Chapter 3 Visualization Models for Multiple Reference Points

Visualization models for multiple reference points have many unique characteristics. They can effectively handle complex information need of a user by using multiple reference points and achieve excellent operation flexibility for the multiple reference points.

A traditional information retrieval system responds to a user search query with a linear results list. Retrieved documents may be ranked based upon their similarities to the query if the system offers a relevance ranking mechanism. However, it does not provide users with a flexible and powerful browsing environment with which users can make a relevance judgment about the retrieved documents. That is because the linear list does not provide users with relevance information among the retrieved documents and it only offers relevance information between the query and the retrieved documents. Originally, the visualization algorithms for multiple reference points were designed to visualize the results of a search query to overcome the weakness of the linear structure of a search results list by projecting the retrieved documents onto a low dimensional visual space. This visual space is defined by the multiple reference points. Basically, a reference point represents a user's information need. We will discuss this concept and its implications in depth in the following section. In this sense, the visualization models for multiple reference points were directly driven or motivated by information retrieval.

The visualization algorithms can be classified into two groups based upon what kind of an information retrieval system it works on: the algorithms that work on a Boolean information retrieval system and the algorithms that work on a vector information retrieval system. The algorithms for multiple reference points can also be categorized into three categories based upon the status of reference points in the visual spaces: the algorithms for fixed reference points in the visual spaces; the algorithms for manual movable reference points; and the algorithm for automatic rotating reference points. The status changes of reference points can not only facilitate users' manipulation of the reference points, but also assist users in clarifying the notorious ambiguity of overlapped documents in the visual space and reveal new visual semantic relationships among displayed documents.

Among the three categories, a large body of research and applications has gravitated to the second category because the algorithms for manual movable reference points are flexible, simple, and also powerful. The representative and pioneering paradigm in this category was *VIBE* (Visual Information Browsing

Environment) (Olsen and Korfhage, 1994). Other related algorithms and applications in this category were derived directly from the *VIBE* algorithm.

The underlying principle of the *VIBE* algorithm is that the position of a displayed document between two reference points indicates its relevance to them in its visual space. The position of a reference point can be any location in the visual space. Due to the algorithm simplicity and flexibility, it has been widely used and adapted to many application domains. For instance, Web pages as objects were visualized in the visual environment *WebVIBE* (Morse and Lewis, 2002). The model was integrated in a multilingual and multimodal system to visualize video objects (Lyu et al., 2002). A visualization environment and a geographic environment were combined to generate a new environment so that users could search and browse geographic information in both *Geo-VIBE* (Cai, 2002) and *Visual Digest* (Christel, 1999; Christel and Huang, 2001). *Radial* (Carey et al., 2003) provided a two dimensional circular space where terms or reference points were limited to positions on its boundary and documents were situated within the circle. Virtual reviewer applied the *VIBE* algorithm for visualization of movies reviews where reference points were virtual reviewers of movies while displayed objects were movies (Tatemura, 2000). Experimental studies on the *VIBE* systems were also conducted (Koshman, 2004; Morse and Lewis, 2002).

3.1 Multiple reference points

A reference point (*RP*), or a point of interest (*POI*), is a search criterion against which database documents or document surrogates are matched and search results are generated and presented to users. In a broad sense, a reference point represents users' information needs and any information related to users' needs. For instance, it can range from general information such as a user's past/current research interests, a user's previous search histories, a user's reading preferences, a user's research projects involved, user's affiliation and educational background; to specific information like search terms from a complicated query, browsed documents, and a group of user's queries as well. A reference point may correspond to either a term or a group of terms. Terms in a reference point can be weighted or not weighted. Since a reference point can cover all aspects of users, it is also called a user profile.

Now let us discuss the implication of multiple reference points on information retrieval.

The primary implication of multiple reference points is that they can form a low dimensional visual space and documents can be mapped onto the space based upon their attractions to the reference points.

Contribution of each individual keyword of a query to a final retrieved result is hardly observed in a given linear results list which reflects combinative impacts of all terms involved in a query. A search process is further compounded when a user formulates a multi-faceted query. The user may want to see not only the result set of the original query but also result sets based upon component terms of the query. This would give the user some idea of the contribution of the query parts to the full query (Havre et al. 2001). This is important for a searcher because searching is an iterative process and each process needs to optimize a search query by adjusting search terms based upon search feedbacks. If users are not satisfied with returned search results, a query reformulation follows. A query reformulation may include removing useless search terms, adding new related terms, revising weights assigned to the terms, and so on. All of these actions need a better understanding of the degree to which an individual term affects on the returned search results. That is, if users can understand how and which search terms affect the search results, users can revise and modify a search strategy more accurately to perform a better search. For instance, if the size of a search result set is too small, users may want to know exactly which search terms led to the results. After identifying the terms, users can either change weights assigned to the terms, or replace them with other terms to increase the result size. If search terms are assigned to reference points respectively and presented in the visual space, the impact of each individual term on search results can be easily and intuitively perceived. Therefore, users can make a judgmental decision on query reformulation based upon the provided visual display.

Multiple search terms instead of a single search term can also be grouped and assigned to a reference point. Previous search queries and current search query can serve as reference points respectively and they are present in the same visualization environment. So, both the degree to which the revised search strategies affect on the final results and comparisons of the impact of these strategies are visually displayed. Similarly, a user's preferences, user research interests or other related information can also serve as reference points if users want to know their impacts upon search results.

3.2 Model for fixed multiple reference points

The representative two-dimensional visualization model for fixed multiple reference points was *InfoCrystal* (Spoerri, 1993 a and b). The model was originally designed as a visual query language to visualize a query result from a Boolean based information retrieval system. In the Boolean contexts each reference point may be equivalent to a term or a sub-Boolean logic expression from a Boolean query. The visual space is a polygon where reference points constitute vertices of the polygon and visual results are displayed. The side lengths of the polygon are equal so that the reference points are evenly configured in the visual space. For instance, if there are three, four, and five reference points, the corresponding polygons are an equilateral triangle, square, and pentagon respectively. There are two basic types of icons within the polygon: the criterion icons that are reference points and interior result icons that show the retrieval results. The polygon is partitioned by N exclusive tiers. N is the number of reference points. The tiers are represented by concentric rings and have different radii. Each of these rings defines a special interior display area where interior result icons between certain reference points/ criterion icons are placed. In other words, result icons in the same tier share some commonality with respect to related reference points. The first tier covers result icons that represent documents related to only one inclusive reference point, the second tier covers result icons that represent only documents related to two inclusive reference points,…, and the last tier(the circle) covers one result icon that represents documents related to all *N* reference points. Shapes, directions, and sizes of these result icons vary in different tiers so that users may easily distinguish and identify them. Each side of a result icon is designed and placed to face one of the related criterion icons it meets. Positions of result icons within a tier are fixed and the number of documents satisfying the criteria is displayed on a corresponding result icon. The extent of related documents to the related reference points can also be visualized in result icons by partitioning the icons proportionally and coloring them differently.

For example, Fig. 3.1 shows an example of the configuration for four reference points where *r1*, *r2*, *r3*, and *r4* are reference points located at four corners of the large square as criterion icons. In this figure, there are 4 tiers due to the 4 reference points. Four circles as interior results icons are situated in the first tier, 8 rectangles in the second tier, 4 triangles in the third tier, and 1 square in the fourth tier. The circle close to *r1* in the first tier shows the results of the related document meeting the criterion *r1* AND NOT (*r2* AND *r3* AND *r4*) because it is located at the first tier, and the related documents are relevant to *r1* but *r2, r3*, and *r4*. The rectangle icon between reference points *r1* and *r2* in the second tier indicates the results of the related documents satisfying the criterion *r1* AND *r2* AND NOT (*r3* AND *r4*). The triangle icon between the center and reference point *r1* in the third tier illustrates the results of the related documents meeting the criterion *r1* AND *r2* AND *r4* AND NOT *r3*. The square icon in the fourth tier demonstrates the results of the related documents meeting the criterion *r1* AND *r2* AND *r3* AND *r4*. In this model result icons between two not adjacent reference points may appear more than one time. For instance, both the rectangle icon between the center icon and reference point *r1* and the rectangle icon between the center icon and reference point *r3* in the second tier show the same results satisfying the criterion *r1* AND *r3* AND NOT(*r2* AND *r4*).

If a criterion icon is a sub-Boolean expression (For instance, in a Boolean expression *A* AND (*B* OR *C*), *B* OR *C* is a sub-Boolean expression), it can be connected to another independent visual display that represents the sub-Boolean expression and has a similar display structure to its parent. In this way, the visualization model can be extended to display a hierarchy for a very complicated nested Boolean query.

This model can easily be applied to visualize documents in a vector space. If it is adapted to a vector space, the display framework or structure of the visual space has to be changed accordingly. Reference points are still situated at the corners of

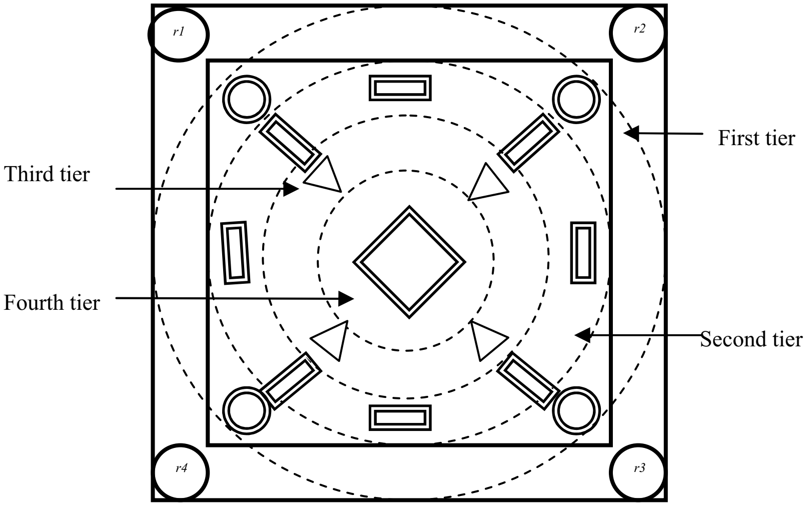

Fig. 3.1. Display of 4 reference points in a fixed reference point environment. (Spoerri, 1993a). © 2003 IEEE. Reprinted with permission

the polygon. The center of the polygon is defined as the origin of the new display coordinate system. In this case, objects in the visual space are individual document icons instead of interior result icons that present a set of related documents. Documents are mapped within the polygon. The position of a document in the visual space is determined by two basic parameters: direction and distance. For instance, if a document is relevant to a reference point, then it is located on the segment defined by the origin of the visual space and the reference point (criterion icon). A document is placed on the segment based upon such a principle that the document with a high similarity to the reference point is close to the origin, and vice versa. So, the position of a projected document is affected not only by its attractions to the criterion icons/ reference points but also by the degree to which the document is relevant to the reference points. As we know, in a vector space such a relevance degree between a document and a reference point can be measured and calculated by using any of the similarity measures discussed in Chap. 2.

It is apparent that the visualization models for both Boolean and vector information retrieval systems support multiple reference points. Reference point icons are fixed and valid display areas are the same in both models. However, visualized objects in these two models are quite different. One visualizes the interior result icons which reflect a result set of retrieved documents while the other displays individual documents. Interior result icon positions are fixed and the number of the result icons is constant in the Boolean based model while the positions of projected documents are not fixed and the number of document icons is a variable in the vector-based model.

Notice that in the Boolean-based model, an interior result icon within a tier represents results of all reference points which are either inclusive (AND) or exclusive (NOT) in a criterion. When the number of exclusive (NOT) reference points in a criterion increases, it could easily lead to an empty set of retrieved documents. It may happen when inclusive (AND) reference points are highly associated with exclusive (NOT) reference points. To avoid this phenomenon, the system should allow users to enable and disable exclusive (NOT) reference points in a criterion.

The uniqueness of the visualization models for fixed multiple reference points is that they may be applied to both a Boolean-based retrieval model and a vector-space-based retrieval model. Due to the fact that all reference points are evenly placed and fixed in the visual space, it results in a symmetrical and well-balanced visual area layout and therefore its interface achieves an aesthetically appealing effect.

3.3 Models for movable multiple reference points

The *VIBE* model (Olsen et al., 1993 a), one model for movable multiple reference points, is distinguished from other visualization models by the fact that the ratio-based similarity scales make displayed objects movable while semantic connections of these objects are still maintained in the visual space. The similarity between a displayed object and a reference point is not directly assigned to any Cartesian coordinates of the display space like other visualization models. The Cartesian system is usually employed to present objects for a visualization model. Cartesian coordinates can be two dimensional or three dimensional. Each dimension corresponds to an axis that is linear. These axes are mutual orthogonal or perpendicular to each other. Any of axes can range from $-\infty$ to ∞. The algorithm uniqueness suggests that the logic relationships among the displayed objects and reference points are independent of their physical locations in the visual space. Position changes of reference points may result in the reconfiguration of projected objects/documents in the visual space. The primary benefit of this uniqueness is that users may arbitrarily place a reference point in the visual space to any interest area (for instance, another reference point, an interest document, a cluster of documents), and observe the impact of that reference point on the area.

3.3.1 Description of the original VIBE algorithm

Since the *VIBE* model works in a vector space, a document D_s can be described by a group indexing keywords $D_s(k_1, k_2, ..., k_i, ..., k_g)$ where k_i is a keyword and g is the number of the index keywords for the document. $R_j(k_1, k_2, ..., k_i, ..., k_m)$ is a reference point (j=1, …, q) where k_i is a keyword, q is the number of the reference points predefined by users, and m is the number of keywords for the reference point R_j. According to the algorithm the position of a document is strongly related to similarities between the document and a group of predefined reference points. The impact of these reference points on the document are defined by a document reference point vector $DRPV_s(r_1, r_2, \ldots, r_i, \ldots, r_q)$ where r_i is the relevance value between document D_s and a reference point $R_j\,(k_1, k_2, \ldots, k_i, \ldots, k_m)$ ($j=1, \ldots, q$).

$$r_i = \frac{|D_s \cap R_i|}{\sum_{j=1}^{q} |D_s \cap R_j|} \tag{3.1}$$

Eq. (3.1) shows that the relevance between document D_s and a reference point R_j is determined by ratio of the number of keywords shared by both the document D_s and the reference point R_j to sum of shared keywords by the document D_s and all reference points. Function $|X|$ indicates the number of elements in a set X. Notice that there may be many other ways to calculate the relevance between a document and a reference point. Because documents and reference points are described in a vector space, the Euclidean distance similarity model, the cosine similarity model, and other models discussed in Chap. 2 are all applicable.

If document index keywords and reference point keywords are associated with weights, the relevance between the document D_s and a reference point R_j can also be defined as:

$$r_i = \frac{\sum_{a=1}^{|D_s \cap R_j|} \left(MIN(WD_s(k_a), WR_i(k_a)) \right)}{\sum_{j=1}^{q} \left(\sum_{a=1}^{|D_s \cap R_j|} \left(MIN(WD_s(k_a), WR_j(k_a)) \right) \right)} \tag{3.2}$$

In Eq. (3.2), i is always between 1 and q, $WD_s(k_a)$ and $WR_i\,(k_a)$ are weights of a shared keyword k_a in document D_s and reference point R_j. Keyword k_a is an element from a joint set between keywords of document D_s and keywords of reference point R_j ($D_s \cap R_j$). In this equation, the sum of the minimum weights of shared keywords in the document D_s and reference point R_j replaces the number of shared keywords in the document D_s and reference point R_j. The equation is also normalized by the total of the minimum weights of shared keywords in document D_s and all reference points R_j($j=1, \ldots, q$) to avoid an unnecessary scale effect.

