

Foreword

These notes revolve around three similarity problems, appearing in three different contexts, but all dealing with the space $B(H)$ of all bounded operators on a complex Hilbert space H . The first one deals with group representations, the second one with C^* -algebras and the third one with the disc algebra. We describe them in detail in the introduction which follows. The first two are still *open*, the third one was solved only recently, actually after the first edition of this book appeared. We include its solution in the present edition.

This volume is devoted to the background necessary to understand these three problems, to the solutions that are known in some special cases and to numerous related concepts, results, counterexamples or extensions which their investigation has generated.

For instance, we are naturally lead to study various Banach spaces formed by the matrix coefficients of group representations. Furthermore, we discuss the closely connected Schur multipliers and Grothendieck's striking characterization of those which act boundedly on $B(H)$.

While the three problems seem different, it is possible to place them in a common framework using the key concept of "*complete boundedness*", which we present in detail. In some sense, completely bounded maps can also be viewed as "coefficients" of C^* -algebraic representations, if we allow " $B(H)$ -valued coefficients", this is the content of the fundamental factorization property of these maps, which plays a central rôle in this volume.

Using this notion, the three problems can all be formulated as asking whether "boundedness" implies "complete boundedness" for linear maps satisfying certain additional algebraic identities.

Finally, a word on the structure of this book: this is definitely a lecture notes volume. Each chapter corresponds roughly to a lecture. In each one, we try to reach quickly some main point, without too many side remarks. This usually corresponds to the actual lecture. Once this point has been made, we then allow ourselves to develop all sorts of additional comments and a guide to the literature (the "notes and remarks") which expand on the first part, and which (for lack of time usually) the audience of the lecture is invited to read.

The main body of the notes is essentially self contained and can be read by anyone familiar with basic Functional and Harmonic Analysis, as presented for example in Rudin's books [R2, R4]. We believe this volume may be used as the basis for an advanced graduate course in Functional Analysis.

These notes are originally based on a course given during a summer school organized by S. Negrepointis in Spetses (Greece) in July 92, and on courses given in Texas A&M and in Paris VI in 92/93. I am very grateful to all those who subsequently provided me with more information, additional references and corrected misprints or errors of all kind. In particular, it is a pleasure to thank M. Bozejko, A. Hess, J. Holbrook, C. Le Merdy, V. Mascioni, V. Paulsen, V. Peller, G. Popescu, F. Wattbled and Q. Xu.

In this new edition, two chapters have been added at the end, and some additional material has been inserted, notably in chapter 2. However, to keep the same numbering of statements as in the first edition, we have chosen to put an asterisk to the newly inserted statements.

Special thanks are due to those who pointed out misprints in the first edition, so that I could remove them from the present one, no doubt adding some in the new chapters!

The research for this work was partially supported by the N. S. F.

In conclusion, I would like to thank Robin Campbell, who typed most of the book, for her outstanding job.