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Annotated Bibliography

1 General Remarks

Historians distinguish between primary and secondary or even ternary sources. A primary source for, say, a biography would be a birth or death record, personal letters, handwritten drafts of papers by the subject of the biography, or even a published paper by the subject. A secondary source could be a biography written by someone who had examined the primary sources, or a non-photographic copy of a primary source. Ternary sources are things pieced together from secondary sources— encyclopædia or other survey articles, term papers, etc.[1] The historian's preference is for primary sources. The further removed from the primary, the less reliable the source: errors are made and propagated in copying; editing and summarising can omit relevant details, and replace facts by interpretations; and speculation becomes established fact even though there is no evidence supporting the "fact".[2]

1.1 Exercise. Go to the library and look up the French astronomer Camille Flammarion in as many reference works as you can find. How many different birthdays does he have? How many days did he die? If you have access to *World Who's Who in Science,* look up Carl Auer von Welsbach under "Auer" and "von Welsbach". What August day of 1929 did he die on?

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[1] As one of the referees points out, the book before you is a good example of a ternary source.

[2] G.A. Miller's "An eleventh lesson in the history of mathematics", *Mathematics Magazine* 21 (1947), pp. 48 - 55, reports that Moritz Cantor's groundbreaking German language history of mathematics was eventually supplied with a list of 3000 errors, many of which were carried over to Florian Cajori's American work on the subject before the corrections were incorporated into a second edition of Cantor.

Answers to the Flammarion question will depend on your library. I found 3 birthdates and 4 death dates.[3] As for Karl Auer, the *World Who's Who in Science* had him die twice— on the 4th and the 8th. Most sources I checked let him rest in peace after his demise on the 4th. In my researches I also discovered that Max Planck died three nights in a row, but, unlike the case with von Welsbach, this information came from 3 different sources. I suspect there is more than mere laziness involved when general reference works only list the years of birth and death. However, even this is no guarantee of correctness: according to my research, the 20th century French pioneer of aviation Clément Ader died in 1923, again in 1925, and finally in 1926.

1.2 Exercise. Go to your favourite encyclopædia and read the article on Napoleon Bonaparte. What is Napoleon's Theorem?

In a general work such as an encyclopædia, the relevant facts about Napoleon are military and political. That he was fond of mathematics and discovered a theorem of his own is not a relevant detail. Indeed, for the history of science his importance is as a patron of the art and not as a a contributor. For a course on the history of mathematics, however, the existence of Napoleon's Theorem becomes relevant, if hardly central.

Translations, by their very nature, are interpretations. Sometimes in translating mathematics, a double translation is made: from natural language to natural langauge and then into mathematical language. That the original was not written in mathematical language could be a significant detail that is omitted. Consider only the difference in impressions that would be made by two translations of al-Khwarezmi's algebra book, one faithfully symbol-less in which even the number names are written out (i.e., "two" instead of "2") and one in which modern symbolism is supplied for numbers, quantities, and arithmetic operations. The former translation will be very heavy going and it will require great concentration to wade through the problems. You will be impressed by al-Khwarezmi's mental powers, but not by his mathematics as it will be hard to survey it all in your mind. The second translation will be easy going and you shouldn't be too impressed unless you mistakenly believe, from the fact that the word "algebra" derived from the Arabic title of his book, that the symbolic approach originated here as well.

The first type of translation referred to is the next best thing to the primary source. It accurately translates the contents and allows the reader to interpret them. The second type accurately portrays the problems treated, as well as the abstract principles behind the methods, possibly more as a concession to readability than a conscious attempt at analysis, but in doing so it does not accurately portray the actual practice and may lead one to overestimate the original author's level of understanding. Insofar as a small shift in one's

[3] I only found them in 4 different combinations. However, through clever footnoting and the choice of different references for the birth and death dates, I can justify $3 \times 4 = 12$ pairs!

perspective can signify a major breakthrough, such a translation can be a significant historical distortion.

It is important in reading a translation to take the translator's goal into account, as revealed by the following quotation from Samuel de Fermat (son of *the* Fermat) in his preface to a 1670 edition of Diophantus:

> Bombelli in his *Algebra* was not acting as a translator for Diophantus, since he mixed his own problems with those of the Greek author; neither was Viète, who, as he was opening up new roads for algebra, was concerned with bringing his own inventions into the limelight rather than with serving as a torch-bearer for those of Diophantus. Thus it took Xylander's unremitting labours and Bachet's admirable acumen to supply us with the translation and interpretation of Diophantus's great work.[4]

And, of course, there is always the possibility of a simple mistranslation. My favourite example was reported by the German mathematical educator Herbert Meschkowski.[5] The 19th century constructivist mathematician Leopold Kronecker, in criticising abstract mathematical concepts, declared, "Die ganzen Zahlen hat der liebe Gott gemacht. Alles andere ist Menschenwerk." This translates as "The Good Lord made the whole numbers. Everything else is manmade", though something like "God created the integers; all the rest is man's work" is a bit more common. The famous theologian/mystery novelist Dorothy Sayers quoted this in one of her novels, which was subsequently translated into German. Kronecker's remark was rendered as "Gott hat die Integralen erschaffen. Alles andere ist Menschenwerk", or "God has created the integrals. All the rest is the work of man"!

Even more basic than translation is transliteration. When the matchup between alphabets is not exact, one must approximate. There is, for example, no equivalent to the letter "h" in Russian, whence the Cyrillic letter most closely resembling the Latin "g" is used in its stead. If a Russian paper mentioning the famous German mathematician David Hilbert is translated into English by a nonmathematician, Hilbert's name will be rendered "Gilbert", which, being a perfectly acceptable English name, may not immediately be recognised by the reader as "Hilbert". Moreover, the outcome will depend on the nationality of the translator. Thus the Russian mathematician Chebyshev's name can also be found written as Tchebichev (French) and Tschebyschew (German). Even with a fixed language, transliteration is far from unique, as schemes for transliteration change over time as the reader will see when we get to the chapter on the Chinese word problem. But we are digressing.

We were discussing why primary sources are preferred and some of the ways references distant from the source can fail to be reliable. I mentioned

[4] Quoted in André Weil, *Number Theory; An Approach Through History, From Hammurapi to Legendre,* Birkhäuser, Boston, 1984, p. 32.

[5] *Mathematik und Realität, Vorträge und Aufsätze,* Bibliographisches Institut, Mannheim, 1979, p. 67.

above that summaries can be misleading and can replace facts by interpretation. A good example is the work of Diophantus, whose *Arithmetica* was a milestone in Greek mathematics. Diophantus essentially studied the problem of finding positive rational solutions to polynomial equations. He introduced some symbolism, but not enough to make his reasoning easily accessible to the modern reader. Thus one can find summary assessments— most damningly expressed in Eric Temple Bell's *Development of Mathematics,*[6]— to the effect that Diophantus is full of clever tricks, but possesses no general methods. Those who read Diophantus 40 years after Bell voiced a different opinion: Diophantus used techniques now familiar in algebraic geometry, but they are hidden by the opacity of his notation. The facts that Diophantus solved this problem by doing this, that one by doing that, etc., were replaced in Bell's case by the interpretation that Diophantus had no method, and in the more modern case, by the diametrically opposed interpretation that he had a method but not the language to describe it.

Finally, as to speculation becoming established fact, probably the quintessential example concerns the Egyptian rope stretchers. It is, I believe, an established fact that the ancient Egyptians used rope stretchers in surveying. It is definitely an established fact that the Pythagorean Theorem and Pythagorean triples like 3, 4, 5 were known to many ancient cultures. Putting 2 and 2 together, the German historian Moritz Cantor speculated that the rope stretchers used knotted ropes giving lengths 3, 4, and 5 units to determine right angles. To cite Bartel van der Wærden,[7]

> ...How frequently it happens that books on the history of mathematics copy their assertions uncritically from other books, without consulting the sources...In 90% of all the books, one finds the statement that the Egyptians knew the right triangle of sides 3, 4, and 5, and that they used it for laying out right triangles. How much value has this statement? None!