A reference point can be arbitrarily located in any meaningful point in the two dimensional visual space. $P(R_j)$ denotes the location of the reference point R_j in the visual space.

$$P(R_j) = V_j(x_j, y_j), \quad j = 1, \ldots, q \tag{3.3}$$

In fact, $V(x, y)$ defines a point which is the reference point icon position in the two dimensional visual space.

The positions of all related reference points in the visual space play a very important role in positioning a projected document. In addition, the relevance between a document and related reference points can also dominate the position of the document in the visual space. When these two factors are considered, the ultimate position of a document is determined.

Given a document D_s is related to two reference points R_1 and R_2. The positions of these two reference points in the two dimensional visual space are $P(R_1)= V_1(x_1, y_1)$ and $P(R_2)= V_2(x_2, y_2)$, respectively. Position of D_s is $P(D_s)= V_s(x_s, y_s)$. The similarities between document D_s and the two reference points R_1 and R_2 are expressed as $DRPV_s\,(r_1, r_2)$ and we assume that both r_1 and r_2 are available by using either Eq. (3.1) or (3.2). Then, the location of document D_s is supposed to be located on a segment between R_1 and R_2 in the visual space. Distances between R_1 and R_2, R_1 and D_s, R_2 and D_s are d_1, d_2 and d_3, respectively. The exact location of D_s depends on its similarities to the two reference points $DRPV_s(r_1, r_2)$. The more similar to a reference point, the closer it is located to the reference point, and vice versa (See Fig. 3.2).

Let us discuss the calculation of the document projection position $V_s(x_s, y_s)$ in the visual space. It is clear that d_1, d_2 and d_3 maintain relationships in Eqs. (3.4) and (3.5).

$$d_1 = \left((x_2 - x_1)^2 + (y_2 - y_1)^2\right)^{1/2} \tag{3.4}$$

$$d_1 = d_2 + d_3 \tag{3.5}$$

In Fig. 3.2, both $V_3\,(x_1, y_s)$ and $V_4\,(x_1, y_2)$ are two temporary points, the right triangle $\Delta V_1V_sV_3$ is similar to the right triangle $\Delta V_1V_2V_4$ because they share the same angle $V_2V_1V_4$, and both angles $V_1V_3V_s$ *and* $V_1V_4V_2$ are right triangles. Therefore, we have the following equations.

$$\frac{d_2}{d_1} = \frac{x_1 - x_s}{x_1 - x_2} \tag{3.6}$$

$$\frac{d_2}{d_1} = \frac{y_1 - y_s}{y_1 - y_2} \tag{3.7}$$

For $DRPV_s(r_1, r_2)$, as we know that both r_1 and r_2 are similarities between document D_s and R_1, and document D_s and R_2 respectively. In the visual space, the larger a similarity value between a document and a reference point, the smaller the distance between them should be. This suggests that the relationship between similarity r and its corresponding distance d in the visual space should be reverse. The reverse relationships are described in on Eqs. (3.6) and (3.8).

$$\frac{d_2}{d_1} = \frac{d_2}{d_2 + d_3} = \frac{(r_1 + r_2) - r_1}{r_1 + r_2} = \frac{r_2}{r_1 + r_2} \tag{3.8}$$

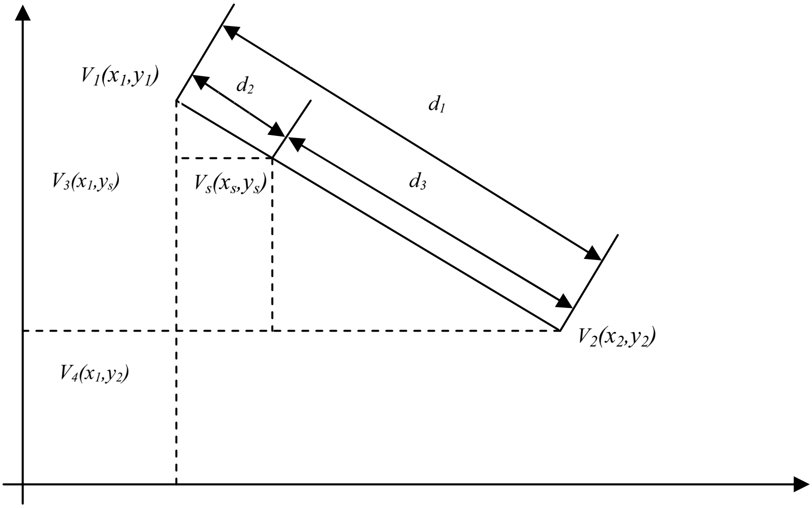

Fig. 3.2. Display of a projected document and two related reference points

$$\frac{d_3}{d_1} = \frac{d_3}{d_2 + d_3} = \frac{(r_1 + r_2) - r_2}{r_1 + r_2} = \frac{r_1}{r_1 + r_2} \tag{3.9}$$

Based upon Eqs. (3.6) and (3.8), we have:

$$x_s = \left(\left(\frac{r_2}{r_1 + r_2} \right) \times (x_2 - x_1) + x_1 \right) \tag{3.10}$$

$$x_s = \frac{r_1 x_1 + r_2 x_2}{r_1 + r_2} \tag{3.11}$$

Similarly, based on Eqs. (3.7) and (3.8), we have:

$$y_s = \frac{r_1 y_1 + r_2 y_2}{r_1 + r_2} \tag{3.12}$$

When r_1 (r_2) is equal to 0, it implies that the document D_s is irrelevant to the reference point R_1 (R_2). Both Eqs. (3.11) and (3.12) tell us that position of the document D_s would be the same as that of R_2 (R_1). In other words, it is overlapped with reference point R_2 (R_1) in the visual space. And the smaller r_1 (r_2) is, the farther away the document D_s is from R_1 (R_2).

In the above equations, ($r_2/(r_1+r_2)$) or ($r_1/(r_1+r_2)$) is called the partition coefficient of document D_s with respect to the reference points R_1 and R_2. It is a ratio of similarity between a document and a reference point to the sum of similarities between the document and the two involved reference points. It underlies where the document is located on the segment defined by the two reference points in the visual space.

Now let us discuss the scenario where multiple reference points are involved. Basically, the *VIBE* algorithm requires three or more reference points to support document projection (Olsen et al., 1993 b). According the original algorithm description (Olsen et al., 1993 a), if a document is related or similar to multiple reference points and similarities between this document and related reference points are available, then first two related reference points from the multiple reference point set are selected and its position on the segment defined by the two reference points is calculated by Eqs. (3.11) and (3.12). The new position of the document on the segment serves as an intermediate reference point for further consideration. The relevance of the newly generated intermediate reference point and the document is the sum of similarities between the document and the two newly merged reference points. The next round position of the document (or next intermediate reference point) is determined by the intermediate reference point and another unprocessed reference point from the multiple reference point set. This process stops when all related reference points from the multiple reference point set are considered and processed. The final intermediate reference point is the ultimate position of the projected document with respect to the multiple reference point set in the visual space.

It can be proved that the sequence of taking reference points from the multiple reference point set into consideration does not affect the final result, which is the ultimate location of the document in the visual space. For simplicity, use three reference points R_1, R_2, and R_3. The positions of these three reference points are $P(R_1)= V_1(x_1, y_1)$, $P(R_2)= V_2(x_2, y_2)$ and $P(R_3)= V_3(x_3, y_3)$, respectively. Similarities of these three reference points to the document D_s are in $DRPV_s$ (r_1, r_2, r_3).

If the reference points R_1 and R_2 are considered first, then position of newly generated intermediate reference point ($R_4(x_4, y_4)$) sees Eqs. (3.13) and (3.14). The similarity value of the intermediate reference point is equal to ($r_1 + r_2$).

$$x_4 = \frac{r_1 x_1 + r_2 x_2}{r_1 + r_2} \tag{3.13}$$

$$y_4 = \frac{r_1 y_1 + r_2 y_2}{r_1 + r_2} \tag{3.14}$$

Take the intermediate reference points R_4 and R_3 to form another intermediate reference point $R_5(x_5, y_5)$.

$$x_5 = \frac{r_3 x_3 + \left(\dfrac{r_1 x_1 + r_2 x_2}{r_1 + r_2} \right) \times (r_1 + r_2)}{(r_1 + r_2) + r_3} \tag{3.15}$$

$$x_5 = \frac{r_3x_3 + r_1x_1 + r_2x_2}{r_1 + r_2 + r_3} \tag{3.16}$$

Similarly the *Y*-axis value of the reference point R_5 can be calculated and the result is displayed in Eq. (3.17).

$$y_5 = \frac{r_3y_3 + r_1y_1 + r_2y_2}{r_1 + r_2 + r_3} \tag{3.17}$$

Using the same approach, merge R_3 and R_2 first, then merge R_1; we would get the same results as Eqs. (3.16) and (3.17).

In general, a position $V_s(x_s, y_s)$ of a document D_s in a two-dimensional visual space can be computed from Eqs. (3.18) and (3.19) in terms of multiple reference points. Position of a reference point R_j (j=1, …, q) denotes $V_j(x_j, y_j)$, and similarities between the document D_s and the reference point R_j (j=1, …, q) are $DRPV_s$ $(r_1,\ldots,r_q)$. In fact, both Eqs. (3.18) and (3.19) show that the sequence of reference point considerations has nothing to do with the ultimate position of a projected document in the visual space.

$$x_s = \frac{\sum_{i=1}^{q} r_i x_i}{\sum_{i=1}^{q} r_i} \tag{3.18}$$

$$y_s = \frac{\sum_{i=1}^{q} r_i y_i}{\sum_{i=1}^{q} r_i} \tag{3.19}$$

Assume that there are three reference points R_1, R_2 and R_3. Their initial positions in the visual space are $V_1(5, 5)$, $V_2(30, 5)$, and $V_3(15, 25)$, respectively. The position of a document D_s is $V_s(x_s, y_s)$. Assume the similarities between this document and the three reference points are shown in Eq. (3.20).

$$\mathrm{DRPV_s}(r_1, r_2, r_3) = \mathrm{DRPV_s}(0.2, 0.4, 0.6) \tag{3.20}$$

And therefore the coordinates of the document D_s are:

$$x_s = \frac{r_1x_1 + r_2x_2 + r_3x_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 30 + 0.6 \times 15}{0.2 + 0.4 + 0.6} = 18.3 \tag{3.21}$$

$$y_s = \frac{r_1y_1 + r_2y_2 + r_3y_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 5 + 0.6 \times 25}{0.2 + 0.4 + 0.6} = 15 \tag{3.22}$$

As the reference point R_3 moves from its original position to a new position $V'_3(30, 25)$, the document D_s is pulled to a new position D'_s by R'_3.

And the coordinates of the document D'_s are:

$$x_{s1} = \frac{r_1 x_1 + r_2 x_2 + r_3 x'_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 30 + 0.6 \times 30}{0.2 + 0.4 + 0.6} = 25.8 \tag{3.23}$$

$$y_{s1} = \frac{r_1 y_1 + r_2 y_2 + r_3 y'_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 5 + 0.6 \times 25}{0.2 + 0.4 + 0.6} = 15 \tag{3.24}$$

Fig. 3.3 gives both the position of the document before reference point R_3 moves and the position of the document after R_3 moves to R'_3.

