Table of Contents

Preface V

1. Basic Graph Theory 1
- 1.1 Graphs, Subgraphs and Factors 2
- 1.2 Paths, Cycles, Connectedness, Trees 5
- 1.3 Euler Tours 13
- 1.4 Hamiltonian Cycles 15
- 1.5 Planar Graphs 21
- 1.6 Digraphs 26
- 1.7 An Application: Tournaments and Leagues 29

2. Algorithms and Complexity 35
- 2.1 Algorithms 36
- 2.2 Representing Graphs 38
- 2.3 The Algorithm of Hierholzer 42
- 2.4 How to Write Down Algorithms 44
- 2.5 The Complexity of Algorithms 46
- 2.6 Directed Acyclic Graphs 50
- 2.7 NP-Complete Problems 53
- 2.8 HC is NP-Complete 56

3. Shortest Paths 63
- 3.1 Shortest Paths 63
- 3.2 Finite Metric Spaces 65
- 3.3 Breadth First Search and Bipartite Graphs 67
- 3.4 Bellman's Equations and Acyclic Digraphs 72
- 3.5 An Application: Scheduling Projects 75
- 3.6 The Algorithm of Dijkstra 79
- 3.7 An Application: Train Schedules 84
- 3.8 The Algorithm of Floyd-Warshall 87
- 3.9 Cycles of Negative Length 92
- 3.10 Path Algebras 93

4. Spanning Trees ... 99
4.1 Trees and Forests ... 99
4.2 Incidence Matrices ... 101
4.3 Minimal Spanning Trees ... 105
4.4 The Algorithms of Prim, Kruskal and Boruvka ... 108
4.5 Maximal Spanning Trees ... 115
4.6 Steiner Trees ... 117
4.7 Spanning Trees with Restrictions ... 120
4.8 Arborescences and Directed Euler Tours ... 123

5. The Greedy Algorithm ... 129
5.1 The Greedy Algorithm and Matroids ... 129
5.2 Characterizations of Matroids ... 131
5.3 Duality of Matroids ... 137
5.4 The Greedy Algorithm as a Technique for Approximation ... 139
5.5 Minimization in Independence Systems ... 145
5.6 Accessible Set Systems ... 150

6. Flows ... 155
6.1 The Theorems of Ford and Fulkerson ... 155
6.2 The Algorithm of Edmonds and Karp ... 161
6.3 Layered Networks and Phases ... 171
6.4 Constructing Blocking Flows ... 177
6.5 Zero-One Flows ... 188
6.6 The Algorithm of Goldberg and Tarjan ... 192

7. Applications in Combinatorics ... 209
7.1 Disjoint Paths: The Theorem of Menger ... 209
7.2 Matchings: The Theorem of König ... 213
7.3 Partial Transversals: The Marriage Theorem ... 217
7.4 Combinatorics of Matrices ... 223
7.5 Dissections: The Theorem of Dilworth ... 227
7.6 Parallelisms: The Theorem of Baranyai ... 231
7.7 Supply and Demand: The Theorem of Gale and Ryser ... 234

8. Colourings ... 239
8.1 Comparability Graphs and Interval Graphs ... 239
8.2 Colourings ... 242
8.3 Edge Colourings ... 245
8.4 Cayley Graphs ... 248

9. Circulations ... 253
9.1 Circulations and Flows ... 253
9.2 Feasible Circulations ... 256
9.3 Elementary Circulations ... 263

9.4 Minty's Painting Lemma ... 266
9.5 The Algorithm of Klein ... 269
9.6 The Algorithm of Busacker and Gowen ... 273
9.7 Potentials and ε-Optimality ... 276
9.8 Determining Optimal Circulations by Successive Approximation ... 285
9.9 A Polynomial Procedure REFINE ... 290
9.10 The Algorithm of Klein II ... 297
9.11 Some Further Problems ... 302

10. Synthesis of Networks ... 305
10.1 Symmetric Networks ... 305
10.2 Synthesis of Equivalent Flow Trees ... 308
10.3 Synthesizing Minimal Networks ... 316
10.4 Cut Trees ... 322
10.5 Increasing the Capacities ... 326

11. Connectivity ... 331
11.1 k-Connected Graphs for $k \geq 2$ ... 331
11.2 Depth First Search ... 334
11.3 2-Connected Graphs ... 338
11.4 Depth First Search for Directed Graphs ... 345
11.5 Strongly Connected Directed Graphs ... 347
11.6 Edge Connectivity ... 351

12. Matchings ... 355
12.1 The 1-Factor Theorem ... 355
12.2 Augmenting Paths ... 358
12.3 Alternating Trees and Flowers ... 363
12.4 The Algorithm of Edmonds ... 371
12.5 Matching Matroids ... 387

13. Weighted Matchings ... 389
13.1 The Bipartite Case ... 389
13.2 The Hungarian Algorithm ... 391
13.3 Matchings, Linear Programs and Polytopes ... 400
13.4 The General Case ... 404
13.5 The Chinese Postman ... 408
13.6 Matchings and Shortest Paths ... 413
13.7 Further Problems Concerning Matchings ... 420

14. A Hard Problem: The TSP ... 423
14.1 The Problem ... 423
14.2 Lower Bounds: Relaxations ... 426
A. The Assignment Relaxation ... 426

B. The MST Relaxation ... 427
C. The 1-Tree Relaxation ... 428
D. The LP Relaxation ... 430
14.3 Lower Bounds: Subgradient Optimization ... 431
14.4 Algorithms for Approximation ... 435
14.5 Upper Bounds: Heuristics ... 441
14.6 Upper Bounds: Post-Optimization ... 444
14.7 Exact Neighbourhoods ... 448
14.8 Optimal Solutions: Branch and Bound ... 453
14.9 Concluding Remarks ... 460
14.10 Appendix: Some NP-Complete Problems ... 462

A. Solutions ... 471
A.1 Solutions for Chapter 1 ... 471
A.2 Solutions for Chapter 2 ... 477
A.3 Solutions for Chapter 3 ... 481
A.4 Solutions for Chapter 4 ... 487
A.5 Solutions for Chapter 5 ... 491
A.6 Solutions for Chapter 6 ... 494
A.7 Solutions for Chapter 7 ... 503
A.8 Solutions for Chapter 8 ... 510
A.9 Solutions for Chapter 9 ... 511
A.10 Solutions for Chapter 10 ... 518
A.11 Solutions for Chapter 11 ... 524
A.12 Solutions for Chapter 12 ... 531
A.13 Solutions for Chapter 13 ... 536
A.14 Solutions for Chapter 14 ... 541

B. List of Symbols ... 543
B.1 General Symbols ... 543
B.2 Special Symbols ... 545

References ... 551

Index ... 577