Cantor's conjecture is an interesting possibility, but it is pure speculation, not backed up by any evidence that the Egyptians had any knowledge of the Pythagorean Theorem at all. Van der Wærden continues

> To avoid such errors, I have checked all the conclusions which I found in modern writers. This is not as difficult as might appear...For reliable translations are obtainable of nearly all texts...
> Not only is it more instructive to read the classical authors themselves (in translation if necessary), rather than modern digests, it also gives much greater enjoyment.

Van der Wærden is not alone in his exhortation to read the classics, but "obtainable" is not the same as "readily available" and one will have to rely on

[6] McGraw-Hill, New York, 1940
[7] *Science Awakening*, 2nd ed., Oxford University Press, New York, 1961, p. 6.

"digests", general reference works, and other secondary and ternary sources for information. Be aware, however, that the author's word is not gospel. One should check if possible the background of the author: does he or she have the necessary mathematical background to understand the material; what sources did he/she consult; and, does the author have his/her own axe to grind?

Modern history of mathematics began to be written in the 19th century by German mathematicians, and several histories were written by American mathematicians in the early 20th century. And today much of the history of mathematics is still written by mathematicians. Professional historians traditionally ignored the hard technical subjects simply because they lacked the understanding of the material involved. In the last several decades, however, a class of professional historians of science trained in history departments has arisen and some of them are writing on the history of mathematics. The two types of writers tend to make complementary mistakes— or, at least, be judged by each other as having made these mistakes.

Some interdisciplinary errors do not amount to much. These can occur when an author is making a minor point and adds some rhetorical flourish without thinking too deeply about it. We saw this with Henle's comment on Euclid in the introduction. I don't know how common it is in print, but its been my experience that historical remarks made by mathematicians in the classroom are often simply factually incorrect. These same people who won't accept a mathematical result from their teachers without proof will accept their mentors' anecdotes as historical facts. Historians' mistakes at this level are of a different nature. Two benign examples come to mind. Joseph Dauben, in a paper[8] on the Chinese approach to the Pythagorean Theorem, compares the Chinese and Greek approaches with the remark that

> ...whereas the Chinese demonstration of the right-triangle theorem involves a rearrangement of areas to show their equivalence, EUCLID's famous proof of the Pythagorean Theorem, Proposition I,47, does not rely on a simple shuffling of areas, moving a to b and c to d, but instead depends upon an elegant argument requiring a careful sequence of theorems about similar triangles and equivalent areas.

The mathematical error here is the use of the word "similar", the whole point behind Euclid's complex proof having been the avoidance of similarity which depends on the more advanced theory of proportion only introduced later in Book V of the *Elements*.[9]

[8] Joseph Dauben, "The 'Pythagorean theorem' and Chinese Mathematics. Liu Hui's Commentary on the Gou-Gu Theorem in Chapter Nine of the *Jin Zhang Suan Shu*", in: S.S. Demidov, M. Folkerts, D.E. Rowe, and C.J. Scriba, eds., *Amphora; Festschrift für Hans Wussing zu seinem 65. Geburtstag*, Birkhäuser-Verlag, Basel, 1992.

[9] Cf. the chapter on the foundations of geometry for a fuller discussion of this point. Incidentally, the use of the word "equivalent" instead of "equal" could also

Another example of an historian making an inconsequential mathematical error is afforded us by Ivor Grattan-Guinness, but concerns more advanced mathematics. When he discovered some correspondence between Kurt Gödel and Ernst Zermelo concerning the former's famous Incompleteness Theorem, he published it along with some commentary[10]. One comment was that Gödel said his proof was nonconstructive. Now anyone who has read Gödel's original paper can see that the proof is eminently constructive and would doubt that Gödel would say such a thing. And, indeed, he didn't. What Gödel actually wrote to Zermelo was that an alternate proof related to Zermelo's initial criticism was— unlike his published proof— nonconstructive. Grattan-Guiness had simply mistranslated and thereby stated something that was mathematically incorrect.

Occasionally, the disagreement between historian and mathematician can be serious. The most famous example concerns the term "geometric algebra", coined by the Danish mathematician Hieronymus Georg Zeuthen in the 1880s to describe the mathematics in one of the books of the *Elements*. One historian saw in this phrase a violation of basic principles of historiography and proposed its banishment. His suggestion drew a heated response that makes for entertaining reading.[11]

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be considered an error by mathematicians. For, areas being numbers they are either equal or unequal, not equivalent.

[10] I. Grattan-Guinness, "In memoriam Kurt Gödel: his 1931 correspondence with Zermelo on his incompletability theorem", *Historia Mathematica* 6 (1979), pp. 294 - 304.

[11] The initial paper and all its responses appeared in the *Archive for the History of the Exact Sciences.* The first, somewhat polemical paper, "On the need to rewrite the history of Greek mathematics" (vol. 15 (1975/76), pp. 67 - 114) was by Sabetai Unguru of the Department of the History of Science at the University of Oklahoma and about whom I know only this controversy. The respondents were Bartel van der Wærden ("Defence of a 'shocking' point of view", vol. 15 (1975), pp. 199 - 210), Hans Freudenthal ("What is algebra and what has been its history?", vol. 16 (1976/77), pp. 189 - 200), and André Weil ("Who betrayed Euclid", vol. 19 (1978), pp. 91 - 93), big guns all. The Dutch mathematician van der Wærden is particularly famous in the history of science for his book *Science Awakening,* which I quoted from earlier. He also authored the classic textbook on modern algebra, as well as other books on the history of early mathematics. Hans Freudenthal, another Dutch mathematician, was a topologist and a colourful character who didn't mince words in the various disputes he participated in during his life. As to the French André Weil, he was one of the leading mathematicians of the latter half of the 20th century. Regarding his historical qualifications, I cited his history of number theory earlier. Unguru did not wither under the massive assault, but wrote a defence which appeared in a different journal: "History of ancient mathematics; some reflections on the state of the art", *Isis* 20 (1979), pp. 555 - 565. Perhaps the editors of the *Archive* had had enough. Both sides had valid points and the dispute was more a clash of perspectives than anyone making major errors. Unguru's *Isis* paper is worth a read. It may be opaque in spots,

On the subject of the writer's motives, there is always the problem of the writer's ethnic, religious, racial, gender, or even personal pride getting in the way of his or her judgement. The result is overstatement.

In 1992, I picked up a paperback entitled *The Miracle of Islamic Science*[12] by Dr. K. Ajram. As sources on Islamic science are not all that plentiful, I was delighted— until I started reading. Ajram was not content to enumerate Islamic accomplishments, but had to ignore earlier Greek contributions and claim priority for Islam. Amidst a list of the "sciences originated by the muslims" he includes trigonometry, apparently ignorant of Ptolemy, whose work on astronomy beginning with the subject is today known by the name given it by the Arabic astronomers who valued it highly. His attempt to denigrate Copernicus by assigning priority to earlier Islamic astronomers simply misses the point of Copernicus's accomplishments, which was not merely to place the sun in the centre of the solar system— which was in fact already done by Aristarchus centuries before Islam or Islamic science existed, a fact curiously unmentioned by Ajram. Very likely most of his factual data concerning Islamic science is correct, but his enthusiasm makes his work appear so amateurish one cannot be blamed for placing his work in the "unreliable" stack.[13]

Probably the most extreme example of advocacy directing history is the Afrocentrist movement, an attempt to declare black Africa to be the source of all Western Culture. The movement has apparently boosted the morale of Africans embarrassed at their having lagged behind the great civilisations of Europe and Asia. I have not read the works of the Afrocentrists, but if one may judge from the responses to it,[14] [15] emotions must run high. The Afrocentrists have low standards of proof (Example: Socrates was black for i. he was not from Athens, and ii. he had a broad nose.) and any criticism is apparently met with a charge of racism. (Example: the great historian of ancient astronomy, Otto Neugebauer described Egyptian astronomy as "primitive" and had better things to say about Babylonian astronomy. The reason for this was declared by one prominent Afrocentrist to be out and out racial prejudice against black

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and not as much fun to read as the attacks, but it does offer a good discussion of some of the pitfalls in interpreting history.