The following algorithm is adapted from the original *VIBE* algorithm (Olsen et al., 1993 b).

```
Begin
    Define multiple reference points $R_j$ ($j=1, ..., p$);
    Get RP positions in the visual space ($V_j(x_j, y_j), j=1, ..., p$);
    Compute the vector (DRPV($r_1,...,r_p$)) for each document based on a selected similarity measure;
    Discard documents that are irrelevant to the reference points;
    While documents with unprocessed DRPVs are still available
        Get a document D and its DRPV ($r_1,...,r_p$);
        If only one $r_i$ in DRPV ($r_1,...,r_p$)($1 \le i \le p$) $\neq 0$
            Then
            Assign the corresponding reference point as the final position of D in the visual space;
            Else
            Select $R_i$ whose $r_i$ is not equal to 0 from DRPV ($r_1,...,r_p$) ($1 \le i \le p$);
            While an unprocessed $R_j$ whose $r_j$ is not equal to 0 in DRPV ($r_1,...,r_p$) is available
                Merge $R_i$ and $R_j$ to form a new intermediate RP, and assign it to $R_i$;
            EndWhile;
            Assign the position of the last intermediate RP $R_i$ as the final position of the document D;
            ElseEnd;
        EndIf;
    EndWhile;
End.
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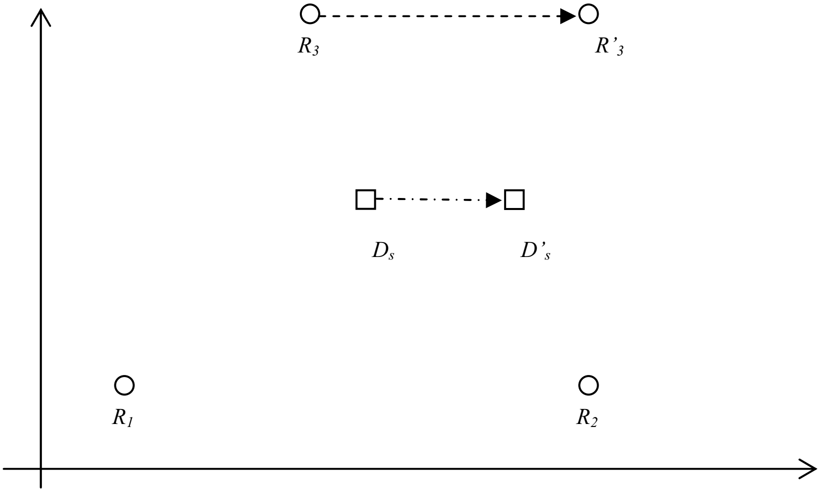

Fig. 3.3. Impact of a moving reference point on a document

L2 to *L7* initialize variables before processing. According to this algorithm, documents which are not relevant to any of the defined reference points are discarded and not considered in the visual space (See *L6* to *L7*). If a document is only relevant to a reference point, its position will overlap with that of the reference point in the visual space (See *L10* to *L13*). If a document is relevant to multiple reference points, a series of intermediate reference points must be generated. A related reference point in conjunction with an intermediate reference point forms a new intermediate reference point. The reference point merging process continues until all relevant reference points are processed. The position of the final intermediate reference point is the position of the projected document in the visual space (See *L14* to *L24*).

Notice that if users change the position of any reference point in the visual space, or add a new reference point to the visual space, or remove a reference point from it, or revise the weights of involved reference points, then it would result in a reconfiguration of projected documents in the visual space. Each of these actions would trigger the algorithm.

Because the algorithm requires calculating a series of intermediate reference points, it is a multiple-step generation algorithm. However, Eqs. (3.18) and (3.19) suggest that calculation of a final document position with respect to multiple reference points in the visual space can be done in a single step. The one-step algorithm would make the algorithm more efficient.

3.3.2 Discussions about the model

Suppose that there are N reference points ($N \geq 3$) in a visual space. These N reference points can form a convex polygon in which all reference points are covered.

The number of polygon vertices is *M,* where *M* is always smaller than or equal to *N*. If some of the reference points are located inside the polygon, then *M* is smaller than *N*. Otherwise, *M* is equal to *N*. The polygon should be constructed in such a way that an angle formed by any vertex of the polygon and its two adjacent vertices should cover the entire polygon.

This polygon will define a valid display area for projected documents if the condition is met for all vertices of the polygon. In other words, all documents should be projected within this special convex polygon generated by the multiple reference points. For a group of reference points, they can be classified into two categories: one includes these reference points which are inside the polygon and the other includes these which form the polygon as its vertices. The above analysis shows that the sequence of taking related reference points into consideration to project a document does not affect the ultimate location of the document in the visual space. We first take consideration of these reference points which are vertices of the polygon. This starts with a vertex point and one of its adjacent reference points in the polygon to calculate the first intermediate reference point with respect to a document. It is apparent that the first intermediate reference point should be located on the side connected by the two selected reference points according to previous analysis. This suggests that the document is not projected outside the polygon. Use the first intermediate reference point and another adjacent reference point (vertex) to generate the second intermediate reference point. Since the second intermediate reference point is situated on the segment between the first intermediate reference point and new selected adjacent vertex, it should be within a triangle area yielded by three reference points already selected. Thus, the document is definitely within the polygon. Then consider the second intermediate reference point and the third adjacent vertex to produce the third intermediate reference point. Because the second intermediate reference point is within the polygon, the segment formed by the second intermediate reference point and the third adjacent vertex should be inside the polygon. This process continues until all vertices of the polygon are considered. This analysis shows that after the reference points in the second category are processed, the last intermediate reference point is still within the polygon. Notice that all reference points in the first category are within the polygon. This implies that the segments formed by any of these reference points and an internal intermediate reference point should also be situated within the polygon. This concludes that all new corresponding intermediate references are within the polygon. According to previous discussion, we know that the location of the final intermediate reference point is the final position of the projected document in the visual space which, of course, is within the polygon. It is important that all reference points should be included in the polygon and the polygon should be a convex polygon which meets the condition we discussed before. These two conditions will ensure that all projected documents are mapped within the polygon.

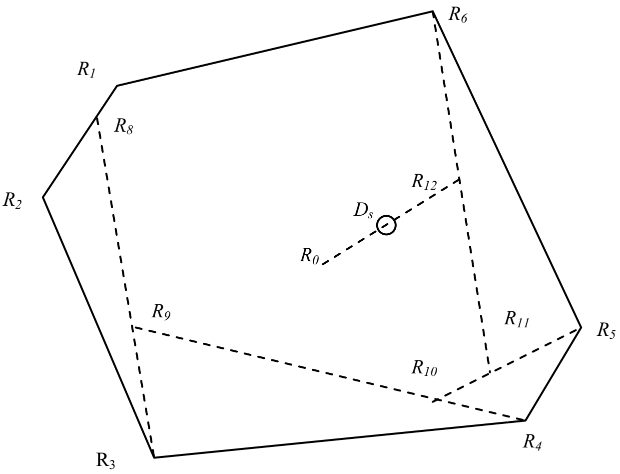

Fig. 3.4. Display of a polygon of a valid display area

For example, there are seven reference points and they are put in the visual space (R_j (j=0, …, 6)). D_s is a document. $\{R_0\}$ is within the first category, and $\{R_1, R_2, R_3, R_4, R_5, R_6\}$ fall in the second category which forms a polygon (See Fig. 3.4). Segments with broken lines are produced by intermediate reference points and adjacent vertices or reference points in the second category. Solid lines are the sides of the polygon. R_8 is the first intermediate reference point produced from both reference points R_1 and R_2. R_9 is the second intermediate reference point produced from the first intermediate reference point R_8 and an adjacent vertex R_3,…. R_{12} is the intermediate reference point produced from the intermediate reference point R_{11} and an adjacent vertex R_6. The final intermediate reference point or projected document D_s is determined by an internal reference point R_0 and the intermediate reference point R_{12} (See the small circle in Fig. 3.4).

After all reference points are positioned in the visual space, some or all of them can actually define a polygon onto which all documents are mapped. Documents within the polygon may be affected by one reference point or many reference points. If a document is only relevant to a certain reference point, it is easy to identify it in the visual space because it is overlapped with the related reference point. However, when a document is relevant to multiple reference points, it is difficult to specify which reference points are related to the document because its position in the visual space is a combination of the impacts of all relevant reference points. To solve this problem, we introduce a concept, reference point monopoly triangle (Korfhage, 1991). A reference point monopoly triangle is a special display area in the visual space that is defined by a vertex of a polygon (a reference

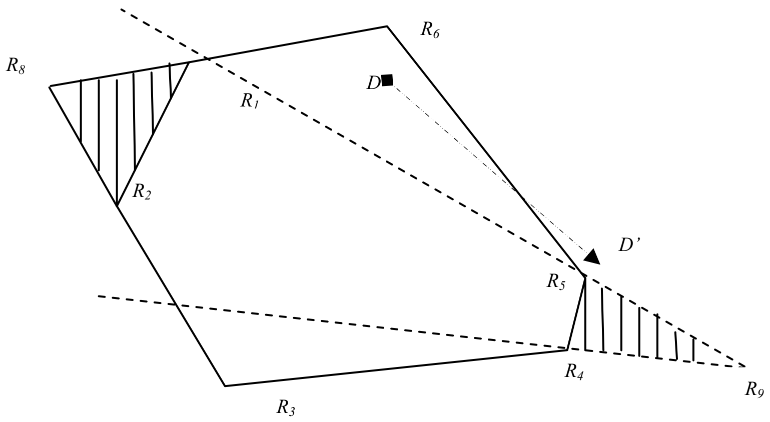

Fig. 3.5. Explanation of reference point monopoly triangle

point) and its two adjacent reference points on the polygon. In the special display triangle area, all projected documents are related to the reference point. As we know, that a group of reference points can define a convex display polygon and all documents are mapped within the polygon. When a new reference point is added to outside the polygon in the visual space, this new reference point will form a new vertex to the polygon. This newly added reference point and its two neighboring reference points constitute a triangle area. After the reference point is added outside the polygon, related documents are reconfigured due to its attraction. A related document to the newly added reference point within the old polygon would move towards the new reference point along the line formed by the newly added reference point and the document. The extent to which the document moves towards the reference point depends upon its similarity to the reference point. It suggests that the related documents may end up within the triangle area if they are strongly attracted by the reference point. In other words, all documents within the triangle are related to the reference point. But documents which are not within the triangle area may also be related to the reference point. If documents are not related to the reference point or they are not very similar to the reference point, they would stay in their original positions or not be "pulled" to the triangle area by it. Users can take advantage of the triangle characteristics to manipulate a reference point, to create a reference point monopoly triangle, and to identify related documents to a special reference point. When creating a reference point monopoly triangle area, make sure that the angle formed by the special reference point and two neighboring vertices of the polygon must be large enough to cover the entire polygon. If this condition is not satisfied, related documents may be "pulled" out of the monopoly triangle area. When a document is located outside the uncovered area and is related to the reference point, it may be pulled out of the monopoly triangle area.

For instance, there are six reference points $\{R_1, R_2, R_3, R_4, R_5, R_6\}$ and they define a polygon (See Fig. 3.5). R_8 and R_9 are two newly added reference points outside the original polygon. They generate two reference point monopoly triangle areas (See the two shaded parts in Fig. 3.5). The triangle formed by reference point R_8 is correctly defined because the angle covers entire polygon area. However, the triangle formed by reference point R_9 is not properly defined because some areas of the polygon are not within the angle area (See the angle formed by two broken lines in Fig. 3.5). Assume that a document D is situated within the polygon but outside the angle area and is related to the reference point R_9. Pulled by R_9, D moves toward R_9 after R_9 is added to the visual space. Obviously, it may end up outside the reference point monopoly triangle area (See the icon D' in Fig. 3.5).

It is worthy to point out that documents situated within a reference point monopoly triangle are definitely related to the reference point and the documents located outside the monopoly triangle area may also be related to the reference point. They are not pulled into the triangle area because the impact of the reference point on these documents is not strong enough compared to other related reference points in the visual space. This occurs often in the visual space. A new question is raised: how can we identify all of the related documents to a specific reference point in the visual space? We must figure out a way to squeeze the polygon and force these documents out of the polygon. A simple way to do this is to put all of the reference points except a selected reference point along a line and position the selected reference point above the line; then all documents related to the selected reference point would be singled out (See Fig. 3.6). In Fig. 3.6, the polygon defined by reference points ($R_1, ..., R_i, ..., R_n$) becomes the reference point monopoly triangle. Therefore, all documents related to a special reference point $R_{selected}$ would be displayed within the triangle.

Generally speaking, it is true that the more similar to a reference point a document is, the closer it is to the reference point in the visual space. However, when a document is compared to another document with respect to the same reference point, we cannot simply use the distances from the two documents to the reference point to judge which one is more relevant to the reference point. That is, if a document (D_1) is closer to a reference point than another document (D_2) in the visual space, it does not conclude that document (D_1) is more relevant to the reference point than document (D_2). This may confuse people a little bit. The reason for this phenomenon is that the similarity between a document and a reference point is not the only factor which affects its position in the visual space. Position of a document in the visual space is a collective effort of multiple factors. These factors range from similarity between the document and the special reference point, and the position of the reference point, to similarities between the document and other related reference points, and positions of other related reference points. Each of these factors plays a role in determining the document's position.

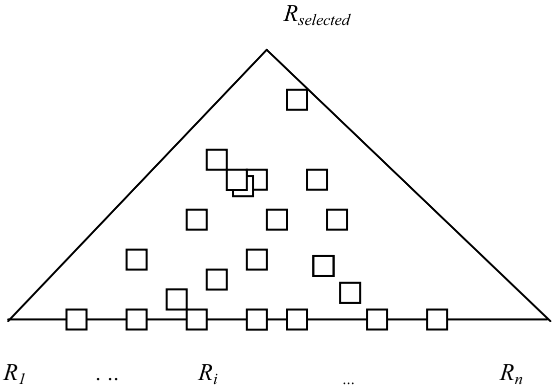

Fig 3.6. Display of all related documents to a special reference point

Therefore, the position of a document is dynamic and relative against the positions of related reference points. It is difficult to judge the similarities of two documents to a special reference point depending upon their distances to the special reference point in the visual space.

On the other hand, comparing two reference points with respect to the same document is relatively easy. Users can neutralize all reference points except the two compared reference points by putting them on a line similar to the one in Fig. 3.6. The user may then put one of the compared reference points above the line and the other below the line. Distances between the two reference points to the line are the roughly same. All related documents would then be spread out around the line to their related reference points. In this way, the impacts of the two reference points on the related documents are shown in the contexts of the other reference points. If a document has an identical similarity to the two reference points, the attractions from the two opposite directions cancel each other because the reference points are located in opposite positions against the line in the visual space. This would cause the document to stay on the line like the other unrelated documents. This problem can be solved by moving any of the two reference points to the line. As a result, the documents would leave the line and move towards the other reference point.

