applying it to an elementary fuzzy optimization problem such as the method for fuzzy linear programming; fuzzy regression and the training of fuzzy neural networks are also presented. Choobineh et al. (2006) proposed an algorithm to deal with a sequencing of *n*-jobs on a single machine with sequence-dependent setup times and *m*-objective functions. The algorithm generates a set of solutions that reflects the objectives' weights and close to the best observed values of the objectives. In addition, the authors formulated a mixed integer linear program to obtain the optimal solution of a triple-objective functions problem. Most of the published research works have not focused on FMCDM problems.

1.4.4 Simulated Annealing (SA)

The SA algorithm is a search technique designed to look for a global minimum among many local minima. The algorithm simulates the thermodynamic process of annealing metals by slow cooling where at high temperatures, molecules in metal move rapidly with respect to each other. If the metal is slow cooled sufficiently, then thermal mobility is lost. The resulting arrangement of atoms tends to form a pure crystal that is completely ordered. This ordered state occurs when the system has achieved minimum energy by an annealing process that must be cooled sufficiently slowly to reach thermal equilibrium.

The SA search method is a powerful tool to provide excellent solutions of single objective optimization problems to reduce the computational cost. Later, this approach was adapted for the multi-objective framework by Serafini (1985), Czyżak et al. (1994) and Ulungu et al. (1995). But they examined only the notion of the probability in the multi-objective framework. Serafini (1985) used simulated annealing on the multi-objective framework. Czyżak and Jaszkiewicz (1998) and Ulungu et al. (1998) designed a complete MOSA algorithm and tested it with a multi-objective combinatorial optimization problem. Ulungu et al. (1999) presented an interactive version of MOSA to solve an industrial application problem. Suppapitnarm et al. (2000) proposed a different simulated annealing approach to handle multi-objective problems. Czyżak et al. (1994) hybridized both SA and GA to provide efficient solutions of multi-objective optimization problems. Loukil et al. (2006) proposed a multi-objective SA algorithm to tackle a production scheduling problem in a flexible job-shop with particular constraints such as batch production; production of several sub-products followed by assembly of the final product, and possible overlaps for the processing periods of two successive operations of the same job. For more details in this area of research, see both Suman (2002) and (2003).

In the literature, there are some research works regarding MCDM problems, and the available fuzzy research works are under the general title "fuzzy optimization" not specific FMCDM problems. So, this area of research is ripe for more investigations.

1.4.5 Genetic Algorithms and FMCDM

The GA is a search algorithm that mimics the processes of natural evolution. The problem addressed by GA is searching the solution space is to identify the best problems that are combinatorial or large scale or ill-structured in general. GA encodes the variables of problems in either binary or real-valued vectors. Each code is called a chromosome. In binary coding there are two decoding functions to convert from real to binary and vice versa. In addition, mutation, crossover, and selection are the three important operators used for generating a new solution within the solution space. For example, the mutation operator introduces new genetic material into the population. Crossover recombines individuals to create new individuals. The selection process elects the next generation by using 1) tournament selection, 2) proportional selection, 3) ranking selection, 4) steady-state selection, and 5) manual selection. An evaluation function called the "fitness function" is generated to test the result solution. In the case of constrained optimization problems, Lagrange multipliers are used to transform the problem into an unconstrained optimization problem to be used as a fitness function. The general flowchart of a GA for solving an optimization problem is shown in Figure 3.