An even earlier clash between historian and mathematician occurred in the pages of the *Archive* when Freudenthal pulled no punches in his response ("Did Cauchy plagiarize Bolzano?", 7 (1971), pp. 375 - 392) to a paper by Grattan-Guinness ("Bolzano, Cauchy, and the 'new analysis' of the early nineteenth century", 6 (1969/70), pp. 372 - 400).

[12] Knowledge House Publishers, Cedar Rapids, 1992

[13] The referee points out that "the best example of distortion due to nationalist advocacy is early Indian science". I have not looked into this.

[14] Robert Palter, "*Black Athena,* Afrocentrism and the History of Science," *History of Science* 31 (1993), pp. 227 - 287.

[15] Mary Lefkowitz, *Not Out of Africa; How Afrocentrism Became an Excuse to Teach Myth as History,* New Republic Books, New York, 1996.

Egyptians and preference for the white Babylonians. The fact of the greater sophistication and accuracy of the Babylonian practice is irrelevant.)

Let me close with a final comment on an author's agenda. He may be presenting a false picture of history because history is not the point he is trying to get across. Samuel de Fermat's remarks on Bombelli and Viète cited earlier are indications. These two authors had developed techniques the usefulness of which they wanted to demonstrate. Diophantus provided a stock of problems. Their goal was to show how their techniques could solve these problems and others, not to show how Diophantus solved them. In one of my own books, I wanted to discuss Galileo's confusions about infinity. This depended on two volume calculations which he did geometrically. I replaced these by simple applications of the Calculus on the grounds that my readers would be more familiar with the analytic method. The relevant point here was the shared value of the volumes and not how the result was arrived at, just as for Bombelli and Viète the relevant point would have been a convenient list of problems. These are not examples of bad history, because they are not history at all. Ignoring the context and taking them to be history would be the mistake here.

So there we have a discussion of some of the pitfalls in studying the history of mathematics. I hope I haven't convinced anyone that nothing one reads can be taken as true. This is certainly not the case. Even the most unreliable sources have more truth than fiction to them. The problem is to sort out which statements are indeed true. For this course, the best guarantee of the reliability of information is endorsement of the author by a trusted authority (e.g., your teacher). So without further ado, I present the following annotated bibliography.

2 General Reference Works

Encyclopædia Britannica

This is the most complete encyclopædia in the English language. It is very scholarly and generally reliable. However, it does not always include scientific information on scientifically marginal figures.

Although the edition number doesn't seem to change these days, new printings from year to year not only add new articles, but drop some on less popular subjects. It is available in every public library and also online.

Any university worthy of the name will also have the earlier 11th edition, called the "scholar's edition". Historians of science actually prefer the even earlier 9th edition, which is available in the libraries of the better universities. However, many of the science articles of the 9th edition were carried over into the 11th.

Enciclopedia Universal Ilustrada Europeo-Americana

The Spanish encyclopædia originally published in 70 volumes, with a 10 volume appendix, is supplemented each year.
I am in no position to judge its level of scholarship. However, I do note that it seems to have the broadest selection of biographies of any encyclopædia, including, for example, an English biologist I could find no information on anywhere else. In the older volumes especially, birth and death dates are unreliable. These are occasionally corrected in the later supplements.

Great Soviet Encyclopedia, 3rd Edition, MacMillan Inc., New York, 1972 - 1982.

Good source for information on Russian scientists. It is translated volume by volume, and entries are alphabetised in each volume, but not across volumes. Thus, one really needs the index volume or a knowledge of Russian to look things up in it. It is getting old and has been removed from the shelves of those few suburban libraries I used to find it in. Thus one needs a university library to consult it.

3 General Biography

J.C. Poggendorff, *Biographisch-literarisches Handwörterbuch zur Geschichte der exacten Wissenschaften*

This is the granddaddy of scientific biography. Published in the mid-19th century with continuing volumes published as late as 1926, the series received an American *Raubdruck*[16] edition in 1945 and is consequently available in some of the better universities. The entries are mostly short, of the *Who's Who* variety, but the coverage is extensive. Birth and death dates are often in error, occasionally corrected in later volumes.

Allen G. Debus, ed., *World Who's Who of Science; From Antiquity to the Present*

Published in 1968 by the producers of the *Who's Who* books, it contains concise *Who's Who* styled entries on approximately 30000 scientists. Debus is an historian of science and the articles were written by scholars under his direction. Nonetheless, there are numerous incorrect birth and death dates and coordination is lacking as some individuals are given multiple, non-cross-referenced entries under different names.

[16] That is, the copyright was turned over to an American publisher by the US Attorney General as one of the spoils of war.

The preface offers a nice explanation of the difficulties involved in creating a work of this kind and the errors that are inherent in such an undertaking.
I have found the book in some municipal libraries and not in some university libraries.

Charles Gillespie, ed., *Dictionary of Scientific Biography*, Charles Scribner's Sons, New York, 1970 - 1991.

This encyclopædia is the best first place to find information on individual scientists who died before 1972. It consists of 14 volumes of extensive biographical articles written by authorities in the relevant fields, plus a single volume supplement, and an index. Published over the years 1970 - 1980, it was augmented in 1991 by an additional 2 volumes covering those who died before 1981.
The *Dictionary of Scientific Biography* is extremely well researched and most reliable. As to the annoying question of birth and death dates, the only possible error I found is Charles Darwin's birthdate, which disagrees with all other references I've checked, including Darwin's autobiography. I suspect Darwin was in error and all the other sources relied on his memory...
The *Dictionary of Scientific Biography* is available in all university and most local libraries.
A *Biographical Dictionary of Mathematicians* has been culled from the *Dictionary of Scientific Biography* and may interest those who would like to have their own copy, but cannot afford the complete set.

Eric Temple Bell, *Men of Mathematics*, Simon and Schuster, New York, 1937.

First published in 1937, this book is still in print today. It is a popularisation, not a work of scholarship, and Bell gets important facts wrong. However, one does not read Bell for information, but for the sheer pleasure of his impassioned prose.

Julian Lowell Coolidge, *The Mathematics of Great Amateurs,* Oxford University Press, Oxford, 1949.

A Dover paperback edition appeared in 1963, and a new edition edited by Jeremy Gray was published by Oxford University Press in 1990. What makes this book unique are i) the choice of subjects and ii) the mathematical coverage. The subjects are people who were not primarily mathematicians— the philosophers Plato and Pascal, the artists Leonardo da Vinci and Albrecht Dürer, a politician, some aristocrats, a school teacher, and even a theologian. The coverage is unusual in that Coolidge discusses the mathematics of these great amateurs. In the two chapters I read carefully I found errors.

Isaac Asimov, *Asimov's Biographical Encyclopedia of Science and Technology*, Doubleday, New York, 1982.

> This is a one-volume biographical dictionary, not an encyclopædia, with entries chronologically organised.
> One historian expressed horror to me at Asimov's methodology. So he would be an acceptable source as a reference for a term paper, but his use in a thesis would be cause for rejection. The problem is that the task he set for himself is too broad for one man to perform without relying on references far removed from the primary sources.