The *VIBE* algorithm was used for a full-text environment (Korfhage and Olsen, 1994). In this case, the visual environment was limited to a single full-text based document instead of a group of documents. Displayed objects were replaced by meaningful semantic logic units within a document. The semantic logic units may be defined as sentences, paragraphs, sections, or chapters within that document. Segmentation of a full text should depend upon the nature and length of the full text. A semantic unit is indexed by keywords extracted from that semantic unit. Keyword weight can be assigned by simply using its raw occurrence in that semantic unit. The weight of a term can also be calculated by a more complicated approach. For instance, we can apply *TF*×*IDF* (Term Frequency × Inverse Document Frequency) to the measuring weight of a term.

$$w_i = f_i \times Log\left(\frac{N}{d_i}\right) \tag{3.25}$$

In this equation, w_i is the weight of the term i, f_i is the frequency of the term i in a semantic unit such as a paragraph, N is the number of all semantic units segmented from a full-text, and d_i is the number of semantic units that contain the term i.

Each of the semantic units is assigned a sequential number based upon its appearance sequence in a full-text. When all semantic units are visualized in the visual space, two adjacent semantic units are connected by a sequence link. Therefore, another dimension is added to the visual space to facilitate analyzing the relationships of semantic units in a full-text analysis in addition to the existing visualization features provided in *VIBE*.

The original two-dimension-based *VIBE* model was also adapted to support three-dimension-based visual environments for greater interaction in two distinctive approaches. The first approach added the third dimension which was the overall significance of a document to the two dimensional *VIBE* space (Benford et al., 1995; Benford et al., 1997). This new dimension was used to demonstrate the relevance between a document and all reference points. The overall significance between a document D_s and all involved reference points R_j (j=1, …, q) was defined as the sum of its similarities to all reference points and used as the Y-axis value of the document in the visual space:

$$z_s = \sum_{i=1}^{q} r_i \tag{3.26}$$

In this equation, the similarities between document D_s and reference point R_j (j=1, …, q) are described in $DRPV_s\ (r_1,\dots,r_q)$.

Definitions of the X-axis and Y-axis for the document D_s are still the same as those in both Eqs. (3.18) and (3.19).

Similarly, the Z-axis of a reference point in the three-dimensional space is defined as the sum of similarities between this reference point R_j to all related documents:

$$z_{R_j} = \sum_{i=1}^{n} r_{iR_j} \tag{3.27}$$

In this equation, r_{iRj} is similarity between the document i and reference point R_j, and n is the number of the involved documents in the visual space.

The X-axis value and Y-axis value for a reference point R_j are dynamic and determined by users while the Z-axis is obviously not.

It is evident that the newly added *Z*-axis value for a certain document or reference point is an absolute value and is no longer dynamic and relative like the *X*-axis and *Y*-axis values. As long as the involved reference points and documents are determined, the *Z*-axis values for these documents and reference points are unchanged. The significance of adding a fixed *Z*-axis value for a document or reference point rests upon that it can alleviate the ambiguity phenomenon of projected documents to some degree in the visual space, and therefore it more accurately reveals relationships among the projected documents. For instance, given two reference points R_1 and R_2, and two documents D_1 and D_2, similarity between D_1 and R_1, and similarity between D_1 and R_2 are 0.4 and 0.8, respectively. Similarity between D_2 and R_1, and similarity between D_2 and R_2 are 0.1 and 0.2, respectively. Without the third overall significance dimension, these two documents are projected onto the same location between the two reference points because the position of a document is determined by relative attraction that is the ratio of the document to all reference points multiplying their location factors to the overall similarities of the document to all reference points. Users simply cannot tell to what extent each document is related to the reference points if the ambiguity phenomenon occurs. With the third overall significance dimension, the two overlapped documents are separated in the third axis because they have different overall significance values, which can distinguish them in the visual space.

The second three-dimensional model was *LyberWorld* (Hemmje et al., 1994). Unlike the first model, the *Z*-axis of a document D_s in this model (See Eq. (3.28)) is similar to both the *X*-axis and the *Y*-axis (See Eqs. (3.18) and (3.19)).

$$z_s = \frac{\sum_{i=1}^{q} r_i z_i}{\sum_{i=1}^{q} r_i} \tag{3.28}$$

All documents are located on the so-called relevance sphere boundary. The indexing terms of a document are always situated within the sphere. In this case, the indexing terms replace the referenced points in the visual space. Users may manipulate the indexing terms like reference points, rotate the relevance sphere, and zoom in/out of the space at will.

3.4 Model for automatic reference point rotation

In this section, a new model for multiple reference points is introduced (Zhang and Nguyen, 2005). The model for automatic reference point rotation is a similarity ratio based model to some extent. The uniqueness of this model is that it adds a new feature reference point automatic rotation to the two-dimensional visual space. It enables users to observe the relevance between a rotating reference point and its related documents in the visual space.

3.4.1 Definition of the visual space

The visual space is two dimensional and is built on a polar coordinate system which is a plane with a point O (the pole) and a ray from O (the polar axis). Each point (P) in the plane is assigned polar coordinates: the directed distance from O to P and the directed angle whose initial side is on the polar axis and whose terminal side is on the line PO. Notice that the polar coordinate system can be converted to the Cartesian coordinate system easily. The valid display area is a sphere in the space. All reference points are positioned on the sphere. Reference points R_j (j=1, …, q) are evenly distributed on the sphere as their default positions. Once a reference point is activated by users, it automatically rotates counterclockwise around the sphere. In other words, the boundary of the sphere is the orbit of a moving reference point. The radius of the sphere is MR. The center of the sphere (O) is the focus point. The focus point can be defined as different meanings in different contexts. This issue will be discussed later. All of the documents are basically scattered within the visual space. The position of a document D_i denotes DP_i (l_i, α_i). Here l_i and α_i are the projection distance/ directed distance and projection angle of a document/ directed angle, respectively. These two parameters determine its position in the visual space.

$$l_i = MR \times (1 - S_{oi}) \tag{3.29}$$

$$S_{oi} = c^{\left|(\sum_{t=1}^{n}(a_t)^{1/2})^2 - (\sum_{t=1}^{n}(x_{it})^{1/2})^2\right|} \times \frac{\sum_{t=1}^{n} a_t \times x_{ti}}{(\sum_{t=1}^{n} a_t^2)^{1/2} \times (\sum_{t=1}^{n} x_{ti}^2)^{1/2}} \tag{3.30}$$

In this equation, $O(a_1, a_2, \dots, a_n)$ is the focus point vector, $D_i(x_{i1}, x_{i2}, \dots, x_{in})$ is the document vector, both a_j and x_{ij} are keyword weights($0 \le j \le n$), and n is dimensionality of the vector space. It is clear that l_i basically reflects the relationship between the document D_i and the focus point O. Eq. (3.30) calculates the similarity between D_i and the focus point O and c is a control constant. The characteristics of this similarity measure were discussed in Chap. 2. In Eq. (3.29) factor(1-S_{oi}) is used to convert the similarity to reflect such a relation that the more relevant the document D_i is to the focus point O, the closer they are in the visual space, and vice versa. Notice that the valid value of S_{oi} is between 0 and 1. MR defines the radius of the sphere. By changing MR, users may zoom in/out of the visual sphere at will.

The projection angle of the document D_i is defined in Eq. (3.31). β_j is the angle which is formed by the reference point R_j and the polar axis against the origin of the visual space or the focus point. S_{jk} is the similarity between reference point R_j and document D_k. If a document D_i is irrelevant to any of the reference points ($\sum_{k=1}^{q} S_{ki} = 0$), then the angle α_i is defined as zero to avoid a meaningless α_i.

The equation shows whether a reference point is irrelevant to any reference points, it would be situated on the polar axis and the distance to the origin is determined by its similarity to the focus point. The equation demonstrates that the projection angle of a document is dominated by a similarity ratio rather than by an absolute sum of the angles multiplied by similarities. This characteristic underlies the reference point rotation feature. The display sphere, reference point rotation direction, the projection distance, and the projection angle are shown in Fig. 3.7. In the figure R_1, R_2, R_3, R_4, and R_5 are five reference points.

$$\alpha_i = \begin{cases} \dfrac{\sum_{j=1}^{q}(\beta_j \times S_{ji})}{\sum_{j=1}^{q} S_{ji}}, & \sum_{k=1}^{q} S_{ki} \neq 0 \\ 0, \quad \sum_{j=1}^{q} S_{ji} = 0 & \end{cases} \tag{3.31}$$

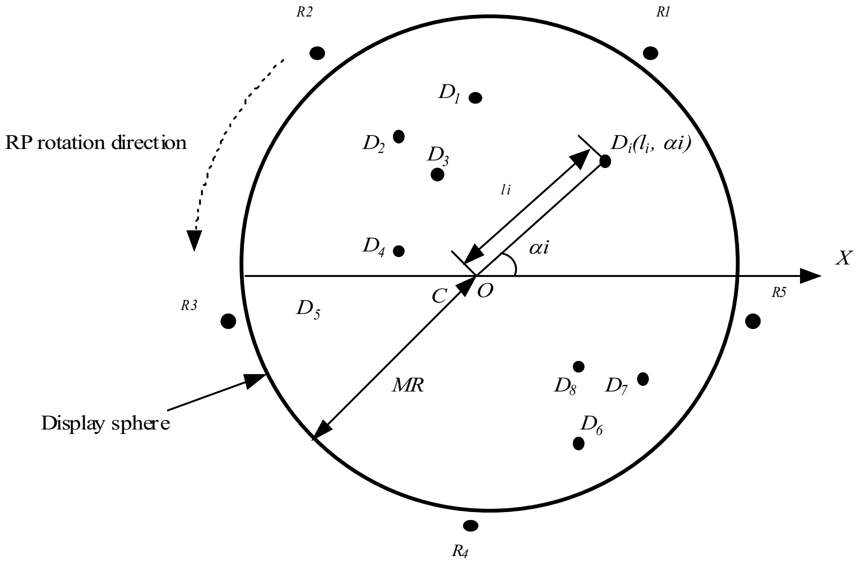

Fig. 3.7. Display of the *WebStar* visual space
Source: Zhang and Nguyen (2005)

3.4.2 Rotation of a reference point

Eq. (3.31) describes the projection angle of a document against static reference points. However, as a selected reference point is activated to rotate around the sphere, related documents no longer stay static. Attracted by the moving reference point, the related documents would also rotate around the orbit whose radius is defined by Eq. (3.29). The speed of a related document primarily depends upon its relevance to the moving reference point and it can be calculated in Eq. (3.32). In this equation, given R_r is the activated reference point, β_r is the initial angle in the visual space, θ is the speed of the activated reference point R_r, t is a time variable, S_{ri} is the similarity between R_r and D_i, and α'_i is the speed of the related document D_i.

$$\alpha_i' = \frac{\sum_{j=1, j\neq r}^{q} (\beta_j \times S_{ji}) + (\theta \times t + \beta_r) \times S_{ri}}{\sum_{j=1}^{q} S_{ji}} \tag{3.32}$$

From Eq. (3.32) we have the speed of any related document.

$$\frac{d\alpha'_i}{dt} = \frac{\theta \times S_{ri}}{\sum_{j=1}^{q} S_{ji}} \tag{3.33}$$

Eq. (3.33) suggests that if a document is not relevant to the rotating reference point it would stay unchanged in the visual space. The more relevant the document is to the rotating reference point, the faster the document rotates around the center, and vice versa. The maximum speed of the document is equal to that of the rotating reference point. The similarities between the related document and other reference points also play a role in the document speed due to the divisor $\sum_{k=1}^{q} S_{ki}$ in Eq. (3.33).

Fig. 3.8 gives a series of the *WebStar* snapshots of a rotating reference point. The four reference points are sport, research, international, and library. Starting from the upper left corner figure clockwise, the first figure is the initial status, the second figure is the status when the reference point international rotates to about 85 degrees, the third figure is the status when the reference point international rotates to 0 degrees, and the fourth figure is the status when the reference point international is activated and rotates to 270 degrees. As the reference point international orbits, all of the related documents rotate accordingly along the same direction but at different speeds.

Differences between this model and other similarity ratio based visualization models are that this model requires a focus point located at the origin of the visual space, it supports single reference point projection, only one of the location parameters (projection angle) is similarity-ratio-based while the other (projection distance) is not, and the reference point rotation feature utilizes the movement of

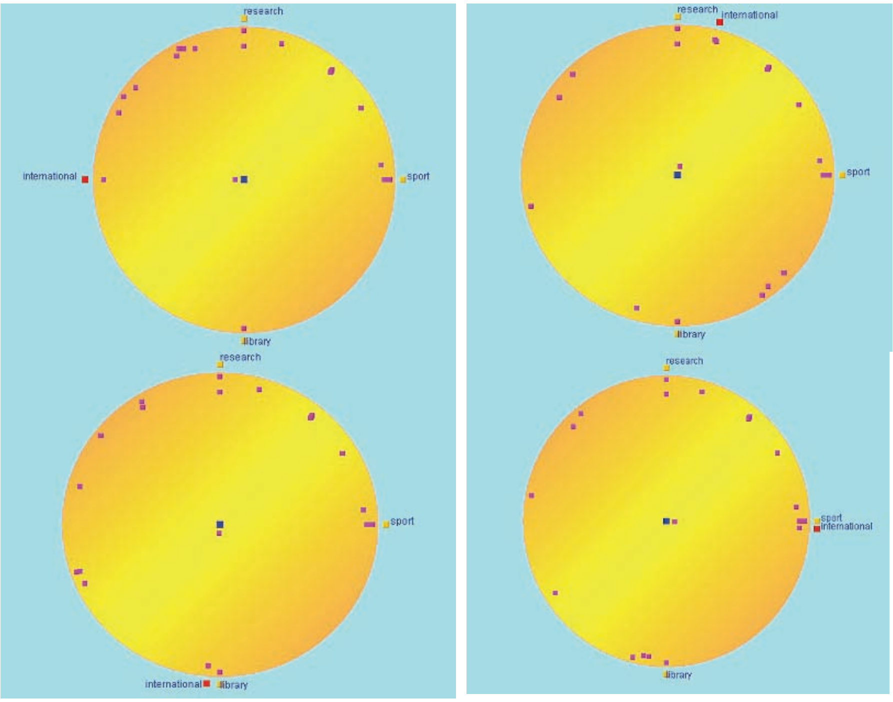

Fig. 3.8. A series of the *WebStar* snapshots of a rotating reference point

objects as an indicator of relevance between a rotating reference point and related documents.

3.5 Implication of information retrieval

The visualization models for multiple reference points have a very natural and direct relationship to information retrieval. The model for fixed multiple reference points was designed to visualize results from a Boolean query. The original models for movable multiple reference points were aimed at visualizing the results of a query from a vector-based space. In fact, a query can be regarded as a special reference point in a broader sense. Documents related to these reference points are displayed in the visual space. Even within the *VIBE* visual space, it can integrate a search mechanism where users submit their queries, and matched document icons in the visual space are highlighted so that they can easily be distinguished from other un-retrieved document icons. In the *WebStar* visual space, users may employ the retrieval contour to retrieve documents in the visual space. The retrieval contour is a circle which shares the same center with the display sphere. The radius of

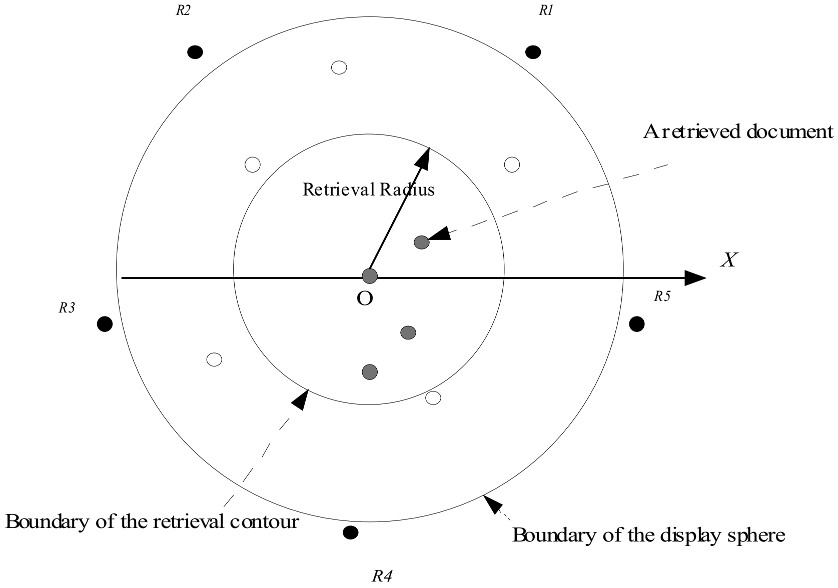

Fig. 3.9. Display of the retrieval contour
Source: Zhang and Nguyen (2005)

the retrieval contour can be manipulated by users to control the size of a retrieved document set (See Fig. 3.9). As the radius increases, more documents are retrieved, and vice versa. Notice that when the number of retrieved documents increases, the similarities of the newly added documents and the focus point decrease because they are getting farther away from it.

The focus point in the model for automatic reference point rotation is extremely important. The visual contexts change if the focus point changes. If it is assigned as a website, then the projected documents are outgoing Web pages. If it is assigned as a search query submitted to an information retrieval system, then the mapped documents are returned results. If it is assigned as a regular paper, then the displayed documents are its citations. If it is assigned as a root of a subject hierarchy, then the projected objects are its children nodes. The focus point enables users to narrow down their interests. As long as the focus point is clearly defined, its relationships to the projected objects are illustrated by observing the distances from the origin to the displayed objects in the visual space. Of course, like other multiple reference point based visualization models it demonstrates the relationships between reference points and documents in the visual space. That is, in the static status a document is located near to reference points which are relevant to it. In addition, the model reveals which documents are related to a rotating reference point and the extent to which these documents are similar to the moving reference point.

A similarity-ratio-based visualization model may be used for term discriminative analysis (Dubin, 1995). In the *VIBE* space, each of the tested keywords is assigned to a reference point, and all of the reference points are scattered onto the visual space to form a circle. Terms with poor discriminative capacity tend to stay around the center of the circle because they are not affected by any reference points. Terms with good discriminative capacity tend to be spread out in the visual space because a good discriminative term will attract related documents. Based upon a distribution of documents, people can make a correct judgment about whether a term is a discriminative term or not.

