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Séminaire de Probabilités 1967–1980

A Selection in Martingale Theory



Springer

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Cataloging-in-Publication Data applied for.

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Séminaire de probabilités 1967 - 1980 : a selection in martingale theory /
M. Émery ; M. Yor (ed.). - Berlin ; Heidelberg ; New York ; Barcelona ; Hong
Kong ; London ; Milan ; Paris ; Tokyo : Springer, 2002
(Lecture notes in mathematics ; 1771)
ISBN 3-540-42813-5

Mathematics Subject Classification (2000):
01A60; 01A75, 60G07, 60G42, 60G48, 60H05, 60H10

ISSN 0075-8434

ISBN 3-540-42813-5 Springer-Verlag Berlin Heidelberg New York

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Science + Business Media GmbH

<http://www.springer.de>

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Printed in Germany

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Typesetting: Camera-ready TeX output by the authors

SPIN: 10855562 41/3142/LK - 543210 - Printed on acid-free paper

À Paul André Meyer

qui, une fois encore, s'est
effacé au moment de signer

FOREWORD

Thirty five volumes of the *Séminaire de Probabilités*, originated in Strasbourg in 1967, have now been published in the Lecture Notes in Mathematics, thanks to the editorial efficiency of Springer-Verlag.

Most volumes in the first half of the *Séminaire* series are now out of print. For several reasons, it seems timely and worthwhile to publish again a selection of articles pertaining to martingale theory. The current trend in the study of stochastic processes is towards more and more sophistication, and many recent applications use sharp results of stochastic integration; so the full generality of the *general theory of processes* (a name still well deserved!) is no longer a luxury, but often a basic need.¹ Young mathematicians, or users of the theory coming from other fields, have a hard time finding their way through what may at first sight look like a jungle, due to the historical process of piling up new results on top of former ones faster than the old ones are made simpler. Practitioners of the theory may be interested in the historical origin of their daily tools. We hope that this volume will be useful to those who wish to get acquainted with martingale theory, as well as to those who, already familiar with it, are curious about its history.

Twenty-five articles have been selected, not without hesitation, from the first fourteen *Séminaires de Probabilités*. This set of articles ranging from 1967 to 1980 is organized in six parts, which, although deeply intertwined, represent essential building blocks in the theory of stochastic processes. They are:

- A. General theory of processes
- B. Stochastic integration
- C. Martingale inequalities
- D. Previsible representation
- E. Semimartingales
- F. Stochastic differential equations

There was much to choose from! We thank P.-A. Meyer for his invaluable contribution to this selection process. The criteria were two-fold: on the one hand, we try to provide a coherent and well-founded exposition of stochastic calculus for general semimartingales; on the other hand, this volume aims to help a reader interested in chapters XX–XXIII on stochastic calculus of *Probabilités et potentiel E* (Dellacherie-Maisonneuve-Meyer, Hermann 1992). The unavoidable core of our selection is, of course, Meyer's "Cours sur les intégrales stochastiques" (B3 in this volume); however, chronologically, stochastic calculus was developed earlier in the

¹ Note that, historically speaking, the general theory of processes stemmed more from the theory of Markov processes than from martingale theory, which only later became the leading point of view.

fundamental paper of Kunita and Watanabe (Nagoya Math. J., 1967), which is expounded by Meyer in B1. A careful reading of B3 necessitates some acquaintance with the general theory of processes, presented in A. Likewise, the remaining four parts of the volume, C, D, E and F, are of constant use nowadays, for both theoretical and practical purposes.

We begin with a short presentation where each selected article is put in historical and mathematical perspective. Wherever possible, the literature we refer to is in English. To the best of our knowledge, only two books in that language provide a self-contained account of stochastic calculus, with a complete proof of the optional and previsible section theorems:

Dellacherie-Meyer, *Probabilities and potential A and B*, North-Holland 1978 and 1982;

He-Wang-Yan, *Semimartingale Theory and Stochastic Calculus*, CRC Press 1992.

Here is also a short list of other books in English where calculus with càdlàg semimartingales is expounded:

Elliott, *Stochastic Calculus and Applications*, Springer 1982;

Ikeda-Watanabe, *Stochastic Differential Equations and Diffusion Processes*, North-Holland 1981;

Jacod-Shiryayev, *Limit Theorems for Stochastic Processes*, Springer 1987;

Kallenberg, *Foundations of Modern Probability*, Springer 1997;

Liptser-Shiryayev, *Theory of Martingales*, Kluwer 1989;

Métivier, *Semimartingales: A course on Stochastic Processes*, de Gruyter 1982;

Prohorov-Shiryayev, *Probability Theory III Stochastic Calculus*, Springer 1998;

Protter, *Stochastic Integration and Differential Equations*, Springer 1990;

Rogers-Williams, *Diffusions, Markov Processes and Martingales I and II*, Wiley 1994 and 1987;

Shiryayev, *Essentials of Stochastic Finance*, World Scientific 1999.

If this selection is well accepted by the mathematical readership, and if Springer, our fellow traveller for thirty five years, agrees, the next stage will be either a sequel, including many items that could reasonably have been included here (for instance, in Vol. XIII alone, Doléans-Meyer's weighted norm inequalities, the series on balayage, Jeulin-Yor's faux-amis and/or Jeulin's work on enlarged filtrations), or a similar volume of articles on Markov processes and related topics, also chosen from the early Séminaires.

Another project, which also aims to facilitate access to the content of past volumes of the Séminaire, is the creation of a data base that describes, in the same historical-mathematical spirit as here, all articles published in the Séminaire, from Volume I onwards. This work is in progress; the data base in its current state can already be consulted on the web site

http://www-irma.u-strasbg.fr/irma/semproba/e_index.shtml

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