This list could be endlessly multiplied. There are several small collections like Bell's of chapter-sized biographies of a few mathematicians, as well as several large collections like Asimov's of short entry biographies of numerous mathematicians and scientists. For the most part, one is better off sticking to the *Dictionary of Scientific Biography* or looking for a dedicated biography of the individual one is interested in. That said, I note that works like E.G.R. Taylor's *The Mathematical Practitioners of Tudor and Stuart England* (Cambridge University Press, 1954) and *The Mathematical Practitioners of Hanoverian England* (Cambridge University Press, 1966), with their 3500 mini-biographies and essays on mathematical practice other than pure mathematical research are good sources for understanding the types of uses mathematics was being put to in these periods.

4 General History of Mathematics

Florian Cajori, *History of Mathematics,* Macmillan and Company, New York, 1895.

—, *A History of Elementary Mathematics, with Hints on Methods of Teaching,* The Macmillan Company, New York, 1917.

—, *A History of Mathematical Notations,* 2 volumes, Open Court Publishing Company, Lasalle (Ill), 1928 - 29.

> The earliest of the American produced comprehensive histories of mathematics is Cajori's, which borrowed a lot from Moritz Cantor's monumental four volume work on the subject, including errors. Presumably most of these have been corrected through the subsequent editions. The current edition is a reprint of the 5th published by the American Mathematical Society.
> Cajori's history of elementary mathematics was largely culled from the larger book and is no longer in print.
> Cajori's history of mathematical notation is a cross between a reference work and a narrative. A paperback reprint by Dover Publishing Company exists.

David Eugene Smith, *History of Mathematics,* 2 volumes, 1923, 1925.

—, *A Source Book in Mathematics,*, 1929.

—, *Rara Arithmetica,* Ginn and Company, Boston, 1908.

All three books are in print in inexpensive Dover paperback editions. The first of these was apparently intended as a textbook, or a history for mathematics teachers as it has "topics for discussion" at the end of each chapter. Most of these old histories do not have much actual mathematics in them. The second book complements the first with a collection of excerpts from classic works of mathematics.
Rara Arithmetica is a bibliographic work, describing a number of old mathematics books, which is much more interesting than it sounds.

Eric Temple Bell, *Development of Mathematics,* McGraw-Hill, New York, 1940.

Bell is one of the most popular of American writers on mathematics of the first half of the 20th century and his books are still in print. There is nothing informational in this history to recommend it over the others listed, but his style and prose beat all the rest hands down.

Dirk Struik, *A Concise History of Mathematics,* revised edition, Dover, New York, 1967.

This is considered by some to be the finest short account of the history of mathematics, and it very probably is. However, it is a bit too concise and I think one benefits most in reading it for additional insight after one is already familar with the history of mathematics.

Howard Eves, *An Introduction to the History of Mathematics,* Holt, Rinehart, and Winston, New York, 1953.

Carl Boyer, *A History of Mathematics,* John Wiley and Sons, New York, 1968.

Both books have gone through several editions and, I believe, are still in print. They were written specifically for the class room and included genuine mathematical exercises. Eves peppers his book (at least, the edition I read) with anecdotes that are most entertaining and reveal the "human side" of mathematicians, but add nothing to one's understanding of the development of mathematics. Boyer is much more serious. The first edition was aimed at college juniors and seniors in a post-Sputnik age of higher mathematical expectations; if the current edition has not been watered down, it should be accessible to some seniors and to graduate students. Eves concentrates on elementary mathematics, Boyer on calculus.
Both author's have written other books on the history of mathematics. Of particular interest are Boyer's separate histories of analytic geometry and the calculus.

David M. Burton, *The History of Mathematics; An Introduction,* McGraw-Hill, New York, 1991.

Victor J. Katz, *A History of Mathematics; An Introduction,* Harper Collins, New York, 1993.

> These appear to be the current textbooks of choice for the American market and are both quite good. A publisher's representative for McGraw-Hill informs me Burton's is the best-selling history of mathematics textbook on the market, a claim supported by the fact that, as I write, it has just come out in a 6th edition. Katz is currently in its second edition. One referee counters with, "regardless of sales, Katz is considered the standard textbook at its level by professionals".[17] Another finds Burton "systematically unreliable". I confess to having found a couple of howlers myself.
> Both books have a lot of history, and a lot of mathematical exercises. Katz's book has more mathematics and more advanced mathematics than the other textbooks cited thus far.

Roger Cooke, *The History of Mathematics; A Brief Course*, Wiley Interscience, 1997.

> I haven't seen this book, which is now in its second edition (2005). Cooke has excellent credentials in the history of mathematics and I would not hesitate in recommending his book sight unseen. The first edition was organised geographically or culturally— first the Egyptians, then Mesopotamians, then Greeks, etc. The second edition is organised by topic— number, space, algebra, etc. Both are reported strong on discussing the cultural background to mathematics.

Morris Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York, 1972.

> This is by far the best single-volume history of general mathematics in the English language that I have seen. It covers even advanced mathematical topics and 20th century mathematics. Kline consulted many primary sources and each chapter has its own bibliography.

5 History of Elementary Mathematics

Otto Neugebauer, *The Exact Sciences in Antiquity,* Princeton University Press, Princeton, 1952.

B.L. van der Wærden, *Science Awakening,* Oxford university Press, New York, 1961.

[17] The referee did not say whether these are professional historians, mathematicians, or teachers of the history of mathematics.

These are the classic works on mathematics and astronomy from the Egyptians through the Hellenistic (i.e. post-Alexander) period. Van der Wærden's book contains more mathematics and is especially recommended. It remains in print in a Dover paperback edition.

Lucas N.H. Bunt, Phillip S. Jones, and Jack D. Bedient, *The Historical Roots of Elementary Mathematics,* Prentice Hall, Englewood Cliffs (New Jersey), 1976.

This is a textbook on the subject written for a very general audience, presupposing only high school mathematics. It includes a reasonable number of exercises. A Dover reprint exists.

Asger Aaboe, *Episodes From the Early History of Mathematics,* Mathematical Association of America, 1964.

This slim volume intended for high school students includes expositions of some topics from Babylonian and Greek mathematics. A small number of exercises is included.

Richard Gillings, *Mathematics in the Time of the Pharoahs,* MIT Press, Cambridge (Mass), 1973

This and Gilling's later article on Egyptian mathematics published in the *Dictionary of Scientific Biography* offer the most complete treatments of the subject readily available. It is very readable and exists in an inexpensive Dover paperback edition.

Euclid, *The Elements*

Proclus, *A Commentary on the First Book of Euclid's Elements*, translated by Glenn Morrow, Princeton University Press, Princeton, 1970.

The three most accessible American editions of *The Elements* are Thomas Heath's translation, available in the unannotated *Great Books of the Western World* edition, an unannotated edition published by Green Lion Press, and a super-annotated version published in 3 paperback volumes from Dover. The Dover edition is the recommended version because of the annotations. If one doesn't need or want the annotations, the Green Lion Press edition is the typographically most beautiful of the three and repeats diagrams on successive pages for greater ease of reading. But be warned: Green Lion Press also published an abbreviated outline edition not including the proofs.
Proclus is an important historical document in the history of Greek mathematics for a variety of reasons. Proclus had access to many documents no longer available and is one of our most detailed sources of early Greek geometry. The work is a good example of the commentary that replaced original mathematical work in the later periods of Greek mathematical supremacy. And, of course, it has much to say about Euclid's *Elements.*

Howard Eves, *Great Moments in Mathematics (Before 1650),* Mathematics Association of America, 1980.

> This is a book of short essays on various developments in mathematics up to the eve of the invention of the Calculus (which is covered in a companion volume). It includes historical and mathematical exposition as well as exercises. I find the treatments a bit superficial, but the exercises counter this somewhat.