3.6 Summary

Reference point is an important concept in information retrieval. Multiple reference points were introduced not only to visualize documents but also to solve the inherent problems of a traditional information retrieval system.

In this chapter, visualization models based on multiple reference points were classified into three categories: the models for fixed multiple reference points, the models for movable multiple reference points, and the model for automatic reference point rotation. Each of them has its own unique qualities. The models for fixed multiple reference points can be used for both a vector-based information retrieval system and Boolean based information retrieval system. They can handle a complex Boolean query. But the algorithm is not based upon the similarity ratio method, even though it can be applied to a vector-based information retrieval system. Most models for movable multiple reference points are similarity ratio based, except for the *VR-VIBE* model, whose *Z*-axis of the visual space is sum of similarities of all related reference points for a document unlike the *X*-axis and *Y*-axis. The visualization model for automatic reference point rotation is based upon the similarity ratio in part because only the projection angle of a document, one of the two projection parameters, is calculated based upon similarity ratio of related reference points while projection distance is not. These two parameters of a document dominate its position in the visual space. The power of the similarity ratio based algorithms relies on the manipulative flexibility for reference points in the visual space. The position of any reference points can be controlled and manipulated by users at will. It is the manipulative flexibility that enables users to compare and analyze the impact of two reference points on documents, and identify good/poor discriminative terms. Both the models for fixed multiple reference points and the models for movable multiple reference points require at least three reference points to project documents in their visual spaces, while the model for automatic reference point rotation requires at least one reference point in conjunction with the focus point to construct its visual space. Visualization models for multiple reference points can be two-dimensional or three-dimensional. They can be applied to either Boolean based information systems or vector based information systems. They can be used to visualize Internet hyperlinks, search results from an information retrieval system, a full-text, and term discriminative analysis.

Chapter 3 Visualization Models for Multiple Reference Points

Visualization models for multiple reference points have many unique characteristics. They can effectively handle complex information need of a user by using multiple reference points and achieve excellent operation flexibility for the multiple reference points.

A traditional information retrieval system responds to a user search query with a linear results list. Retrieved documents may be ranked based upon their similarities to the query if the system offers a relevance ranking mechanism. However, it does not provide users with a flexible and powerful browsing environment with which users can make a relevance judgment about the retrieved documents. That is because the linear list does not provide users with relevance information among the retrieved documents and it only offers relevance information between the query and the retrieved documents. Originally, the visualization algorithms for multiple reference points were designed to visualize the results of a search query to overcome the weakness of the linear structure of a search results list by projecting the retrieved documents onto a low dimensional visual space. This visual space is defined by the multiple reference points. Basically, a reference point represents a user's information need. We will discuss this concept and its implications in depth in the following section. In this sense, the visualization models for multiple reference points were directly driven or motivated by information retrieval.

The visualization algorithms can be classified into two groups based upon what kind of an information retrieval system it works on: the algorithms that work on a Boolean information retrieval system and the algorithms that work on a vector information retrieval system. The algorithms for multiple reference points can also be categorized into three categories based upon the status of reference points in the visual spaces: the algorithms for fixed reference points in the visual spaces; the algorithms for manual movable reference points; and the algorithm for automatic rotating reference points. The status changes of reference points can not only facilitate users' manipulation of the reference points, but also assist users in clarifying the notorious ambiguity of overlapped documents in the visual space and reveal new visual semantic relationships among displayed documents.

Among the three categories, a large body of research and applications has gravitated to the second category because the algorithms for manual movable reference points are flexible, simple, and also powerful. The representative and pioneering paradigm in this category was *VIBE* (Visual Information Browsing

Environment) (Olsen and Korfhage, 1994). Other related algorithms and applications in this category were derived directly from the *VIBE* algorithm.

The underlying principle of the *VIBE* algorithm is that the position of a displayed document between two reference points indicates its relevance to them in its visual space. The position of a reference point can be any location in the visual space. Due to the algorithm simplicity and flexibility, it has been widely used and adapted to many application domains. For instance, Web pages as objects were visualized in the visual environment *WebVIBE* (Morse and Lewis, 2002). The model was integrated in a multilingual and multimodal system to visualize video objects (Lyu et al., 2002). A visualization environment and a geographic environment were combined to generate a new environment so that users could search and browse geographic information in both *Geo-VIBE* (Cai, 2002) and *Visual Digest* (Christel, 1999; Christel and Huang, 2001). *Radial* (Carey et al., 2003) provided a two dimensional circular space where terms or reference points were limited to positions on its boundary and documents were situated within the circle. Virtual reviewer applied the *VIBE* algorithm for visualization of movies reviews where reference points were virtual reviewers of movies while displayed objects were movies (Tatemura, 2000). Experimental studies on the *VIBE* systems were also conducted (Koshman, 2004; Morse and Lewis, 2002).

3.1 Multiple reference points

A reference point (*RP*), or a point of interest (*POI*), is a search criterion against which database documents or document surrogates are matched and search results are generated and presented to users. In a broad sense, a reference point represents users' information needs and any information related to users' needs. For instance, it can range from general information such as a user's past/current research interests, a user's previous search histories, a user's reading preferences, a user's research projects involved, user's affiliation and educational background; to specific information like search terms from a complicated query, browsed documents, and a group of user's queries as well. A reference point may correspond to either a term or a group of terms. Terms in a reference point can be weighted or not weighted. Since a reference point can cover all aspects of users, it is also called a user profile.

Now let us discuss the implication of multiple reference points on information retrieval.

The primary implication of multiple reference points is that they can form a low dimensional visual space and documents can be mapped onto the space based upon their attractions to the reference points.

Contribution of each individual keyword of a query to a final retrieved result is hardly observed in a given linear results list which reflects combinative impacts of all terms involved in a query. A search process is further compounded when a user formulates a multi-faceted query. The user may want to see not only the result set of the original query but also result sets based upon component terms of the query. This would give the user some idea of the contribution of the query parts to the full query (Havre et al. 2001). This is important for a searcher because searching is an iterative process and each process needs to optimize a search query by adjusting search terms based upon search feedbacks. If users are not satisfied with returned search results, a query reformulation follows. A query reformulation may include removing useless search terms, adding new related terms, revising weights assigned to the terms, and so on. All of these actions need a better understanding of the degree to which an individual term affects on the returned search results. That is, if users can understand how and which search terms affect the search results, users can revise and modify a search strategy more accurately to perform a better search. For instance, if the size of a search result set is too small, users may want to know exactly which search terms led to the results. After identifying the terms, users can either change weights assigned to the terms, or replace them with other terms to increase the result size. If search terms are assigned to reference points respectively and presented in the visual space, the impact of each individual term on search results can be easily and intuitively perceived. Therefore, users can make a judgmental decision on query reformulation based upon the provided visual display.

Multiple search terms instead of a single search term can also be grouped and assigned to a reference point. Previous search queries and current search query can serve as reference points respectively and they are present in the same visualization environment. So, both the degree to which the revised search strategies affect on the final results and comparisons of the impact of these strategies are visually displayed. Similarly, a user's preferences, user research interests or other related information can also serve as reference points if users want to know their impacts upon search results.

3.2 Model for fixed multiple reference points

The representative two-dimensional visualization model for fixed multiple reference points was *InfoCrystal* (Spoerri, 1993 a and b). The model was originally designed as a visual query language to visualize a query result from a Boolean based information retrieval system. In the Boolean contexts each reference point may be equivalent to a term or a sub-Boolean logic expression from a Boolean query. The visual space is a polygon where reference points constitute vertices of the polygon and visual results are displayed. The side lengths of the polygon are equal so that the reference points are evenly configured in the visual space. For instance, if there are three, four, and five reference points, the corresponding polygons are an equilateral triangle, square, and pentagon respectively. There are two basic types of icons within the polygon: the criterion icons that are reference points and interior result icons that show the retrieval results. The polygon is partitioned by N exclusive tiers. N is the number of reference points. The tiers are represented by concentric rings and have different radii. Each of these rings defines a special interior display area where interior result icons between certain reference points/ criterion icons are placed. In other words, result icons in the same tier share some commonality with respect to related reference points. The first tier covers result icons that represent documents related to only one inclusive reference point, the second tier covers result icons that represent only documents related to two inclusive reference points,…, and the last tier(the circle) covers one result icon that represents documents related to all *N* reference points. Shapes, directions, and sizes of these result icons vary in different tiers so that users may easily distinguish and identify them. Each side of a result icon is designed and placed to face one of the related criterion icons it meets. Positions of result icons within a tier are fixed and the number of documents satisfying the criteria is displayed on a corresponding result icon. The extent of related documents to the related reference points can also be visualized in result icons by partitioning the icons proportionally and coloring them differently.

For example, Fig. 3.1 shows an example of the configuration for four reference points where *r1*, *r2*, *r3*, and *r4* are reference points located at four corners of the large square as criterion icons. In this figure, there are 4 tiers due to the 4 reference points. Four circles as interior results icons are situated in the first tier, 8 rectangles in the second tier, 4 triangles in the third tier, and 1 square in the fourth tier. The circle close to *r1* in the first tier shows the results of the related document meeting the criterion *r1* AND NOT (*r2* AND *r3* AND *r4*) because it is located at the first tier, and the related documents are relevant to *r1* but *r2, r3*, and *r4*. The rectangle icon between reference points *r1* and *r2* in the second tier indicates the results of the related documents satisfying the criterion *r1* AND *r2* AND NOT (*r3* AND *r4*). The triangle icon between the center and reference point *r1* in the third tier illustrates the results of the related documents meeting the criterion *r1* AND *r2* AND *r4* AND NOT *r3*. The square icon in the fourth tier demonstrates the results of the related documents meeting the criterion *r1* AND *r2* AND *r3* AND *r4*. In this model result icons between two not adjacent reference points may appear more than one time. For instance, both the rectangle icon between the center icon and reference point *r1* and the rectangle icon between the center icon and reference point *r3* in the second tier show the same results satisfying the criterion *r1* AND *r3* AND NOT(*r2* AND *r4*).

If a criterion icon is a sub-Boolean expression (For instance, in a Boolean expression *A* AND (*B* OR *C*), *B* OR *C* is a sub-Boolean expression), it can be connected to another independent visual display that represents the sub-Boolean expression and has a similar display structure to its parent. In this way, the visualization model can be extended to display a hierarchy for a very complicated nested Boolean query.

This model can easily be applied to visualize documents in a vector space. If it is adapted to a vector space, the display framework or structure of the visual space has to be changed accordingly. Reference points are still situated at the corners of

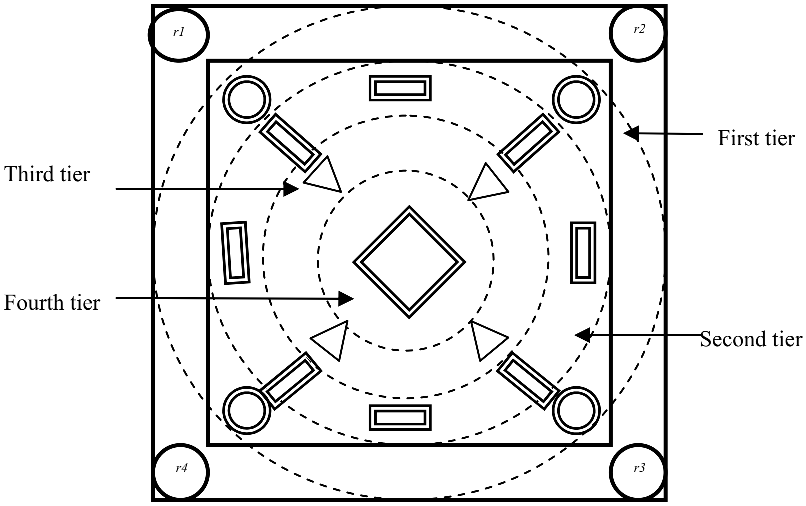

Fig. 3.1. Display of 4 reference points in a fixed reference point environment. (Spoerri, 1993a). © 2003 IEEE. Reprinted with permission