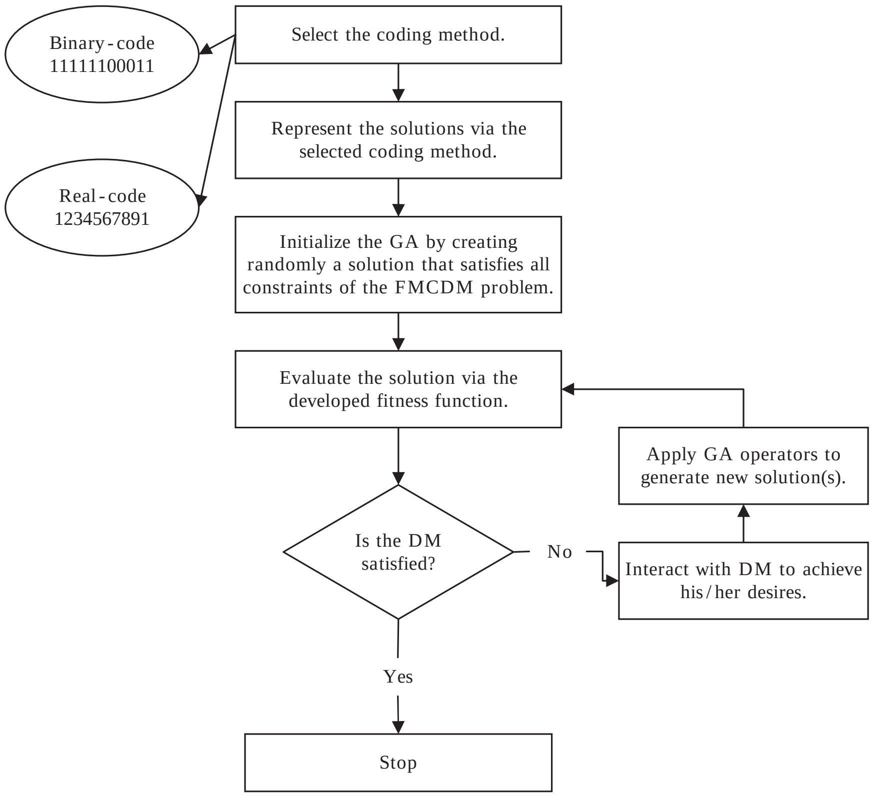

Figure 3. General schema of GA to solve FMCDM problems

GAs seem desirable for solving MOOPs because they deal simultaneously with a set of solutions (the so-called population) that allows the problem solver to find several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs, such as with the traditional mathematical programming techniques. Additionally, GAs are less susceptible to the shape or continuity of the Pareto front, whereas these two issues are a real concern for mathematical programming techniques. The integration between GA and MOOPs can be classified in the following two categories:

♦**Non-Pareto Techniques**

Under this category, we will consider approaches that do not incorporate directly the concept of Pareto optimality. Although these approaches are efficient, most of them enable us to produce certain portions of the Pareto front. However, their simplicity has made them popular among a certain sector of researchers. These approaches are as follows:

1. Aggregating approaches,
2. Lexicographic ordering,
3. The ε-constraint method, and
4. Target-vector approaches.

♦ **Pareto-Based Techniques**

In this category, the main idea is finding the set of strings in the population that are Pareto nondominated by the rest of the population. These strings are assigned the highest rank and are eliminated from additional considerations. Another set of Pareto nondominated strings are determined from the remaining population and are assigned the next highest rank. Some of the approaches that implement this idea are:

1. Pure Pareto ranking,
2. Multi-objective genetic algorithm (MOGA),
3. Nondominated sorting genetic algorithm (NSGA), and
4. Nondominated pareto genetic algorithm (NPGA).

In the context of this chapter, some works have been found and can be classified into the following categories:

1.4.5.1 Interactive FMCDM-Based GA

Sakawa and others presented a series of papers in this category. The ideas of these works can be summarized in the following:

- Kato et al. (1997) introduce an interactive satisfying method using GA for getting the satisfying solution for a decision maker from an extended Pareto optimal solution set. In this method, for a certain value of α-level cut and reference membership function, the solution of large-scale multi-objective 0-1 programming is obtained by adopting a GA with decomposition procedures.
- Sakawa and Yauchi (1999) highlight the multi-objective, nonconvex, nonlinear programming problems with fuzzy goals and solve it by applying an interactive fuzzy satisfying method. In this method, the Pareto optimal solution is obtained by solving the augmented mini-max problem for which the floating point GA called GENOCOP III is applicable.

- Sakawa and Yauchi (2000) proposed an interactive decision-making method for solving multi-objective, nonconvex programming problems with fuzzy numbers through co-evolutionary GAs. In this paper, the authors were trying to overcome the drawbacks of GENCOP III by introducing a method to generate an initial feasible point and a bisection method. This modification leads to a new GENCOP called revised GENCOP III.
- Sakawa and Kubota (2000) solved an application in job shop scheduling with fuzzy processing time and fuzzy due date by using GA.
- Sakawa and Kato (2002) deal with the general multi-objective 0-1 programming problems that involve positive and negative coefficients. The extended GA with double strings is implemented with a new decoding algorithm for individuals. The double strings map each individual to a feasible solution based on backtracking and individual modification. For more details about the GA and FMCDM, see Sakawa (2002).
- Basu (2004) applied an interactive fuzzy satisfying method based on an evolutionary programming technique for short-term multi-objective hydrothermal scheduling. The multi-objective problem is formulated by assuming that the decision maker has fuzzy goals for each of the objective functions and that the evolutionary programming technique-based fuzzy satisfying method is applied for generating a corresponding optimal noninferior solution for the decision maker’s goals.
- Wahed et al. (2005) presented a contribution in this area by suggesting an interactive approach to determine the preferred compromise solution for the MCDM problems in the presence of fuzzy preferences. Here, the decision maker evaluates the solution by using a defined set of linguistic variables, and consequently, the achievement membership function can be constructed for each objective function. The used non-negative differential weights are determined based on the entropy degree of each objective function to support transforming the MCDM into a single objective function.