6 Source Books

A source book is a collection of extracts from primary sources. The first of these, still in print, was Smith's mentioned earlier:

David Eugene Smith, *A Source Book in Mathematics*

At a more popular level is the following classic collection.

James R. Newman, *The World of Mathematics,* Simon and Schuster, New York, 1956.

> This popular 4 volume set contains a wealth of material of historical interest. It is currently available in a paperback edition.

Ivor Thomas, *Selections Illustrating the History of Greek Mathematics, I; Thales to Euclid,* Harvard University Press, Cambridge (Mass), 1939.

—, *Selections Illustrating the History of Greek Mathematics, II; From Aristarchus to Pappus,* Harvard University Press, Cambridge (Mass), 1941.

> These small volumes from the Loeb Classical Library are presented with Greek and English versions on facing pages. There is not a lot, but the assortment of selections was judiciously made.

Morris R. Cohen and I.E. Drabkin, *A Source Book in Greek Science,* Harvard University Press, Cambridge (Mass), 1966.

Edward Grant, *A Source Book in Medieval Science,* Harvard University Press, Cambridge (Mass), 1974.

Dirk Struik, *A Source Book in Mathematics, 1200 - 1800,* Harvard University Press, Cambridge (Mass), 1969.

> In the 1960s and 1970s, Harvard University Press published a number of fine source books in the sciences. The three listed are those most useful for a general course on the history of mathematics. More advanced readings can be found in the specialised source books in analysis and mathematical logic. I believe these are out of print, but I would expect them to be available in any university library.

Ronald Calinger, *Classics of Mathematics,* Moore Publishing Company, Oak Park (Ill), 1982.

For years this was the only general source book for mathematics to include twentieth century mathematics. The book is currently published by Prentice-Hall.

Douglas M. Campbell and John C. Higgins, *Mathematics; People, Problems, Results,* Wadsworth International, Belmont (Cal), 1984.

This three volume set was intended to be an up-to-date replacement for Newman's *World of Mathematics.* It's extracts, however, are from secondary sources rather than from primary sources. Nonetheless it remains of interest.

John Fauvel and Jeremy Gray, *The History of Mathematics; A Reader,* McMillan Education, Ltd, London, 1987.

This is currently published in the US by the Mathematical Association of America. It is probably the nicest of the source books. In addition to extracts from mathematical works, it includes extracts from historical works (e.g., comments on his interpretation of the Ishango bone by its discoverer, and extracts from the debate over Greek geometric algebra) and some cultural artefacts (e.g., Alexander Pope and William Blake on Newton).

Stephen, Hawking, *God Created the Integers; The Mathematical Breakthroughs that Changed History*, Running Press, Philadelphia, 2005.

The blurb on the dust jacket and the title page announce this collection was edited with commentary by Stephen Hawking. More correctly stated, each author's works are preceded by an essay by the renowned physicist titled "His life and work"; explanatory footnotes and, in the case of Euclid's *Elements*, internal commentary are lifted without notice from the sources of the reproduced text. This does not make the book any less valuable, but if one doesn't bear this in mind one might think Hawking is making some statement about our conception of time when one reads the reference (which is actually in Thomas Heath's words) to papers published in 1901 and 1902 as having appeared "in the last few years". Aside from this, it is a fine collection, a judicious choice that includes some twentieth century mathematics with the works of Henri Lebesgue, Kurt Gödel, and Alan Turing.

Jean-Luc Chabert, ed., *A History of Algorithms, From the Pebble to the Microchip,* Springer-Verlag, Berlin, 1999.

Originally published in French in 1994, this is a combination history and source book. I list it under source books rather than special historical topics because of the rich variety of the excerpts included and the breadth of the coverage, all areas of mathematics being subject to algorithmic pursuits.

7 Multiculturalism

George Gheverghese Joseph, *The Crest of the Peacock; Non-European Roots of Mathematics,* Penguin Books, London, 1992.

> A very good account of non-European mathematics which seems to be quite objective and free of overstatement.

Yoshio Mikami, *The Development of Mathematics in China and Japan,* 2nd. ed., Chelsea Publishing Company, New York, 1974.

Joseph Needham, *Science and Civilization in China, III; Mathematics and the Sciences of the Heavens and the Earth,* Cambridge University Press, Cambridge, 1959.

Lǐ Yan and Dù Shíràn, *Chinese Mathematics; A Concise History,* Oxford University Press, Oxford, 1987.

> Mikami's book was first published in German in 1913 and is divided into two parts on Chinese and Japanese mathematics, respectively. Needham's series of massive volumes on the history of science in China is the standard. The third volume covers mathematics, astronomy, geography, and geology and is not as technical as Mikami or the more recent book by Lǐ Yan and Dù Shíràn, for which Needham wrote the Foreword.
> Needham's book is still in print. The other two books are out of print.

David Eugene Smith and Yoshio Mikami, *A History of Japanese Mathematics*

> I haven't seen this book, but in the introductory note of his book on Chinese and Japanese mathematics, Mikami announces that the book was to be written at a more popular level. It is in print in 2 or 3 editions, including a paperback one by Dover.

Seyyed Hossein Nasr, *Science and Civilization in Islam,* Harvard University Press, Cambridge (Mass), 1968.

> Nasr borrowed the title from Needham, but his work is much shorter—only about 350 pages. It does not have much technical detail, and the chapter on mathematics is only some 20 odd pages long. The book is still in print in a paperback edition.

J.L. Berggren, *Episodes in the Mathematics of Medieval Islam,* Springer-Verlag, NY, 1986.

> This appears to be the best source on Islamic mathematics. It even includes exercises. The book is still in print.

8 Arithmetic

Louis Charles Karpinski, *The History of Arithmetic,* Rand McNally and Company, Chicago, 1925.

> This is the classic American study of numeration and computation by hand. It includes history of early number systems, the Hindu-Arabic numerals, and even a brief study of textbooks from Egypt to America and Canada. The book is out of print.

Karl Menninger, *Number Words and Number Symbols; A Cultural History of Mathematics,* MIT Press, Cambridge (Mass), 1969.

> This large volume covers the history of numeration and some aspects of the history of computation, e.g. calculation with an abacus. The book is currently in print by Dover.

9 Geometry

Adrien Marie Legendre, *Geometry*

> One of the earliest rivals to Euclid (1794), this book in English translation was the basis for geometry instruction in United States in the 19th century wherever Euclid was not used. Indeed, there were several translations into English, including a famous one by Thomas Carlisle, usually credited to Sir David Brewster who oversaw the translation. The book is available only through antiquariat book sellers and in some of the older libraries. It is a must have for those interested in the history of geometry teaching in the United States.

Lewis Carroll, *Euclid and His Modern Rivals,* Dover, New York, 1973.

> Originally published in 1879, with a second edition in 1885, this book argues, in dialogue form, against the replacement of Euclid by numerous other then modern geometry textbooks at the elementary level. Carroll, best known for his Alice books, was a mathematician himself and had taught geometry to schoolboys for almost a quarter of a century when he published the book, which has recently been reprinted by Dover.

David Eugene Smith, *The Teaching of Geometry,* Ginn and Company, Boston, 1911.

> This is not a history book *per se,* but it is of historical interest in a couple of ways. First, it includes a brief history of the subject. Second, it gives a view of the teaching of geometry in the United States at the beginning of the twentieth century. It is currently out of print, but might be available in the better university libraries.

Julian Lowell Coolidge, *A History of Geometrical Methods,* Oxford University Press, 1940.

> This is a rather advanced history of the whole of geometry requiring a knowledge of abstract algebra and the calculus. Publication was taken over by Dover in 1963 and it remains in print.

Felix Klein, *Famous Problems of Elementary Geometry,* Dover.

Wilbur Richard Knorr, *The Ancient Tradition of Geometric Problems,* Dover.