the polygon. The center of the polygon is defined as the origin of the new display coordinate system. In this case, objects in the visual space are individual document icons instead of interior result icons that present a set of related documents. Documents are mapped within the polygon. The position of a document in the visual space is determined by two basic parameters: direction and distance. For instance, if a document is relevant to a reference point, then it is located on the segment defined by the origin of the visual space and the reference point (criterion icon). A document is placed on the segment based upon such a principle that the document with a high similarity to the reference point is close to the origin, and vice versa. So, the position of a projected document is affected not only by its attractions to the criterion icons/ reference points but also by the degree to which the document is relevant to the reference points. As we know, in a vector space such a relevance degree between a document and a reference point can be measured and calculated by using any of the similarity measures discussed in Chap. 2.

It is apparent that the visualization models for both Boolean and vector information retrieval systems support multiple reference points. Reference point icons are fixed and valid display areas are the same in both models. However, visualized objects in these two models are quite different. One visualizes the interior result icons which reflect a result set of retrieved documents while the other displays individual documents. Interior result icon positions are fixed and the number of the result icons is constant in the Boolean based model while the positions of projected documents are not fixed and the number of document icons is a variable in the vector-based model.

Notice that in the Boolean-based model, an interior result icon within a tier represents results of all reference points which are either inclusive (AND) or exclusive (NOT) in a criterion. When the number of exclusive (NOT) reference points in a criterion increases, it could easily lead to an empty set of retrieved documents. It may happen when inclusive (AND) reference points are highly associated with exclusive (NOT) reference points. To avoid this phenomenon, the system should allow users to enable and disable exclusive (NOT) reference points in a criterion.

The uniqueness of the visualization models for fixed multiple reference points is that they may be applied to both a Boolean-based retrieval model and a vector-space-based retrieval model. Due to the fact that all reference points are evenly placed and fixed in the visual space, it results in a symmetrical and well-balanced visual area layout and therefore its interface achieves an aesthetically appealing effect.

3.3 Models for movable multiple reference points

The *VIBE* model (Olsen et al., 1993 a), one model for movable multiple reference points, is distinguished from other visualization models by the fact that the ratio-based similarity scales make displayed objects movable while semantic connections of these objects are still maintained in the visual space. The similarity between a displayed object and a reference point is not directly assigned to any Cartesian coordinates of the display space like other visualization models. The Cartesian system is usually employed to present objects for a visualization model. Cartesian coordinates can be two dimensional or three dimensional. Each dimension corresponds to an axis that is linear. These axes are mutual orthogonal or perpendicular to each other. Any of axes can range from $-\infty$ to ∞. The algorithm uniqueness suggests that the logic relationships among the displayed objects and reference points are independent of their physical locations in the visual space. Position changes of reference points may result in the reconfiguration of projected objects/documents in the visual space. The primary benefit of this uniqueness is that users may arbitrarily place a reference point in the visual space to any interest area (for instance, another reference point, an interest document, a cluster of documents), and observe the impact of that reference point on the area.

3.3.1 Description of the original VIBE algorithm

Since the *VIBE* model works in a vector space, a document D_s can be described by a group indexing keywords $D_s(k_1, k_2, ..., k_i, ..., k_g)$ where k_i is a keyword and g is the number of the index keywords for the document. $R_j(k_1, k_2, ..., k_i, ..., k_m)$ is a reference point (j=1, …, q) where k_i is a keyword, q is the number of the reference points predefined by users, and m is the number of keywords for the reference point R_j. According to the algorithm the position of a document is strongly related to similarities between the document and a group of predefined reference points. The impact of these reference points on the document are defined by a document reference point vector $DRPV_s(r_1, r_2, ..., r_i, ..., r_q)$ where r_i is the relevance value between document D_s and a reference point $R_j(k_1, k_2, ..., k_i, ..., k_m)$ (j=1, …, q).

$$r_i = \frac{|D_s \cap R_i|}{\sum_{j=1}^{q} |D_s \cap R_j|} \tag{3.1}$$

Eq. (3.1) shows that the relevance between document D_s and a reference point R_j is determined by ratio of the number of keywords shared by both the document D_s and the reference point R_j to sum of shared keywords by the document D_s and all reference points. Function $|X|$ indicates the number of elements in a set X. Notice that there may be many other ways to calculate the relevance between a document and a reference point. Because documents and reference points are described in a vector space, the Euclidean distance similarity model, the cosine similarity model, and other models discussed in Chap. 2 are all applicable.

If document index keywords and reference point keywords are associated with weights, the relevance between the document D_s and a reference point R_j can also be defined as:

$$r_i = \frac{\sum_{a=1}^{|D_s \cap R_j|} \left(MIN(WD_s(k_a), WR_i(k_a)) \right)}{\sum_{j=1}^{q} \left(\sum_{a=1}^{|D_s \cap R_j|} \left(MIN(WD_s(k_a), WR_j(k_a)) \right) \right)} \tag{3.2}$$

In Eq. (3.2), i is always between 1 and q, $WD_s(k_a)$ and WR_i (k_a) are weights of a shared keyword k_a in document D_s and reference point R_j. Keyword k_a is an element from a joint set between keywords of document D_s and keywords of reference point R_j ($D_s \cap R_j$). In this equation, the sum of the minimum weights of shared keywords in the document D_s and reference point R_j replaces the number of shared keywords in the document D_s and reference point R_j. The equation is also normalized by the total of the minimum weights of shared keywords in document D_s and all reference points R_j(j=1, …, q) to avoid an unnecessary scale effect.

A reference point can be arbitrarily located in any meaningful point in the two dimensional visual space. $P(R_j)$ denotes the location of the reference point R_j in the visual space.

$$P(R_j) = V_j(x_j, y_j), \quad j = 1, ..., q \tag{3.3}$$

In fact, $V(x, y)$ defines a point which is the reference point icon position in the two dimensional visual space.

The positions of all related reference points in the visual space play a very important role in positioning a projected document. In addition, the relevance between a document and related reference points can also dominate the position of the document in the visual space. When these two factors are considered, the ultimate position of a document is determined.

Given a document D_s is related to two reference points R_1 and R_2. The positions of these two reference points in the two dimensional visual space are $P(R_1)= V_1(x_1, y_1)$ and $P(R_2)= V_2(x_2, y_2)$, respectively. Position of D_s is $P(D_s)= V_s(x_s, y_s)$. The similarities between document D_s and the two reference points R_1 and R_2 are expressed as $DRPV_s\ (r_1, r_2)$ and we assume that both r_1 and r_2 are available by using either Eq. (3.1) or (3.2). Then, the location of document D_s is supposed to be located on a segment between R_1 and R_2 in the visual space. Distances between R_1 and R_2, R_1 and D_s, R_2 and D_s are d_1, d_2 and d_3, respectively. The exact location of D_s depends on its similarities to the two reference points $DRPV_s(r_1, r_2)$. The more similar to a reference point, the closer it is located to the reference point, and vice versa (See Fig. 3.2).

Let us discuss the calculation of the document projection position $V_s(x_s, y_s)$ in the visual space. It is clear that d_1, d_2 and d_3 maintain relationships in Eqs. (3.4) and (3.5).

$$d_1 = \left((x_2 - x_1)^2 + (y_2 - y_1)^2\right)^{1/2} \tag{3.4}$$

$$d_1 = d_2 + d_3 \tag{3.5}$$

In Fig. 3.2, both $V_3\ (x_1, y_s)$ and $V_4\ (x_1, y_2)$ are two temporary points, the right triangle $\Delta V_1V_sV_3$ is similar to the right triangle $\Delta V_1V_2V_4$ because they share the same angle $V_2V_1V_4$, and both angles $V_1V_3V_s$ *and* $V_1V_4V_2$ are right triangles. Therefore, we have the following equations.

$$\frac{d_2}{d_1} = \frac{x_1 - x_s}{x_1 - x_2} \tag{3.6}$$

$$\frac{d_2}{d_1} = \frac{y_1 - y_s}{y_1 - y_2} \tag{3.7}$$

For $DRPV_s(r_1, r_2)$, as we know that both r_1 and r_2 are similarities between document D_s and R_1, and document D_s and R_2 respectively. In the visual space, the larger a similarity value between a document and a reference point, the smaller the distance between them should be. This suggests that the relationship between similarity r and its corresponding distance d in the visual space should be reverse. The reverse relationships are described in on Eqs. (3.6) and (3.8).

$$\frac{d_2}{d_1} = \frac{d_2}{d_2 + d_3} = \frac{(r_1 + r_2) - r_1}{r_1 + r_2} = \frac{r_2}{r_1 + r_2} \tag{3.8}$$

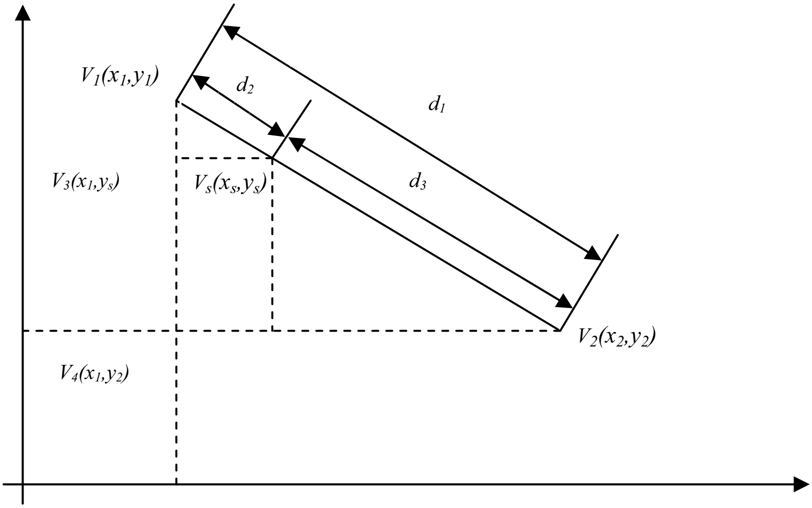

Fig. 3.2. Display of a projected document and two related reference points

$$\frac{d_3}{d_1}=\frac{d_3}{d_2+d_3}=\frac{(r_1+r_2)-r_2}{r_1+r_2}=\frac{r_1}{r_1+r_2} \tag{3.9}$$

Based upon Eqs. (3.6) and (3.8), we have:

$$x_s=\left(\left(\frac{r_2}{r_1+r_2}\right)\times(x_2-x_1)+x_1\right) \tag{3.10}$$

$$x_s=\frac{r_1x_1+r_2x_2}{r_1+r_2} \tag{3.11}$$

Similarly, based on Eqs. (3.7) and (3.8), we have:

$$y_s=\frac{r_1y_1+r_2y_2}{r_1+r_2} \tag{3.12}$$

When r_1 (r_2) is equal to 0, it implies that the document D_s is irrelevant to the reference point R_1 (R_2). Both Eqs. (3.11) and (3.12) tell us that position of the document D_s would be the same as that of R_2 (R_1). In other words, it is overlapped with reference point R_2 (R_1) in the visual space. And the smaller r_1 (r_2) is, the farther away the document D_s is from R_1 (R_2).

In the above equations, ($r_2/(r_1+r_2)$) or ($r_1/(r_1+r_2)$) is called the partition coefficient of document D_s with respect to the reference points R_1 and R_2. It is a ratio of similarity between a document and a reference point to the sum of similarities between the document and the two involved reference points. It underlies where the document is located on the segment defined by the two reference points in the visual space.

Now let us discuss the scenario where multiple reference points are involved. Basically, the *VIBE* algorithm requires three or more reference points to support document projection (Olsen et al., 1993 b). According the original algorithm description (Olsen et al., 1993 a), if a document is related or similar to multiple reference points and similarities between this document and related reference points are available, then first two related reference points from the multiple reference point set are selected and its position on the segment defined by the two reference points is calculated by Eqs. (3.11) and (3.12). The new position of the document on the segment serves as an intermediate reference point for further consideration. The relevance of the newly generated intermediate reference point and the document is the sum of similarities between the document and the two newly merged reference points. The next round position of the document (or next intermediate reference point) is determined by the intermediate reference point and another unprocessed reference point from the multiple reference point set. This process stops when all related reference points from the multiple reference point set are considered and processed. The final intermediate reference point is the ultimate position of the projected document with respect to the multiple reference point set in the visual space.

It can be proved that the sequence of taking reference points from the multiple reference point set into consideration does not affect the final result, which is the ultimate location of the document in the visual space. For simplicity, use three reference points R_1, R_2, and R_3. The positions of these three reference points are $P(R_1)= V_1(x_1, y_1)$, $P(R_2)= V_2(x_2, y_2)$ and $P(R_3)= V_3(x_3, y_3)$, respectively. Similarities of these three reference points to the document D_s are in $DRPV_s$ (r_1, r_2, r_3).

If the reference points R_1 and R_2 are considered first, then position of newly generated intermediate reference point ($R_4(x_4, y_4)$) sees Eqs. (3.13) and (3.14). The similarity value of the intermediate reference point is equal to ($r_1 + r_2$).

$$x_4 = \frac{r_1 x_1 + r_2 x_2}{r_1 + r_2} \tag{3.13}$$

$$y_4 = \frac{r_1 y_1 + r_2 y_2}{r_1 + r_2} \tag{3.14}$$

Take the intermediate reference points R_4 and R_3 to form another intermediate reference point $R_5(x_5, y_5)$.

$$x_5 = \frac{r_3 x_3 + \left(\frac{r_1 x_1 + r_2 x_2}{r_1 + r_2}\right) \times (r_1 + r_2)}{(r_1 + r_2) + r_3} \tag{3.15}$$

$$x_5 = \frac{r_3x_3 + r_1x_1 + r_2x_2}{r_1 + r_2 + r_3} \tag{3.16}$$

Similarly the *Y*-axis value of the reference point R_5 can be calculated and the result is displayed in Eq. (3.17).

$$y_5 = \frac{r_3y_3 + r_1y_1 + r_2y_2}{r_1 + r_2 + r_3} \tag{3.17}$$

Using the same approach, merge R_3 and R_2 first, then merge R_1; we would get the same results as Eqs. (3.16) and (3.17).

In general, a position $V_s(x_s, y_s)$ of a document D_s in a two-dimensional visual space can be computed from Eqs. (3.18) and (3.19) in terms of multiple reference points. Position of a reference point R_j (j=1, …, q) denotes $V_j(x_j, y_j)$, and similarities between the document D_s and the reference point R_j (j=1, …, q) are $DRPV_s$ $(r_1,\ldots,r_q)$. In fact, both Eqs. (3.18) and (3.19) show that the sequence of reference point considerations has nothing to do with the ultimate position of a projected document in the visual space.

$$x_s = \frac{\sum_{i=1}^{q} r_i x_i}{\sum_{i=1}^{q} r_i} \tag{3.18}$$

$$y_s = \frac{\sum_{i=1}^{q} r_i y_i}{\sum_{i=1}^{q} r_i} \tag{3.19}$$

Assume that there are three reference points R_1, R_2 and R_3. Their initial positions in the visual space are $V_1(5, 5)$, $V_2(30, 5)$, and $V_3(15, 25)$, respectively. The position of a document D_s is $V_s(x_s, y_s)$. Assume the similarities between this document and the three reference points are shown in Eq. (3.20).

$$\mathrm{DRPV_s}(r_1, r_2, r_3) = \mathrm{DRPV_s}(0.2, 0.4, 0.6) \tag{3.20}$$

And therefore the coordinates of the document D_s are:

$$x_s = \frac{r_1x_1 + r_2x_2 + r_3x_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 30 + 0.6 \times 15}{0.2 + 0.4 + 0.6} = 18.3 \tag{3.21}$$

$$y_s = \frac{r_1y_1 + r_2y_2 + r_3y_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 5 + 0.6 \times 25}{0.2 + 0.4 + 0.6} = 15 \tag{3.22}$$

As the reference point R_3 moves from its original position to a new position $V'_3(30, 25)$, the document D_s is pulled to a new position D'_s by R'_3.

And the coordinates of the document D'_s are:

$$x_{s1} = \frac{r_1 x_1 + r_2 x_2 + r_3 x'_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 30 + 0.6 \times 30}{0.2 + 0.4 + 0.6} = 25.8 \qquad (3.23)$$

$$y_{s1} = \frac{r_1 y_1 + r_2 y_2 + r_3 y'_3}{r_1 + r_2 + r_3} = \frac{0.2 \times 5 + 0.4 \times 5 + 0.6 \times 25}{0.2 + 0.4 + 0.6} = 15 \qquad (3.24)$$

Fig. 3.3 gives both the position of the document before reference point R_3 moves and the position of the document after R_3 moves to R'_3.

The following algorithm is adapted from the original *VIBE* algorithm (Olsen et al., 1993 b).

