Preface

This text consists of notes of three courses given during the CIME summer school which took place in 1999 in Martina Franca, Italy. The subject was Iwahori-Hecke algebras and their representation theory. The program consisted of several courses taught by senior faculty and some advanced lectures given by young researchers.

The scheduled courses were

G. Heckman, *Representation theory of affine Hecke algebras*
R. Howe, *Affine-like Hecke algebras and p-adic representations theory*
G. Lusztig *Representations of affine Hecke algebras*
I.Cherednik *Hankel transform via double Hecke algebra*

The specialized lectures on more advanced topics were given by T. Haines, M. Nazarov, C. Kriloff, U. Kulkharni, K. Maktouf, J. Kim and G. Papadoupoulo.

The volume contains the notes of the courses by I. Cherednik, R. Howe, and G. Lusztig. G. Heckman was not able to provide notes for his course. We give some references later in the introduction.

In the remainder of the introduction we give some background material on affine Hecke algebras and extra references that complement the notes.

Two basic problems of representation theory are to classify irreducible representations and decompose representations occuring naturally in some other context. Algebras of Iwahori-Hecke type are one of the tools and were (probably) first considered in the context of representation theory of finite groups of Lie type. For example for $G = GL(2, \mathbb{F}_q)$, consider the question of decomposing the induced module

$$I = Ind_B^G[\mathbb{1}] := \{f : G \longrightarrow \mathbb{C} \ : \ f(gb) = f(g)\}, \qquad \iota(g)f(x) := f(g^{-1}x),$$

where B is the subgroup of upper triangular matrices. One is naturally led to consider the algebra of intertwining operators of the module I. These are linear endomorphisms $T : I \longrightarrow I$ satisfying $T \circ \iota(g) = \iota(g) \circ T$ for all $g \in G$.

This algebra of endomorphisms can be identified with the algebra of B-biinvariant functions on G with multiplication structure given by convolution. In the case of $GL(2, \mathbb{F}_q)$ it is generated (over $\mathbb{C}$) by an element T satisfying the relation $T^2 = (q-1)T + q$, and is called the finite Hecke algebra of type

A_1. Its generalization to type A_n is of independent interest to knot theory because it is a quotient of the braid group.

The Hecke algebras mentioned above play an important role in combinatorics and representation theory of $GL(n)$ and the symmetric group. An account of their role in the theory of finite groups of Lie type is detailed in [Car] and the references therein.

The above algebra is not what Hecke introduced in the context of automorphic forms. He defined certain operators $T(n)$ $(n \in \mathbb{N})$ that act on automorphic forms for congruence groups and studied their eigenvalues and relation to Dirichlet series. An introduction to this theory can be found in [Se]. When one considers automorphic forms in the adelic setting ([Ge], [Bump]) the operators $T(n)$ are very closely related to the previous example. One is led to consider the group of rational points $G(\mathbb{F})$ for a local field $\mathbb{F}$ with residual field of characteristic p and a compact open subgroup K. The operators $T(n)$ are K-biinvariant functions supported on certain cosets. This interpretation has led to a systematic study of the representation theory of groups over totally disconnected fields.

The talks of Howe gave an overview of this research area, particularly the role of these algebras. The notes are in the article of R. Howe and C. Krillof *Affine-like Hecke algebras and p-adic representation theory* The following is a brief list of the topics covered.

– Structure of p-adic groups and their associated affine Hecke algebras.
– Bruhat and Iwahori-Bruhat decompositions from a geometric perspective.
– Application of Hecke algebras in representations of the p-adic groups.

Lusztig's talks were focused on the special case when the compact open subgroup is an Iwahori subgroup. In this case very detailed knowledge of the representation theory is available due to his work partly joint with Kazhdan. His notes, G. Lusztig *Notes on affine Hecke algebras* cover the following topics:

– The affine Hecke algebra
– $\mathcal{H}$ and equivariant K-theory
– Convolution
– Subregular case

A very different field where Hecke algebras play a role is in the area of special functions. MacDonald has made various conjectures about the existence of orthogonal polynomials in several variables attached to root systems. These polynomials are related to the spherical functions of real and p-adic groups. The conjectures have a natural interpretation in the context of representations of Iwahori-Hecke algebras. Much of the work in this area *e.g.* by Cherednik, Heckman and Opdam make essential use of this structure. Heckmann's course provided an introduction to this area. We refer to the Bourbaki talks [He] and [M].

In Cherednik's notes the focus is on the advantages of the operator approach in the theory of Bessel functions and the classical Hankel transform. An account of these results can be found in the article I. Cherednik and Y. Markov, *Hankel transform via double Hecke algebra:*

- L-operator
- Hankel transform
- Dunkl operator
- Double H double prime
- Nonsymmetric eigenfunctions
- Inverse transform and Plancherel formula
- Truncated Bessel functions

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