> There are several books on the geometrical construction problems and the proofs of their impossibility. Klein was a leading mathematician of the 19th century, noted for his fine expositions. The book cited is a bit dated, but worth looking into. Knorr is a professional historian of mathematics, whence I would expect more interpretation and analysis and less mathematics from him; I haven't seen his book.

Robert Bonola, *Non-Euclidean Geometry,* Open Court Publishing Company, 1912.

> Republished by Dover in 1955 and still in print in this edition, Bonola is the classic history of non-Euclidean geometry. It includes translations of the original works on the subject by János Bolyai and Nikolai Lobachevsky.

Marvin Jay Greenberg, *Euclidean and Non-Euclidean Geometries; Development and History,* W.H. Freeman and Company, San Francisco, 1974.

> This textbook serves both as an introduction to and a history of non-Euclidean geometry. It contains numerous exercises. The book is currently in its third edition and remains in print.

10 Calculus

Carl Boyer, *History of Analytic Geometry,* The Scholar's Bookshelf, Princeton Junction (NJ), 1988.

—, *The History of the Calculus and Its Conceptual Development,* Dover, New York, 1959.

> These are two reprints, the former from articles originally published in the now defunct journal *Scripta Mathematica* in 1956 and the second published in book form in 1949. Both books discuss rather than do mathematics, so one gets the results but not the proofs of a given period.

Margaret L. Baron, *The Origins of the Infinitesimal Calculus,* Pergamon Press, Oxford, 1969.

> This is a mathematically more detailed volume than Boyer.

C.H. Edwards, Jr., *The Historical Development of the Calculus,* Springer-Verlag, New York, 1979.

> This is a yet more mathematically detailed exposition of the history of the calculus complete with exercises and 150 illustrations.

Judith V. Grabiner, *The Origins of Cauchy's Rigorous Calculus,* MIT Press, Cambridge (Mass), 1981.

> Today's formal definitions of limit, convergence, etc. were written by Cauchy. This book discusses the origins of these definitions. Most college students come out of calculus courses with no understanding of these definitions; they can neither explain them nor reproduce them. Hence, one must consider this a history of advanced mathematics.

11 Women in Science

Given the composition of this class[18], I thought these books deserved special mention. Since women in science were a rare occurrence, there are no unifying scientific threads to lend some structure to their history. The common thread is not scientific but social— their struggles to get their feet in the door and to be recognised. From a masculine point of view this "whining" grows tiresome quickly, but the difficulties are not imaginary. I've spoken to female engineering students who told me of professors who announced women would not get good grades in their classes, and Julia Robinson told me that she accepted the honour of being the first woman president of the American Mathematical Society, despite her disinclination to taking the position, because she felt she owed it to other women in mathematics.

Several books take the struggle to compete in a man's world as their main theme. Some of these follow.

H.J. Mozans, *Women in Science, with an Introductory Chapter on Woman's Long Struggle for Things of the Mind,* MIT Press, Cambridge (Mass), 1974.

> This is a facsimile reprint of a book originally published in 1913. I found some factual errors and thought it a bit enthusiastic.

P.G. Abir-Am and D. Outram, *Uneasy Careers and Intimate Lives; Women in Science, 1789 - 1979,* Rutgers University Press, New Brunswick, 1987.

> Publishing information is for the paperback edition. The book is strong on the struggle, but says little about the science done by the women.

[18] Mostly female.

H.M. Pycior, N.G. Stack, and P.G. Abir-Am, *Creative Couples in the Sciences,* Rutgers University Press, New Brunswick, 1996.

The title pretty much says it all. Pycior has written several nice papers on the history of algebra in the 19th century. I am unfamiliar with the credentials of her co-authors, other than, of course, noticing that Abir-Am was co-author of the preceding book.

G. Kass-Simon and Patricia Farnes, eds., *Women of Science; Righting the Record,* Indiana University Press, Bloomington,1990.

This is a collection of articles by different authors on women in various branches of science. The article on mathematics was written by Judy Green and Jeanne LaDuke. Both have doctorates in mathematics, and LaDuke also in history of mathematics. With credentials like that, it is a shame their contribution isn't book-length.

There are a few books dedicated to biographies of women of science in general.

Margaret Alic, *Hypatia's Heritage; A History of Women in Science from Antiquity through the Ninetheenth Century,* Beacon Press, 1986.

Margaret Alic is a molecular biologist who taught courses on the history of women in science, so this narrative ought to be considered fairly authoritative.

Martha J. Bailey, *American Women in Science; A Biographical Dictionary,* ABC-CLIO Inc., Santa Barbara, 1994.

As the title says, this is a biographical dictionary of women scientists—including some still living, but limited to Americans. The entries are all about one two-column page in size, with bibliographic references to ternary sources. The author is a librarian.

Marilyn Bailey Ogilvie, *Women in Science; Antiquity through the Nineteenth Century,* MIT Press, Cambridge (Mass), 1986.

Probably the best all-round dictionary of scientific womens' biography.

Sharon Birch McGrayne, *Nobel Women in Science; Their Lives, Struggles and Momentous Discoveries,* Birch Lane Press, New York, 1993.

There being no Nobel prize in mathematics, this book is only of tangential interest to this course. It features chapter-length biographies of Nobel Prize winning women.

Edna Yost, *Women of Modern Science,* Dodd, Mead and Company, New York, 1959.

The book includes 11 short biographies of women scientists, none of whom were mathematicians.

Lois Barber Arnold, *Four Lives in Science; Womens' Education in the Nineteenth Century,* Schocken Books, New York, 1984.

> This book contains the biographies of 4 relatively obscure women scientists and what they had to go through to acquire their educations and become scientists. Again, none of them were mathematicians.

There are also more specialised collections of biographies of women of mathematics.

Lynn M. Osen, *Women in Mathematics,* MIT Press, Cambridge (Mass), 1974.

> Oft reprinted, this work contains chapter-sized biographies of a number of female mathematicians from Hypatia to Emmy Noether.

Miriam Cooney, ed., *Celebrating Women in Mathematics and Science,* National Council of Teachers of Mathematics, Reston (Virginia), 1996.

> This book is the result of a year-long seminar on women and science involving classroom teachers. The articles are short biographical sketches written by the teachers for middle school and junior high school students. They vary greatly in quality and do not contain a lot of mathematics. The chapter on Florence Nighingale, for example, barely mentions her statistical work and does not even exhibit one of her pie charts. Each chapter is accompanied by a nice woodcut-like illustration.

Charlene Morrow and Teri Perl, *Notable Women in Mathematics; A Biographical Dictionary*, Greenwood Press, Westport (Conn.), 1998.

> This is a collection of biographical essays on 59 women in mathematics from ancient to modern times, the youngest having been born in 1965. The essays were written for the general public and do not go into the mathematics (the papers average 4 to 5 pages in length) but are informative nonetheless. Each essay includes a portrait.

There are quite a few biographies of individual female scientists. Marie Curie is, of course, the most popular subject of such works. In America, Maria Mitchell, the first person to discover a telescopic comet (i.e., one not discernible by the naked eye), is also a popular subject. Florence Nightingale, "the passionate statistician" who believed one could read the will of God through statistics, is the subject of several biographies— that make no mention of her mathematical involvement. Biographies of women of mathematics that unflinchingly acknowledge their mathematical activity include the following.

Maria Dzielska, *Hypatia of Alexandria,* Harvard University Press, 1995.

> This is a very scholarly account of what little is known of the life of Hypatia. It doesn't have too much to say about her mathematics, citing but not reproducing a list of titles of her mathematical works.

Nonetheless, the book is valuable for its debunking a number of myths about the subject.

Doris Langley Moore, *Ada, Countess of Lovelace, Byron's Legitimate Daughter,* John Murray, London, 1977.

Dorothy Stein, *Ada; A Life and a Legacy,* MIT Press, Cambridge (Mass), 1985.