1.4.5.2 Goal Programming-Based GAs

Goal programming (GP) is an important technique that is capable of solving a problem with multiple goals. The concept of goal programming (GP) is extended to solve multi-objective decision-making problems because of its ability to transform it into a single-objective programming problem with or without priority through putting the objective functions as goal constraints with predetermined aspiration levels. Also, FGP is extended to solve the complex problems in MCDM/FMCDM problems, especially with implementing GAs. In this case, some research works have been enumerated as follows:

- Zheng et al. (1996) discussed the initialization process, fitness function structure, and the GA operators in the proposed GA for solving nonlinear goal programming (NLGP).
- Gen et al. (1997) developed a GA to solve fuzzy NLGP. They assumed that the implemented membership functions are strictly monotone decreasing (or increasing) and continuous functions with the set of objective functions and certain maximum tolerance limits to the given resources.
- Hu et al. (2007) suggested a method for generating the solution that is consistent with the decision maker's desires where the goal with high priority may have the first level of goal achievement. The method uses a co-evolutionary genetic algorithm to solve the nonlinear, nonconvex problem that results from the original problem. GENCOPIII package is used to handle this problem.

1.4.5.3 Fuzzy Programming-Based GAs

- Li et al. (1997) presented an improved GA for solving a multi-objective solid transportation problem with consideration of the coefficients of the objective function as fuzzy numbers. The selection and evaluation process in GA are done by incorporating ranking of fuzzy numbers with integral value.
- Kim (1998) designed a two-phase genetic algorithm to improve the system performance in nonlinear and complex problems. The first phase is responsible for generating a fuzzy rule base that covers as many of the training examples as possible. The second phase constructed fine-tuned membership functions that minimize the system error.
- Liu and Iwamura (2001) provide a fuzzy simulation-based GA to handle both fuzzy objectives and goal constraints as well as other ideas.
- Jimenez et al. (2003) proposed an evolutionary algorithm to solve fuzzy nonlinear programming as a first step to solving the general nonlinear programming problem.
- Sasaki and Gen (2003) proposed a GA for solving fuzzy multiple objective design problems by implementing a new chromosomes representation that makes the GA more effective.
- Wang et al. (2005) implemented the multi-objective GA to extract interpretable fuzzy rule-based knowledge from data where the genes

are arranged into control genes and parameter genes. This division enables the fuzzy sets and rules to be optimally reduced.

At the end of this section, we can decide that the implementation of GAs in solving the FMCDM problems are occupied a wide interest of the research move so than any other AI searches technique. For more knowledge, see the following website: http://www.jeo.org/emo/EMOOjournals.html. However, there are still some problems in FMCDM problems that have not been studied yet such as:

1. Large-scale FMCDM problems with fuzzy numbers in the objective functions and constraints.
2. Combining both ES and GA to handle the fuzzy preferences in MCDM problems to get a more powerful solution method.
3. Implementing the GA to study both sensitivity and parametric analysis of linear and nonlinear FMCDM.

1.4.6 Ant Colony Optimization

Ant colony optimization (ACO) is a meta-heuristic approach that emulates the foraging behavior of real ants to find the shortest paths between food sources and their nest. This approach is proposed by Dorigo (1992). During the ant's walk from food sources and vice versa, ants deposit a chemical substance called "*Pheromone*" on the ground to guide the rest of ants to the shortest and safest path they should follow. The artificial ants that simulate the real ants perform random walks on a completely connected graph $G = (S, L)$, whose vertices are the solution components S and the connections L. This graph is based on probabilistic model called the "Pheromone model." When a constrained combinatorial optimization problem is considered, the constraints are built into the ants to get the feasible solution(s) only. ACO methods have been successfully applied to diverse combinatorial optimization problems, including traveling salesman, quadratic assignment, vehicle routing, telecommunication networks, graph coloring, constraint satisfaction, Hamiltonian graphs, and scheduling (Cordon et al., 2002). The following chart indicated the mechanism of ACO in solving combinatorial optimization (CO).