```
Begin
    Define multiple reference points $R_j$ ($j=1, ..., p$);
    Get RP positions in the visual space ($V_j(x_j, y_j), j=1, ..., p$);
    Compute the vector (DRPV($r_1,...,r_p$)) for each document based
    on a selected similarity measure;
    Discard documents that are irrelevant to the reference
    points;
    While documents with unprocessed DRPVs are still available
        Get a document D and its DRPV ($r_1,...,r_p$);
        If only one $r_i$ in DRPV ($r_1,...,r_p$)($1 \le i \le p$) $\neq 0$
            Then
            Assign the corresponding reference point as
            the final position of D in the visual space;
            Else
            Select $R_i$ whose $r_i$ is not equal to 0 from
            DRPV ($r_1,...,r_p$) ($1 \le i \le p$);
            While an unprocessed $R_j$ whose $r_j$ is not
            equal to 0 in DRPV ($r_1,...,r_p$) is available
                Merge $R_i$ and $R_j$ to form a new
                intermediate RP, and assign it to $R_i$;
            EndWhile;
            Assign the position of the last intermediate RP
            $R_i$ as the final position of the document D;
            ElseEnd;
        EndIf;
    EndWhile;
End.
```

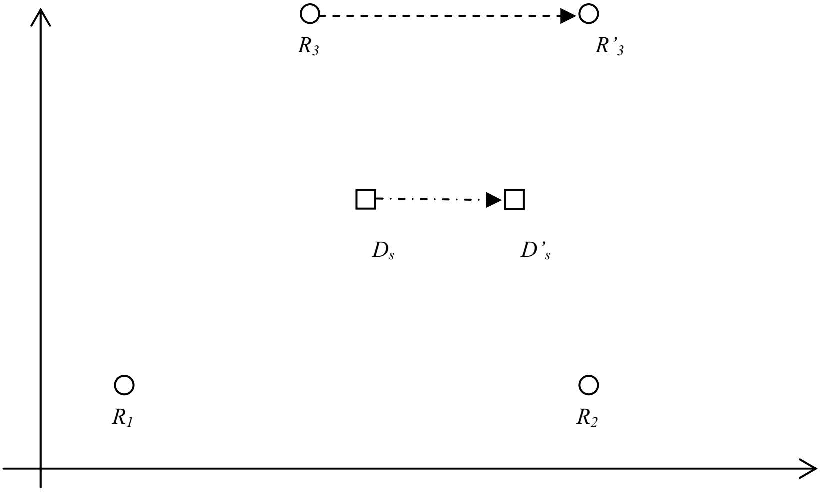

Fig. 3.3. Impact of a moving reference point on a document

L2 to *L7* initialize variables before processing. According to this algorithm, documents which are not relevant to any of the defined reference points are discarded and not considered in the visual space (See *L6* to *L7*). If a document is only relevant to a reference point, its position will overlap with that of the reference point in the visual space (See *L10* to *L13*). If a document is relevant to multiple reference points, a series of intermediate reference points must be generated. A related reference point in conjunction with an intermediate reference point forms a new intermediate reference point. The reference point merging process continues until all relevant reference points are processed. The position of the final intermediate reference point is the position of the projected document in the visual space (See *L14* to *L24*).

Notice that if users change the position of any reference point in the visual space, or add a new reference point to the visual space, or remove a reference point from it, or revise the weights of involved reference points, then it would result in a reconfiguration of projected documents in the visual space. Each of these actions would trigger the algorithm.

Because the algorithm requires calculating a series of intermediate reference points, it is a multiple-step generation algorithm. However, Eqs. (3.18) and (3.19) suggest that calculation of a final document position with respect to multiple reference points in the visual space can be done in a single step. The one-step algorithm would make the algorithm more efficient.

3.3.2 Discussions about the model

Suppose that there are N reference points ($N \geq 3$) in a visual space. These N reference points can form a convex polygon in which all reference points are covered.

The number of polygon vertices is *M,* where *M* is always smaller than or equal to *N*. If some of the reference points are located inside the polygon, then *M* is smaller than *N*. Otherwise, *M* is equal to *N*. The polygon should be constructed in such a way that an angle formed by any vertex of the polygon and its two adjacent vertices should cover the entire polygon.

This polygon will define a valid display area for projected documents if the condition is met for all vertices of the polygon. In other words, all documents should be projected within this special convex polygon generated by the multiple reference points. For a group of reference points, they can be classified into two categories: one includes these reference points which are inside the polygon and the other includes these which form the polygon as its vertices. The above analysis shows that the sequence of taking related reference points into consideration to project a document does not affect the ultimate location of the document in the visual space. We first take consideration of these reference points which are vertices of the polygon. This starts with a vertex point and one of its adjacent reference points in the polygon to calculate the first intermediate reference point with respect to a document. It is apparent that the first intermediate reference point should be located on the side connected by the two selected reference points according to previous analysis. This suggests that the document is not projected outside the polygon. Use the first intermediate reference point and another adjacent reference point (vertex) to generate the second intermediate reference point. Since the second intermediate reference point is situated on the segment between the first intermediate reference point and new selected adjacent vertex, it should be within a triangle area yielded by three reference points already selected. Thus, the document is definitely within the polygon. Then consider the second intermediate reference point and the third adjacent vertex to produce the third intermediate reference point. Because the second intermediate reference point is within the polygon, the segment formed by the second intermediate reference point and the third adjacent vertex should be inside the polygon. This process continues until all vertices of the polygon are considered. This analysis shows that after the reference points in the second category are processed, the last intermediate reference point is still within the polygon. Notice that all reference points in the first category are within the polygon. This implies that the segments formed by any of these reference points and an internal intermediate reference point should also be situated within the polygon. This concludes that all new corresponding intermediate references are within the polygon. According to previous discussion, we know that the location of the final intermediate reference point is the final position of the projected document in the visual space which, of course, is within the polygon. It is important that all reference points should be included in the polygon and the polygon should be a convex polygon which meets the condition we discussed before. These two conditions will ensure that all projected documents are mapped within the polygon.

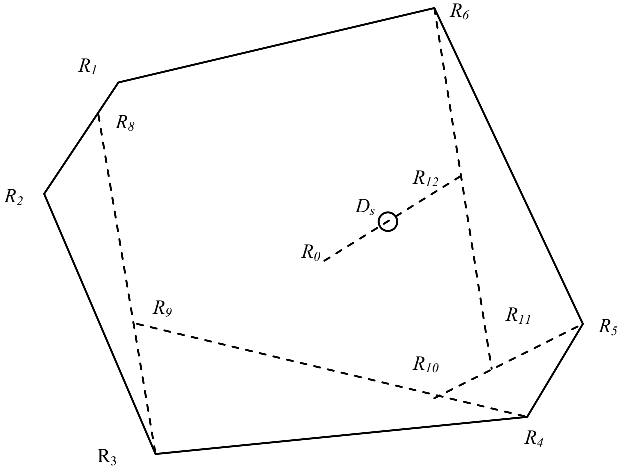

Fig. 3.4. Display of a polygon of a valid display area

For example, there are seven reference points and they are put in the visual space (R_j (j=0, …, 6)). D_s is a document. $\{R_0\}$ is within the first category, and $\{R_1, R_2, R_3, R_4, R_5, R_6\}$ fall in the second category which forms a polygon (See Fig. 3.4). Segments with broken lines are produced by intermediate reference points and adjacent vertices or reference points in the second category. Solid lines are the sides of the polygon. R_8 is the first intermediate reference point produced from both reference points R_1 and R_2. R_9 is the second intermediate reference point produced from the first intermediate reference point R_8 and an adjacent vertex R_3,…. R_{12} is the intermediate reference point produced from the intermediate reference point R_{11} and an adjacent vertex R_6. The final intermediate reference point or projected document D_s is determined by an internal reference point R_0 and the intermediate reference point R_{12} (See the small circle in Fig. 3.4).

After all reference points are positioned in the visual space, some or all of them can actually define a polygon onto which all documents are mapped. Documents within the polygon may be affected by one reference point or many reference points. If a document is only relevant to a certain reference point, it is easy to identify it in the visual space because it is overlapped with the related reference point. However, when a document is relevant to multiple reference points, it is difficult to specify which reference points are related to the document because its position in the visual space is a combination of the impacts of all relevant reference points. To solve this problem, we introduce a concept, reference point monopoly triangle (Korfhage, 1991). A reference point monopoly triangle is a special display area in the visual space that is defined by a vertex of a polygon (a reference

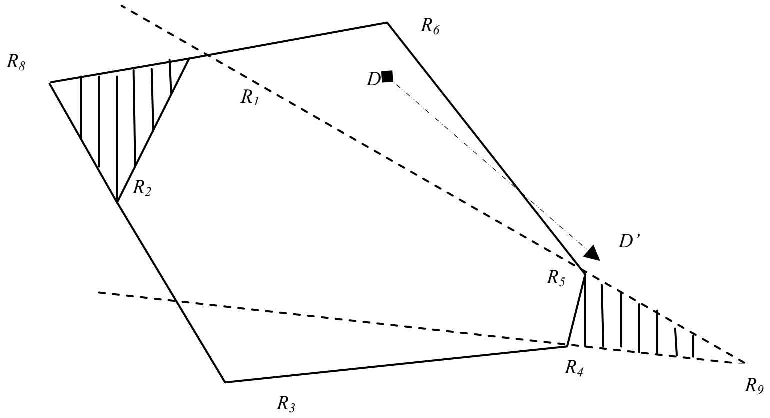

Fig. 3.5. Explanation of reference point monopoly triangle

point) and its two adjacent reference points on the polygon. In the special display triangle area, all projected documents are related to the reference point. As we know, that a group of reference points can define a convex display polygon and all documents are mapped within the polygon. When a new reference point is added to outside the polygon in the visual space, this new reference point will form a new vertex to the polygon. This newly added reference point and its two neighboring reference points constitute a triangle area. After the reference point is added outside the polygon, related documents are reconfigured due to its attraction. A related document to the newly added reference point within the old polygon would move towards the new reference point along the line formed by the newly added reference point and the document. The extent to which the document moves towards the reference point depends upon its similarity to the reference point. It suggests that the related documents may end up within the triangle area if they are strongly attracted by the reference point. In other words, all documents within the triangle are related to the reference point. But documents which are not within the triangle area may also be related to the reference point. If documents are not related to the reference point or they are not very similar to the reference point, they would stay in their original positions or not be "pulled" to the triangle area by it. Users can take advantage of the triangle characteristics to manipulate a reference point, to create a reference point monopoly triangle, and to identify related documents to a special reference point. When creating a reference point monopoly triangle area, make sure that the angle formed by the special reference point and two neighboring vertices of the polygon must be large enough to cover the entire polygon. If this condition is not satisfied, related documents may be "pulled" out of the monopoly triangle area. When a document is located outside the uncovered area and is related to the reference point, it may be pulled out of the monopoly triangle area.

For instance, there are six reference points $\{R_1, R_2, R_3, R_4, R_5, R_6\}$ and they define a polygon (See Fig. 3.5). R_8 and R_9 are two newly added reference points outside the original polygon. They generate two reference point monopoly triangle areas (See the two shaded parts in Fig. 3.5). The triangle formed by reference point R_8 is correctly defined because the angle covers entire polygon area. However, the triangle formed by reference point R_9 is not properly defined because some areas of the polygon are not within the angle area (See the angle formed by two broken lines in Fig. 3.5). Assume that a document *D* is situated within the polygon but outside the angle area and is related to the reference point R_9. Pulled by R_9, *D* moves toward R_9 after R_9 is added to the visual space. Obviously, it may end up outside the reference point monopoly triangle area (See the icon *D'* in Fig. 3.5).

It is worthy to point out that documents situated within a reference point monopoly triangle are definitely related to the reference point and the documents located outside the monopoly triangle area may also be related to the reference point. They are not pulled into the triangle area because the impact of the reference point on these documents is not strong enough compared to other related reference points in the visual space. This occurs often in the visual space. A new question is raised: how can we identify all of the related documents to a specific reference point in the visual space? We must figure out a way to squeeze the polygon and force these documents out of the polygon. A simple way to do this is to put all of the reference points except a selected reference point along a line and position the selected reference point above the line; then all documents related to the selected reference point would be singled out (See Fig. 3.6). In Fig. 3.6, the polygon defined by reference points $(R_1, \ldots, R_i, \ldots, R_n)$ becomes the reference point monopoly triangle. Therefore, all documents related to a special reference point $R_{selected}$ would be displayed within the triangle.

Generally speaking, it is true that the more similar to a reference point a document is, the closer it is to the reference point in the visual space. However, when a document is compared to another document with respect to the same reference point, we cannot simply use the distances from the two documents to the reference point to judge which one is more relevant to the reference point. That is, if a document (D_1) is closer to a reference point than another document (D_2) in the visual space, it does not conclude that document (D_1) is more relevant to the reference point than document (D_2). This may confuse people a little bit. The reason for this phenomenon is that the similarity between a document and a reference point is not the only factor which affects its position in the visual space. Position of a document in the visual space is a collective effort of multiple factors. These factors range from similarity between the document and the special reference point, and the position of the reference point, to similarities between the document and other related reference points, and positions of other related reference points. Each of these factors plays a role in determining the document's position.

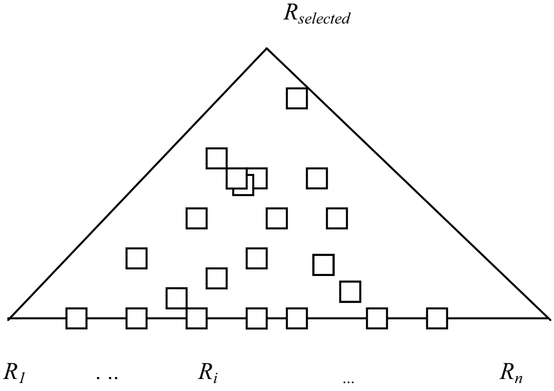

Fig 3.6. Display of all related documents to a special reference point

Therefore, the position of a document is dynamic and relative against the positions of related reference points. It is difficult to judge the similarities of two documents to a special reference point depending upon their distances to the special reference point in the visual space.

On the other hand, comparing two reference points with respect to the same document is relatively easy. Users can neutralize all reference points except the two compared reference points by putting them on a line similar to the one in Fig. 3.6. The user may then put one of the compared reference points above the line and the other below the line. Distances between the two reference points to the line are the roughly same. All related documents would then be spread out around the line to their related reference points. In this way, the impacts of the two reference points on the related documents are shown in the contexts of the other reference points. If a document has an identical similarity to the two reference points, the attractions from the two opposite directions cancel each other because the reference points are located in opposite positions against the line in the visual space. This would cause the document to stay on the line like the other unrelated documents. This problem can be solved by moving any of the two reference points to the line. As a result, the documents would leave the line and move towards the other reference point.

The *VIBE* algorithm was used for a full-text environment (Korfhage and Olsen, 1994). In this case, the visual environment was limited to a single full-text based document instead of a group of documents. Displayed objects were replaced by meaningful semantic logic units within a document. The semantic logic units may be defined as sentences, paragraphs, sections, or chapters within that document. Segmentation of a full text should depend upon the nature and length of the full text. A semantic unit is indexed by keywords extracted from that semantic unit. Keyword weight can be assigned by simply using its raw occurrence in that semantic unit. The weight of a term can also be calculated by a more complicated approach. For instance, we can apply *TF*×*IDF* (Term Frequency × Inverse Document Frequency) to the measuring weight of a term.

$$w_i = f_i \times Log\left(\frac{N}{d_i}\right) \tag{3.25}$$

In this equation, w_i is the weight of the term i, f_i is the frequency of the term i in a semantic unit such as a paragraph, N is the number of all semantic units segmented from a full-text, and d_i is the number of semantic units that contain the term i.

Each of the semantic units is assigned a sequential number based upon its appearance sequence in a full-text. When all semantic units are visualized in the visual space, two adjacent semantic units are connected by a sequence link. Therefore, another dimension is added to the visual space to facilitate analyzing the relationships of semantic units in a full-text analysis in addition to the existing visualization features provided in *VIBE*.

