Preface

The Euro-Summer School on Mathematical Aspects of Evolving Interfaces gathered senior experts and young researchers at the University of Madeira, Funchal, Portugal, in the week July 3-9, 2000. This meeting arose as a joint school of CIM (Centro Internacional de Matemática, Portugal) and CIME (Centro Internazionale Matematico Estivo, Italy).

The school was intended to present an advanced introduction and state of the art of recent analytic, modeling and numerical techniques to the mathematical representation and description of moving interfaces. Five complementary courses were delivered and this volume collects the notes of the lectures.

Interfaces are geometrical objects modeling free or moving boundaries and arise in a wide range of phase change problems in physical and biological sciences, in particular in material technology and in dynamics of patterns. Especially in the end of last century, the rigorous study of evolving interfaces in a number of applied fields becomes increasingly important, so that the possibility of describing their dynamics through suitable mathematical models became one of the most challenging and interdisciplinary problems in applied mathematics.

It was recognized that essential problems related to evolving interfaces can be modelled by means of partial differential equations and systems in domains whose boundary depends on time. In many complicated cases these boundaries are themselves unknown, and correspond, e.g., to a particular level set, or to the discontinuity set, of some physical quantity. In particular, free boundary problems are boundary value problems for differential equations set in a domain where part of the boundary is "free" and further conditions allow to exclude undeterminacy.

Although the first modern work in a free boundary problem was written by Lamé and Clapeyron in 1831, who considered a simple model for the solidification of a liquid sphere, in the last decades of the XXth century this interdisciplinary field developed tremendously with many new computational demands and new problems from industry and applied sciences, as well as with increasing contributions from Mathematics. Indeed, problems of this sort are concerned with several phenomena of high applied interest. Examples include Stefan type problems, where, tipically, the free boundary is the moving interface between liquid and solid, as well as, more general models of phase transitions. Another important example arises in filtration through porous media; here free boundaries occur as fronts between saturated and unsaturated regions. Interesting examples also come from reaction-diffusion models, fluid dynamics, contact mechanics, superconductivity and so on. Several of these problems are also of direct industrial interest, and offer an interesting opportunity of collaboration among theoretical analysts, mathematical physicists and applied scientists.

The Madeira school reported on mathematical advances in some theoretical, modeling and numerical issues concerned with dynamics of interfaces and free boundaries. Specifically, the five courses dealt with an assessment of recent results on the optimal transportation problem (L. Ambrosio), the numerical approximation of moving fronts evolving by mean curvature (G. Dziuk), the dynamics of patterns and interfaces in some reaction-diffusion systems with chemical-biological applications (M. Mimura), evolutionary free boundary problems of parabolic type or for Navier-Stokes equations (V.A. Solonnikov), and a variational approach to evolution problems for the Ginzburg-Landau functional (H.M. Soner).

We expect that these lecture notes will be useful not only to experienced readers, to find a detailed description of results and a presentation of techniques, but also to the beginners that aim to learn some of the mathematical aspects behind the different fields.

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