Joan Baum, *The Calculating Passion of Ada Byron,* Archon Books, Hamden (Conn), 1986.

Betty Alexandra Toole, *Ada, the Enchantress of Numbers; A Selection from the Letters of Lord Byron's Daughter and Her Description of the First Computer,* Strawberry Press, Mill Valley (Calif), 1992.

I've not seen Moore's book, but do not recommend it[19]. For one thing, I've read that it includes greater coverage of Ada Byron's mother than of Ada herself. For another, Dorothy Stein, in defending the publication of her own biography of Ada Byron so soon after Moore's, says in ther preface, "... a second biography within a decade, of a figure whose achievement turns out not to deserve the recognition accorded it, requires some justification. My study diverges from Mrs. Moore's in a number of ways. The areas she felt unable to explore— the mathematical, the scientific, and the medical— are central to my treatment". A psychologist with a background in physics and computer science, Stein is the only one of Ada's biographers with the obvious credentials to pass an informed judgment on Ada's scientific prowess. And her judgment is very negative.
The romantic myth of a pretty, young girl pioneering computer science by writing the first ever computer program has proven far too strong to be exploded by the iconoclastic Stein. According to the blurb on the dust jacket, "Unlike recent writers on the Countess of Lovelace, Joan Baum does justice both to Ada and to her genuine contribution to the history of science". Of course, an author cannot be blamed for the hype on the dust jacket and Baum is no doubt innocent of the out and out false assertion that "Ada was the first to see from mechanical drawings that the machine, in theory, could be programmed". "The machine" in question is Babbage's analytical engine and was designed expressly for the purpose of being programmed. In any event, Baum is a professor of English and her mathematical background is not described. Approach this book with extreme caution, if at all.

[19] The referee, whose comments themselves often display a great deal of respect for authority, admonished me for this remark. However, in the real world, one must decide whether or not to expend the effort necessary to consult one more reference. In the present case, Stein's credentials are impeccable, her writing convincing, and her comments say to me that Moore's book contains nothing of interest to me. This suffices for me.

Toole's book consists of correspondence of Ada Byron "narrated and edited" by a woman with a doctorate in education. The editing is fine, but the narration suspect. At one point she describes as sound a young Ada's speculation on flying— by making herself a pair of wings! I for one have seen enough film clips of men falling flat on their faces after strapping on wings to question this evaluation of Ada's childhood daydreams. Approach with caution.

Louis L. Bucciarelli and Nancy Dworsky, *Sophie Germain; An Essay in the History of the Theory of Elasticity,* D. Reidel Publishing Company, Dordrecht, 1980.

This is an excellent account of the strengths and weaknesses of a talented mathematician who lacked the formal education of her contemporaries.

Sofya Kovalevskaya, *A Russian Childhood,* Springer-Verlag, New York, 1978.

Pelageya Kochina, *Love and Mathematics: Sofya Kovalevskaya,* Mir Publishers, Moscow, 1985. (Russian original: 1981.)

Ann Hibler Koblitz, *A Convergence of Lives; Sofia Kovalevskaia: Scientist, Writer, Revolutionary,* Birkhäuser, Boston, 1983.

Roger Cooke, *The Mathematics of Sonya Kovalevskaya,* Springer-Verlag, New York, 1984.

Before Emmy Noether, Sofia Kovalevskaya was the greatest woman mathematician who had ever lived. She was famous in her day in a way unusual for scientists. *A Russian Childhood* is a modern translation by Beatrice Stillman of an autobiographical account of her youth first published in 1889 in Swedish in the guise of a novel and in the same year in Russian. Over the next several years it was translated into French, German, Dutch, Danish, Polish, Czech, and Japanese. Two translations into English appeared in 1895, both published in New York, one by The Century Company and one by Macmillan and Company. Each of these volumes also included its own translation of Charlotte Mittag-Leffler's biography of her. The original translations are described by the new translator as being "riddled with errors", which explains the need for the new edition, which also includes a short autobiographical sketch completing Kovalevskaya's life story and a short account of her work by Kochina, to whom, incidentally, the book is dedicated.

The volumes by Kochina and Koblitz are scholarly works. Kochina was head of the section of mathematical methods at the Institute of Problems of Mechanics of the Soviet Academy of Sciences and is also known for her work in the history of mathematics. Koblitz's areas of expertise are the history of science, Russian intellectual history, and women in science. Both women are peculiarly qualified to write

a biography of Kovalevskaya. Kochina's book actually includes some mathematics.
Cooke's book discusses Kovalevskaya's mathematical work in detail, placing it in historical context. He includes biographical information as well. This book is quite technical and not for the weak at heart.

Auguste Dick, *Emmy Noether, 1882 - 1935,* Birkhäuser, Boston, 1981.

This is a short biography of the greatest woman mathematician to date. It includes three obituaries by such mathematical notables as B.L. van der Wærden, Hermann Weyl, and P.S. Alexandrov.

Tony Morrison, *The Mystery of the Nasca Lines,* Nonesuch Expeditions Ltd., Woodbridge (Suffolk), 1987.

The author is an English man and not the African American poetess (Ton*i*). The Nasca Lines are lines laid out by prehistoric Indians on a high, dry plateau in Peru. The book has much information on these lines and Maria Reiche's studies of them, as well as biographical information on Reiche. Reiche studied mathematics in Germany before moving to Peru and making a study of the lines her life's work. The book has lots of photographs.
The Nasca Lines and Maria Reiche have been the subjects of televised science specials. According to these, her specific astronomical interpretations of the lines are in dispute, but her demonstrations of the utterly simple geometric constructions that can be used to draw the figures accompanying the lines obviate the need to assume them the work of ancient astronauts *à la* Erich von Däniken. I don't recall this being in the book, which I found at a local library.

Constance Reid, *Julia; A Life in Mathematics,* Mathematical Association of America, 1996.

This is a very pleasant little volume on the life and work of Julia Robinson. It contains an "autobiography" actually written by Robinson's sister Constance Reid, as well as three articles on her mathematical work written by her friend Lisl Gaal and her friends and collaborators Martin Davis and Yuri Matijasevich.
Constance Reid has written a number of popular biographies of mathematicians. She is not a mathematician herself, but had access to mathematicians, in particular, Julia Robinson and her husband Raphæl.

12 Miscellaneous Topics

F.N. David, *Games, Gods and Gambling; A History of Probability and Statistical Ideas,* Dover.

Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra,* Dover.

I haven't seen these books which are listed on Dover's website.

Petr Beckmann, *A History of* π*,* Golem Press, 1971.

Lennart Berggren, Jonathan Borwein, and Peter Borwein, *Pi: A Source Book*, Springer-Verlag, New York, 1997.

> This is a delightful book on π, covering everything from early estimates to the attempt by the Indiana legislature to pass a law making the number rational. The book has been republished by St. Martin's Press and is currently in print in a paperback edition by St. Martin's. As the title says, the book by Berggren *et al.* is a source book, consisting of a broad selection of papers on π of varying levels of difficulty. The book is not annotated, several papers in Latin, German and French are untranslated, some of the small print is illegible (too muddy in Lindemann's paper on the transcendence of π and too faint in Weierstrass's simplification), and the reader is left to his or her own devices. Nonetheless, there is plenty of material accessible to most students. The book is currently in its third (2004) edition. There are other books on π, as well as books on e, the golden ratio, and i, but these books are particularly worthy of one's attention.

Elisha S. Loomis, *The Pythagorean Proposition. Its Demonstrations Analyzed and Classified and Bibliography of Sources for Data of the Four Kinds of "Proofs"*, 2nd. ed., Edwards Brothers, Ann Arbor, 1940.