The original two-dimension-based *VIBE* model was also adapted to support three-dimension-based visual environments for greater interaction in two distinctive approaches. The first approach added the third dimension which was the overall significance of a document to the two dimensional *VIBE* space (Benford et al., 1995; Benford et al., 1997). This new dimension was used to demonstrate the relevance between a document and all reference points. The overall significance between a document D_s and all involved reference points R_j (j=1, …, q) was defined as the sum of its similarities to all reference points and used as the Y-axis value of the document in the visual space:

$$z_s = \sum_{i=1}^{q} r_i \tag{3.26}$$

In this equation, the similarities between document D_s and reference point R_j (j=1, …, q) are described in $DRPV_s$ $(r_1,...,r_q)$.

Definitions of the X-axis and Y-axis for the document D_s are still the same as those in both Eqs. (3.18) and (3.19).

Similarly, the Z-axis of a reference point in the three-dimensional space is defined as the sum of similarities between this reference point R_j to all related documents:

$$z_{R_j} = \sum_{i=1}^{n} r_{iR_j} \tag{3.27}$$

In this equation, r_{iRj} is similarity between the document i and reference point R_j, and n is the number of the involved documents in the visual space.

The X-axis value and Y-axis value for a reference point R_j are dynamic and determined by users while the Z-axis is obviously not.

It is evident that the newly added *Z*-axis value for a certain document or reference point is an absolute value and is no longer dynamic and relative like the *X*-axis and *Y*-axis values. As long as the involved reference points and documents are determined, the *Z*-axis values for these documents and reference points are unchanged. The significance of adding a fixed *Z*-axis value for a document or reference point rests upon that it can alleviate the ambiguity phenomenon of projected documents to some degree in the visual space, and therefore it more accurately reveals relationships among the projected documents. For instance, given two reference points R_1 and R_2, and two documents D_1 and D_2, similarity between D_1 and R_1, and similarity between D_1 and R_2 are 0.4 and 0.8, respectively. Similarity between D_2 and R_1, and similarity between D_2 and R_2 are 0.1 and 0.2, respectively. Without the third overall significance dimension, these two documents are projected onto the same location between the two reference points because the position of a document is determined by relative attraction that is the ratio of the document to all reference points multiplying their location factors to the overall similarities of the document to all reference points. Users simply cannot tell to what extent each document is related to the reference points if the ambiguity phenomenon occurs. With the third overall significance dimension, the two overlapped documents are separated in the third axis because they have different overall significance values, which can distinguish them in the visual space.

The second three-dimensional model was *LyberWorld* (Hemmje et al., 1994). Unlike the first model, the *Z*-axis of a document D_s in this model (See Eq. (3.28)) is similar to both the *X*-axis and the *Y*-axis (See Eqs. (3.18) and (3.19)).

$$z_s = \frac{\sum_{i=1}^{q} r_i z_i}{\sum_{i=1}^{q} r_i} \tag{3.28}$$

All documents are located on the so-called relevance sphere boundary. The indexing terms of a document are always situated within the sphere. In this case, the indexing terms replace the referenced points in the visual space. Users may manipulate the indexing terms like reference points, rotate the relevance sphere, and zoom in/out of the space at will.

3.4 Model for automatic reference point rotation

In this section, a new model for multiple reference points is introduced (Zhang and Nguyen, 2005). The model for automatic reference point rotation is a similarity ratio based model to some extent. The uniqueness of this model is that it adds a new feature reference point automatic rotation to the two-dimensional visual space. It enables users to observe the relevance between a rotating reference point and its related documents in the visual space.

3.4.1 Definition of the visual space

The visual space is two dimensional and is built on a polar coordinate system which is a plane with a point *O* (the pole) and a ray from *O* (the polar axis). Each point (*P*) in the plane is assigned polar coordinates: the directed distance from *O* to *P* and the directed angle whose initial side is on the polar axis and whose terminal side is on the line *PO*. Notice that the polar coordinate system can be converted to the Cartesian coordinate system easily. The valid display area is a sphere in the space. All reference points are positioned on the sphere. Reference points R_j (j=1, …, q) are evenly distributed on the sphere as their default positions. Once a reference point is activated by users, it automatically rotates counterclockwise around the sphere. In other words, the boundary of the sphere is the orbit of a moving reference point. The radius of the sphere is *MR*. The center of the sphere (*O*) is the focus point. The focus point can be defined as different meanings in different contexts. This issue will be discussed later. All of the documents are basically scattered within the visual space. The position of a document D_i denotes DP_i ($l_{i,}$ α_i). Here l_i and α_i are the projection distance/ directed distance and projection angle of a document/ directed angle, respectively. These two parameters determine its position in the visual space.

$$l_i = MR \times (1 - S_{oi}) \tag{3.29}$$

$$S_{oi} = c^{\left|(\sum_{t=1}^{n}(a_t)^{1/2})^2 - (\sum_{t=1}^{n}(x_{it})^{1/2})^2\right|} \times \frac{\sum_{t=1}^{n} a_t \times x_{ti}}{(\sum_{t=1}^{n} {a_t}^2)^{1/2} \times (\sum_{t=1}^{n} {x_{ti}}^2)^{1/2}} \tag{3.30}$$

In this equation, $O(a_{1,}\, a_{2,\dots,}\, a_n)$ is the focus point vector, $D_i(x_{i1,}\, x_{i2,\dots,}\, x_{in})$ is the document vector, both a_j and x_{ij} are keyword weights($0 \le j \le n$), and n is dimensionality of the vector space. It is clear that l_i basically reflects the relationship between the document D_i and the focus point *O*. Eq. (3.30) calculates the similarity between D_i and the focus point *O* and *c* is a control constant. The characteristics of this similarity measure were discussed in Chap. 2. In Eq. (3.29) factor(1-S_{oi}) is used to convert the similarity to reflect such a relation that the more relevant the document D_i is to the focus point *O*, the closer they are in the visual space, and vice versa. Notice that the valid value of S_{oi} is between 0 and 1. *MR* defines the radius of the sphere. By changing *MR*, users may zoom in/out of the visual sphere at will.

The projection angle of the document D_i is defined in Eq. (3.31). β_j is the angle which is formed by the reference point R_j and the polar axis against the origin of the visual space or the focus point. S_{jk} is the similarity between reference point R_j and document D_k. If a document D_i is irrelevant to any of the reference points ($\sum_{k=1}^{q} S_{ki} = 0$), then the angle α_i is defined as zero to avoid a meaningless $\alpha_{i.}$

The equation shows whether a reference point is irrelevant to any reference points, it would be situated on the polar axis and the distance to the origin is determined by its similarity to the focus point. The equation demonstrates that the projection angle of a document is dominated by a similarity ratio rather than by an absolute sum of the angles multiplied by similarities. This characteristic underlies the reference point rotation feature. The display sphere, reference point rotation direction, the projection distance, and the projection angle are shown in Fig. 3.7. In the figure R_1, R_2, R_3, R_4, and R_5 are five reference points.

$$\alpha_i = \begin{cases} \dfrac{\sum_{j=1}^{q}(\beta_j \times S_{ji})}{\sum_{j=1}^{q} S_{ji}}, & \sum_{k=1}^{q} S_{ki} \neq 0 \\ 0, \quad \sum_{j=1}^{q} S_{ji} = 0 & \end{cases} \tag{3.31}$$

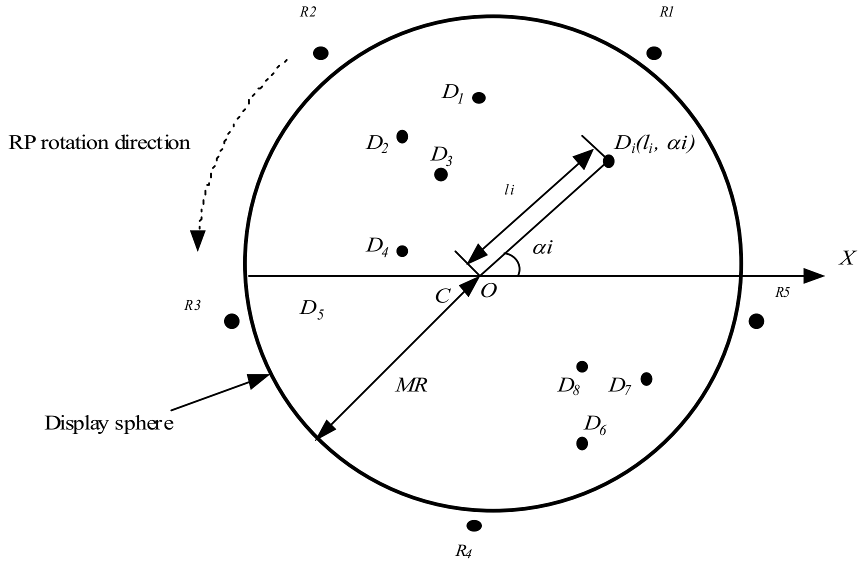

Fig. 3.7. Display of the *WebStar* visual space
Source: Zhang and Nguyen (2005)

3.4.2 Rotation of a reference point

Eq. (3.31) describes the projection angle of a document against static reference points. However, as a selected reference point is activated to rotate around the sphere, related documents no longer stay static. Attracted by the moving reference point, the related documents would also rotate around the orbit whose radius is defined by Eq. (3.29). The speed of a related document primarily depends upon its relevance to the moving reference point and it can be calculated in Eq. (3.32). In this equation, given R_r is the activated reference point, β_r is the initial angle in the visual space, θ is the speed of the activated reference point R_r, t is a time variable, S_{ri} is the similarity between R_r and D_i, and α'_i is the speed of the related document D_i.

$$\alpha_i' = \frac{\sum_{j=1, j\neq r}^{q} (\beta_j \times S_{ji}) + (\theta \times t + \beta_r) \times S_{ri}}{\sum_{j=1}^{q} S_{ji}} \tag{3.32}$$

From Eq. (3.32) we have the speed of any related document.

$$\frac{d\alpha'_i}{dt} = \frac{\theta \times S_{ri}}{\sum_{j=1}^{q} S_{ji}} \tag{3.33}$$

Eq. (3.33) suggests that if a document is not relevant to the rotating reference point it would stay unchanged in the visual space. The more relevant the document is to the rotating reference point, the faster the document rotates around the center, and vice versa. The maximum speed of the document is equal to that of the rotating reference point. The similarities between the related document and other reference points also play a role in the document speed due to the divisor $\sum_{k=1}^{q} S_{ki}$ in Eq. (3.33).

Fig. 3.8 gives a series of the *WebStar* snapshots of a rotating reference point. The four reference points are sport, research, international, and library. Starting from the upper left corner figure clockwise, the first figure is the initial status, the second figure is the status when the reference point international rotates to about 85 degrees, the third figure is the status when the reference point international rotates to 0 degrees, and the fourth figure is the status when the reference point international is activated and rotates to 270 degrees. As the reference point international orbits, all of the related documents rotate accordingly along the same direction but at different speeds.

Differences between this model and other similarity ratio based visualization models are that this model requires a focus point located at the origin of the visual space, it supports single reference point projection, only one of the location parameters (projection angle) is similarity-ratio-based while the other (projection distance) is not, and the reference point rotation feature utilizes the movement of

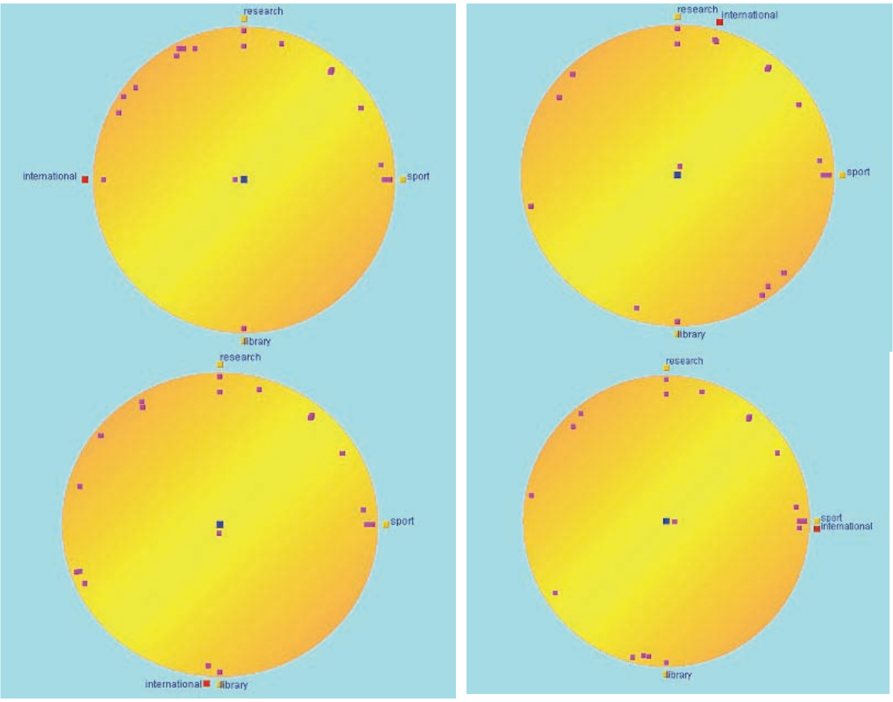

Fig. 3.8. A series of the *WebStar* snapshots of a rotating reference point

objects as an indicator of relevance between a rotating reference point and related documents.

3.5 Implication of information retrieval

The visualization models for multiple reference points have a very natural and direct relationship to information retrieval. The model for fixed multiple reference points was designed to visualize results from a Boolean query. The original models for movable multiple reference points were aimed at visualizing the results of a query from a vector-based space. In fact, a query can be regarded as a special reference point in a broader sense. Documents related to these reference points are displayed in the visual space. Even within the *VIBE* visual space, it can integrate a search mechanism where users submit their queries, and matched document icons in the visual space are highlighted so that they can easily be distinguished from other un-retrieved document icons. In the *WebStar* visual space, users may employ the retrieval contour to retrieve documents in the visual space. The retrieval contour is a circle which shares the same center with the display sphere. The radius of

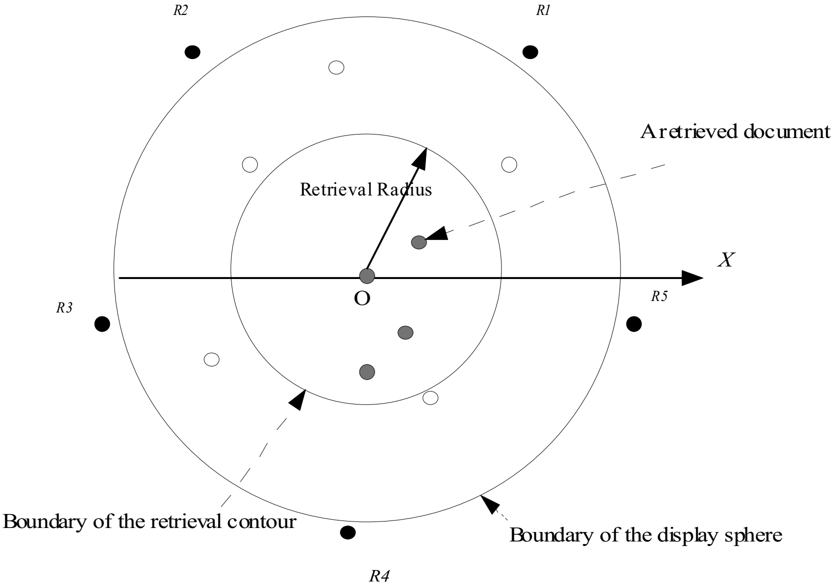

Fig. 3.9. Display of the retrieval contour
Source: Zhang and Nguyen (2005)

the retrieval contour can be manipulated by users to control the size of a retrieved document set (See Fig. 3.9). As the radius increases, more documents are retrieved, and vice versa. Notice that when the number of retrieved documents increases, the similarities of the newly added documents and the focus point decrease because they are getting farther away from it.

The focus point in the model for automatic reference point rotation is extremely important. The visual contexts change if the focus point changes. If it is assigned as a website, then the projected documents are outgoing Web pages. If it is assigned as a search query submitted to an information retrieval system, then the mapped documents are returned results. If it is assigned as a regular paper, then the displayed documents are its citations. If it is assigned as a root of a subject hierarchy, then the projected objects are its children nodes. The focus point enables users to narrow down their interests. As long as the focus point is clearly defined, its relationships to the projected objects are illustrated by observing the distances from the origin to the displayed objects in the visual space. Of course, like other multiple reference point based visualization models it demonstrates the relationships between reference points and documents in the visual space. That is, in the static status a document is located near to reference points which are relevant to it. In addition, the model reveals which documents are related to a rotating reference point and the extent to which these documents are similar to the moving reference point.

A similarity-ratio-based visualization model may be used for term discriminative analysis (Dubin, 1995). In the *VIBE* space, each of the tested keywords is assigned to a reference point, and all of the reference points are scattered onto the visual space to form a circle. Terms with poor discriminative capacity tend to stay around the center of the circle because they are not affected by any reference points. Terms with good discriminative capacity tend to be spread out in the visual space because a good discriminative term will attract related documents. Based upon a distribution of documents, people can make a correct judgment about whether a term is a discriminative term or not.

3.6 Summary

Reference point is an important concept in information retrieval. Multiple reference points were introduced not only to visualize documents but also to solve the inherent problems of a traditional information retrieval system.

In this chapter, visualization models based on multiple reference points were classified into three categories: the models for fixed multiple reference points, the models for movable multiple reference points, and the model for automatic reference point rotation. Each of them has its own unique qualities. The models for fixed multiple reference points can be used for both a vector-based information retrieval system and Boolean based information retrieval system. They can handle a complex Boolean query. But the algorithm is not based upon the similarity ratio method, even though it can be applied to a vector-based information retrieval system. Most models for movable multiple reference points are similarity ratio based, except for the *VR-VIBE* model, whose *Z*-axis of the visual space is sum of similarities of all related reference points for a document unlike the *X*-axis and *Y*-axis. The visualization model for automatic reference point rotation is based upon the similarity ratio in part because only the projection angle of a document, one of the two projection parameters, is calculated based upon similarity ratio of related reference points while projection distance is not. These two parameters of a document dominate its position in the visual space. The power of the similarity ratio based algorithms relies on the manipulative flexibility for reference points in the visual space. The position of any reference points can be controlled and manipulated by users at will. It is the manipulative flexibility that enables users to compare and analyze the impact of two reference points on documents, and identify good/poor discriminative terms. Both the models for fixed multiple reference points and the models for movable multiple reference points require at least three reference points to project documents in their visual spaces, while the model for automatic reference point rotation requires at least one reference point in conjunction with the focus point to construct its visual space. Visualization models for multiple reference points can be two-dimensional or three-dimensional. They can be applied to either Boolean based information systems or vector based information systems. They can be used to visualize Internet hyperlinks, search results from an information retrieval system, a full-text, and term discriminative analysis.