The ACO approach is performing well in combinatorial network optimization problems where the solution space is difficult to enumerate especially in large-scale problems. It has been applied to solve the multi-objective combinatorial optimization problems. For example, Chan and Swarnkar (2006) present a fuzzy goal programming approach to model the

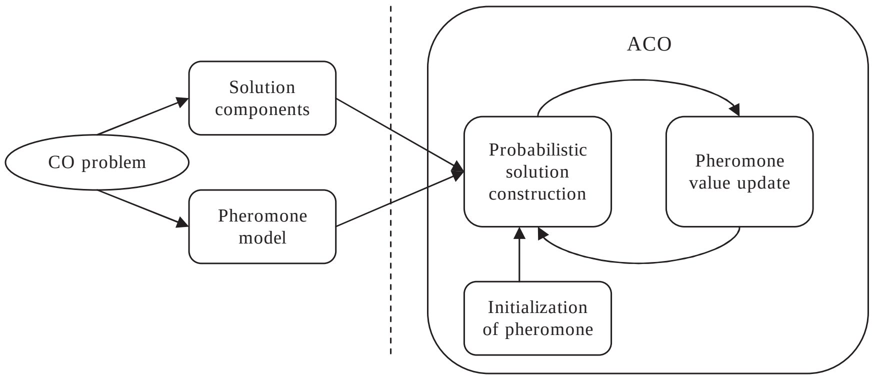

Figure 4. Mechanism of ACO in solving combinatorial optimization (Blum, 2005)

machine tool selection and operation allocation problem of flexible manufacturing systems. The proposed model is optimized by an ant colony algorithm to the computational complexities involved in solving the problem. Doerner et al. (2006) applied Pareto ant colony optimization (P-ACO) that performs particularly well for integer linear programming. The given procedure identifies several efficient portfolio solutions within a few seconds and correspondingly initializes the pheromone trails before running P-ACO. This extension offers a larger exploration of the search space at the beginning of the search with low cost. Marc Gravel et al. (2002) applied the ACO for getting the solution of an industrial scheduling problem in an aluminum casting center. They present an efficient representation scheme of a continuous horizontal casting process that takes into account several objectives that are important to the scheduler.

A little research work has been done in using ACO and MCDM/ FMCDM problems. Most of the research work is done in multi-objective combinatorial optimization problems (MOCOPs) since the meta-heuristics perform much better than the other approaches. So, this area needs more and more research especially in combinatorial FMCDM problems.

1.4.7 Particle Swarm Optimization (PSO)

The basic principles of PSO are represented by a set of moving particles that is initially thrown inside the search space. Each particle is characterized by the following features:

1. A position and a velocity,
2. It knows its position and the objective function value for this position,

3. It knows its neighbors, the best previous position, and the objective function value,
4. It remembers its best previous position,
5. It is considered that the neighborhood of a particle includes this particle itself.

At each time step, the behavior of a given particle is a compromise between three possible choices:

1. Following its own way,
2. Going toward its best previous position,
3. Going toward the best neighbor's best previous position.

The basic equations of PSO can be formalized as follows:

$$\begin{cases} v_{t+1} = c_1 v_t + c_2\left(p_{i,t} - x_t\right) + c_3\left(p_{g,t} - x_t\right) \\ x_{t+1} = x_t + v_{t+1} \end{cases} \tag{13}$$

with

$v_t :$ = velocity at time step t,

$x_t :$ = position at time step t,

$P_{it} :$ = best previous position at time step t,

$P_{gt} :$ = best neighbours previous best, at time step t, (or best neighbor),

$c_1, c_2, c_3 :=$ social/cognitive confidence coefficients.

PSO has been used in solving some real-life applications that involved multi-objectives. For example, Parsopoulos and Vrahatis (2002) presented the first study on MCDM by using PSO algorithm. The authors highlighted some important issues such as:

1. The ability of PSO to obtain the Pareto optimal points as well as the shape of the Pareto front.
2. Applying the weighted sum approach with fixed or adaptive weights.
3. Adopting the well-known GA approach VEGA for MCDM problems to the PSO framework to develop multi-swarm PSO to be implemented in MCDM problems in an effective manner.

The study can be considered the corner stone of applying PSO to solve such MCDM problems. Salman et al. (2002) proposed a PSO to task assignment. The PSO system combines local search methods (through self-experience) with global search methods (through neighboring experience), attempting to balance exploration and exploitation. A scan of some international electronic databases indicated that PSO has not applied yet in solving FMCDM problems.