> Originally published in 1927, the book received the endorsement of the National Council of Teachers of Mathematics when it published a reprint in 1972. I've not seen the book, and paraphrase my friend Eckart Menzler-Trott: 370 proofs are analysed in terms of being algebraic (109), geometric (255), quaternionic (4), or dynamic (2). He jokingly states that it is the Holy Book of esoteric Pythagoreans, having got the book through a religious web site. Indeed, I myself purchased a couple of biographies of Pythagoras at a religious bookstore, and not at a scientific bookseller's.

13 Special Mention

Anon, ed., *Historical Topics for the Mathematics Classroom,* National Council of Teachers of Mathematics, Washington, D.C., 1969.

> This is a collection of chapters on various topics (numbers, computation, geometry, etc. up to and including calculus) with historical information on various aspects of these topics. The discussions do not include a lot of mathematical detail (e.g., it gives the definition and graph of the quadratrix, but does not derive the equations and show how to square the circle with it). Nonetheless, if one is interested in

using history in the classroom, it is a good place to start looking for ideas on just how to do so.

Ludwig Darmstædter, *Handbuch zur Geschichte der Naturwissenschaften und der Technik,* 2nd enlarged edition, Springer-Verlag, Berlin, 1908.

Claire L. Parkinson, *Breakthroughs; A Chronology of Great Achievements in Science and Mathematics 1200 - 1930,* GK Hall and Company, Boston, 1985.

Darmstædter's book is a carefully researched 1070 page chronology of all of science up to 1908. It exists in an authorised reprint by Kraus Reprint Co., Millwood, NY, 1978, and thus ought to be in any respectable American university library.
Parkinson's book is a modern replacement for Darmstædter's. It brings one a bit more up to date, but starts a lot later. Parkinson compiled her dates from more secondary sources than did Darmstædter, and her book is probably best viewed more as a popularisation than as a scholarly reference work. On the other hand, her book does have an extensive bibliography, which Darmstædter's does not. Moreover, neither book is illustrated and they ought not to be confused with some more recent coffee table publications on the subject.

Chronologies are not all that useful, a fact possibly first made manifest by the failure of Darmstædter's massive effort to have had an effect on the history of science[20]. One limitation of the usefulness of such a volume is the breadth of coverage for a fixed number of pages: more exhaustive coverage means shorter entries. For example, we read in Darmstædter that in 1872 Georg Cantor founded "the mathematical theory of manifolds (theory of point sets)", i.e. Cantor founded set theory in 1872. What does this mean? Is this when Cantor started his studies of set theory, when he published his first paper on the subject, a date by which he had most of the elements of the theory in place, or...? Similarly, we read that in 250 A.D., "Diophantus of Alexandria freed arithmetic from the bonds of geometry and founded a new arithmetic and algebra on the Egyptian model". What does this mean? How does the algebra founded by Diophantus compare with the geometric algebra of Euclid, the later algebra of al-Khwarezmi, or the "letter calculus"[21] by Viète in 1580? To answer these questions, one must go elsewhere.

Another problem concerns events we do not have the exact dates of. In compiling a chronology, does one include only those events one can date exactly, or does one give best guesses for the uncertain ones? Darmstædter has done the latter, as evidenced by his dating of Diophantus at 250 AD. Unfortunately, he did not write "c. 250" to indicate this to be only an approximation. Is the year 1872 cited for Cantor an exact date or an estimate, perhaps a

[20] Helge Krogh, *An Introduction to the Historiography of Science*, Cambridge University Press, Cambridge, 1987, pp. 17 and 175.
[21] I.e., the use of letters as variables.

midpoint in Cantor's career? Once again one has to look elsewhere for the answer.

And, of course, a problem with Darmstædter or any older reference is that later research may allow us to place questionably dated events more exactly in time. It may uncover events that were unknown and thus left out of the chronology. Darmstædter himself cites the 1906 discovery of *The Method* of Archimedes by J.L. Heiberg, a result that Darmstædter could not have included in his first edition. And such research may correct other errors: Darmstædter cites Euclid of Megara as the author of the *Elements*, a common misidentification we discussed in Chapter 1.

Ivor Grattan-Guinness, ed., *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences,* 2 vols., Routledge, London and New York, 1992.

> With over 1700 pages not counting the end matter, these two volumes give a very broad but shallow coverage of the whole of mathematics. It has a useful annotated bibliography as well as a chronology.

Augustus de Morgan, *A Budget of Paradoxes,* 2 vols., 2nd ed., Open Court Publishing Company, Chicago and London, 1915.

> The first edition was published in 1872, edited by de Morgan's widow Sophia. The second edition was edited by David Eugene Smith. Some later printings of the second edition appeared unter the title *An Encyclopædia of Eccentrics.*
> The book is an amazing bit of odds and ends— anecdotes, opinion pieces, and even short reviews, some dealing with mathematical subjects and some not. Smith's description of the work as a "curious medley" and reference to its "delicious satire" sum it up nicely.

14 Philately

In connexion with the final chapter of this book, I cite a few references on mathematics and science on stamps.

W.J. Bishop and N.M. Matheson, *Medicine and Science in Postage Stamps*, Harvey and Blythe Ltd., London, 1948.

> This slim volume written by a librarian of a medical museum and a surgeon contains a short 16 page essay, 32 pages of plates sporting 3 to 6 stamps each, a 3 page bibliography, and 23 pages of mini-biographies of the physicians and scientists depicted on the stamps, together with years of issue and face values of the stamps.

R.W. Truman, *Science Stamps*, American Topical Association, Milwaukee (Wisconsin), 1975.

This is a rambling account of science on stamps with 7 pages densely covered with images of stamps. The text is divided into three parts, the first being split into shorter chapters on Physicists, Chemistry, Natural History, Medicine, and Inventors; the second with special chapters on The Curies, Louis Pasteur, Alexander von Humboldt, Albert Schweitzer, and Leonardo da Vinci; and the third with chapters on the stamps of Poland, France, Germany, Russia, and Italy. There are also checklists of stamps by name, country, and scientific discipline. These checklists give years of issue and catalogue numbers from the American *Scott* postage stamp catalogue.

William L. Schaaf, *Mathematics and Science; An Adventure in Postage Stamps*, National Council of Teachers of Mathematics, Reston (Virginia), 1978.

This is the earliest book in my collection to deal primarily with mathematics. It has a narrative history of mathematics illustrated by postage stamps, some reproduced in colour. This is supplemented by two checklists, one by scientist and one by subject. The checklists are based on the American catalogue.

Peter Schreiber, *Die Mathematik und ihre Geschichte im Spiegel der Philatelie*, B.G. Teubner, Leipzig, 1980.

This slim East German paperback contains the customary short history of mathematics illustrated by 16 pages of not especially well printed colour plates featuring about a dozen stamps each. Its checklist is by country, but there is a name index that allows one to look up an individual. The checklist is based on the East German *Lipsius* catalogue, which is quite rare. When I visited the library of the American Philatelic Association some years ago, they didn't have a complete catalogue.

Robert L. Weber, *Physics on Stamps*, A. S. Barnes and Company, Inc., San Diego, 1980.

This book concerns physics, not mathematics, but some of the stamps are of mathematical interest. The book makes no attempt at complete coverage. There is no checklist and the narrative is a sequence of topical essays illustrated by postage stamps, all in black and white despite the promise of colour on the blurb on the dust jacket.

Hans Wussing and Horst Remane, *Wissenschaftsgeschichte en Miniature*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1989

This very attractive volume consists of essays on the development of science through the ages, each page illustrated with 4 to 6 stamps. Wussing is both an historian and a philatelist.

Robin J. Wilson, *Stamping Through Mathematics*, Springer-Verlag, New York, 2001.

This slim volume consists of a collection of short 1 page essays on various topics in the history of mathematics, each illustrated by a page of 6 to 8 beautiful oversize colour reproductions of appropriate stamps.