Preface

Optimal transportation is an old subject.

In fact the issue was raised by Monge (the Monge-Ampère equation), then rediscovered by Kantorovich in the context of economics and it is an important topic within probability (the Wasserstein metric).

It concerns, roughly, how to transport a mass (probability density) from one location (and distribution) to another, in such a way as to keep the transportation cost to a minimum - a very natural and reasonable problem. But, it was in the late '80s that the basic mathematical problem associated to it, and its connection with Monge-Ampère type equations, started to surface in very different environments: in metereology, in the theory of front formation, in the discretization of Euler equations by some sort of Lagrangian approach, in the theory of lubrication, the study of rates of decay for nonlinear evolution equations, etc..

It was found to be a lagrangian version of the div-curl decomposition, to have connection with sand pile dynamics, with statistical mechanics and many other fields.

We feel, thus, that this is a very appropriate moment to bring together a group of researchers in the field, with different views and perspectives in the topic, to create a basic "guide" to those young researchers, that may find this theory challenging and useful.

The C.I.M.E. course on Optimal Transportation and Applications, held in Martina Franca (Italy) from September 2 to September 8, 2001, was designed precisely to fulfil this purpose.

The school was organized into the following five courses:

G. Buttazzo: *Shape optimization problems through the Monge-Kantorovich equation*

Y. Brenier: *Geometric PDEs related to fluids and plasmas*

L. A. Caffarelli: *The Monge-Ampère Equation, Optimal Transportation and periodic media*

L. C. Evans: *Optimal Transportation*

C. Villani: *Mass transportation tools for dissipative PDEs*

We are pleased to express our appreciation to the speakers for their beautiful lectures. The present volume records and completes the material presented in the courses listed above, with an important and original additional contribution from Ambrosio and Pratelli:

L. Ambrosio and A. Pratelli: *Existence and Stability Results in the L^1 Theory of Optimal Transportation.*

Many researchers from Italy and abroad attended the courses; we thank them for their active contribution to the success of the school. We would also like to thank the C.I.M.E. Scientific Committee for the invitation to organize the School in Martina Franca.

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