1.5 Conclusions

From the above analysis, one can conclude that the implementation of AI techniques to handle FMCDM problems has occupied a reasonable amount of attention from the researchers with respect to some AI techniques such as ES, ANN, and GAs. But other techniques have not been opened yet such as SA, TS, PSO, DNA, and parallel hybrid techniques for handling the problems of FMCDM. However, the AI techniques that have been applied proved that they have the following advantages when dealing with FMCDM problems:

1. They have the possibility to consider the qualitative factors in the model structure and the solution procedure.
2. They can handle the decision maker's preferences, which are characterized as fuzzy preferences.
3. They can deal with a large amount of data that can be used in solving FMCDM problems.
4. The availability to estimate the aspiration levels in FMCDM.
5. The ability to estimate (determine) the membership functions that can be implemented to transform the FMCDM problem into a crisp problem to be handled easily.
6. The possibility to search and scan the search space in fuzzy multi-criteria combinatorial optimization problems where the search space is very large.
7. The AI techniques successes in solving different real-life problems such as scheduling, manufacturing, chemical, managerial, and other industrial applications.

1.5.1 Research Directions

The future research direction in this area is viewed from two angles:

1. Improving the performance of intelligent techniques by combining two or more of these techniques to get more powerful ones.
2. Implementing the available techniques to handle the FMCDM problems.

We shall talk about each individual case.

First: Improving the available techniques:

a) The mathematical background of these techniques needs more investigation and analysis.
b) Extending the AI techniques to handle more problems regarding FMCDM.
c) Studying the possibility and validity of combining more than two of these techniques to outperform the original ones.
d) Developing a comparative study between the AI techniques (metaheuristic techniques) to measure the performance of each one with respect to others. On the other side, measuring the performance and/or the quality of the solution(s) when changing the parameters of each technique.
e) Lights should be placed on new hybrid techniques as well as on parallel hybrid techniques that will be probably perform better than the AI techniques themselves.

Second: Intelligent FMCDM research directions:

This area of research still needs intensive research such as the following directions:

a) Large-scale FMCDM with mixed integer decision variables needs more investigation especially by using parallel hybrid intelligent systems to reduce the solution time.
b) Measuring the performance of AI techniques in higher dimensional FMCDM problems where the only test of performance is using benchmark functions. In addition, the theoretical analysis of measuring AI performance needs a look from the researchers.
c) Developing the theoretical analysis to deal with the FMCDM problems in its fuzzy environment without transforming it into crisp model, where the resulting solution may be more reasonable than the solution results from the transformation process.
d) Studying the effect of changing the AI techniques parameters on the solution behavior of FMCDM problems. In other words, understanding

the dynamics of swarm's dynamics (as in PSO) and the Pheromones dynamics (as in ACO) on the behavior of the optimization process.

e) Until now, no one has tried to open the area on doing both parametric and sensitivity analysis of MCDM and/or FMCDM by applying the AI techniques. The time is suitable for performing intelligent parametric analysis of MCDM and/or FMCDM problems. The results may be better than the traditional techniques for both linear and nonlinear FMCDM problems. As an idea, conduct the study of intelligent parametric analysis based on satisfying Kuhn−Tucker conditions or look for another easy way to do that.

f) Developing an intelligent system that combined most AI techniques to deal with FMCDM problems. For example, ES, ANN, SA, GA, and PSO may be combined in the following manner:
   - ES may handle the fuzzy preferences and other qualitative factors that have a great impact on the FMCDM problem behavior. This phase can be used as an evaluation process of the result solution(s).
   - Applying GA as a second phase to scan the solution space to get a satisfactory Pareto optimal solution.
   - Improving the performance of a PSO-based ANN with SA to use the GA output as an initial solution to this phase as a trial to obtain a better solution than the one in step (b).

This is a proposed scenario, and the researchers can change this scenario in different manners. More attention can be paid to measure the performance, and effectiveness should be done to compare the results with the existing techniques.

1. The ANN (for example) can be used to generate a reasonable membership function for solving the FMCDM problems based on the desires of the DM and/or the historical data of the problem.
2. Applying the AI techniques to implement the ranking approaches to deal with FMCDM problems.
3. Developing new approaches based on AI techniques to handle the fuzzy multi-attribute decision-making problems where a little research work has been done in this area.
4. Implementing AI techniques to solve FMCDM in the presence of multiple decision makers with indifference preferences information.
5. Invoking AI techniques in both interactive and goal programming to solve FMCDM. For example, developing an ANN to capture and represent the decision maker's preferences to support the search process for obtaining the most desirable solution.

6. The hybridization of fuzzy logic and evolutionary computation in what is called genetic fuzzy systems became an important research area during the last decade, and the results should be applied to deal with FMCDM to solve the problem without transforming it into a crisp model.

Last but not least, the implementation of AI techniques to solve the different problems of both FMCDM and MCDM will occupy a wide range of research in the next 20 years because of their ability to handle many complicated problems.

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