

Chapter 2

School Mathematics As A Developmental Activity¹

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Abstract: This chapter points out that one of the purposes of having mathematics as a school subject is that it can contribute directly to learners' development of higher psychological functions, and hence to the development of their identity as mature people. It draws attention to the dangers of too narrow an interpretation of situated learning, and makes the case for mathematics in the school context being seen as having a deeper psychological effect than that of acquiring mathematical instruments to solve problems close to life. Rather, activity theory, with its different levels of operations, tasks and complex activities, is shown to enable mathematics in school to be seen as potentially contributing to the development of thinking, motivation and identity.

Key words: epistemology in mathematics education, situated learning, activity theory, 'intellectualisation', cognitive development

1. INTRODUCTION

In this chapter I use an activity theory perspective to draw attention to features of school mathematics which have exceptional developmental potential. My interest is in the role of mathematics within the school curriculum, and this viewpoint highlights some limitations of a situated approach to mathematics learning.

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The Czech Republic has recently approved a new A-level examination (Baccalaureate) in which mathematics has been dropped as a compulsory subject for the first time. This was the result of emotive resistance against compulsory examination by the public and by politicians. In discussions about school curricula, many protagonists imagine mathematics to be an almost emblematic example of school education detached from life. It is still seen by many to be a highly abstract exercise of the mind that serves to classify children as ‘talented’ or not, and which does not prepare children for anything useful which may serve them in their later life – perhaps with the exception of simple calculations similar to everyday protoarithmetic. Recent discussions amongst psychologists, among others, show that the relation between mathematics at school and its influence on the mental development of the individual (child) is far from understood. Various implicit epistemologies of mathematics are shared by didacticians and teachers, and transmitted, through teaching, to pupils and, indirectly, to their parents. In the Czech context, as in many other contexts worldwide, parents, politicians, students, teachers and psychologists argue about the role of mathematics in the school curriculum from different epistemological standpoints, and these each have different consequences for the conception of mathematics in school.

What I want to deal with first are the basic epistemological approaches inherent in educational work in school. Those approaches reveal different answers to essential questions: what is mathematics? Or: what does it mean to be ‘doing mathematics’?

In the recent past this type of implicit questioning gave rise to an often shared answer, namely: to be an efficient mathematics teacher/learner presupposes engaging in active methods, taking a constructivist view of learning, and understanding learning to be situated in particular contexts. Only then do mathematics and the knowledge it communicates make sense to the child. The idea of the child as an active sense-making individual within a social context, for instance engaged in solving problems or in mathematical games, has undoubtedly contributed to the history of teaching the discipline.

Nevertheless, I am going to attempt to show the limits of this approach. The idea that all learning is situated has been interpreted widely to imply that learning mathematics either needs to be based in everyday contexts, or is about recognizing its utility in a range of situations. The activity theory perspective of Leontiev, developed later by others (for example Clot’s work-activity analysis, 1999) reveals the structure of cognitive activity in which mathematical concepts represent the tools to resolve specific tasks. But it goes further than mere utility; it also makes it possible to distinguish an instrumental ‘managing of the situation’ school mathematics task from the

educational shift in which the apprehension of mathematical terms has contributed to the development of mental functions, and mental structures, and of the whole personality. This potential effect of learning mathematics in school is consistently underplayed in the implementation of the situated perspective, and also in attempts to apply Vygotskian theories to mathematics teaching and learning.

2. IMPLICIT EPISTEMOLOGY: WHAT DOES IT MEAN ‘TO BE DOING MATHEMATICS’?

Charlot (1991a) analyzed three implicit epistemologies close to mathematics. It is well known that the most ancient epistemological conception of mathematics is the Platonic version of a certain ‘*celestial mathematics*’ (Desanti, 1968). This conception is based on the idea that mathematical forms pre-exist the grasp of a mathematician, as if they exist ‘in themselves’. This is a widespread conception not only in general but also among teachers; mathematics, they believe, is to be taught and learnt as coming to know universal truths and structures. Mathematical ideas are seen as pure and evident, and the mathematician (and the mathematics teacher) discovers them (their relations, structure, etc.). This world of mathematical ideas is basically independent of the mathematician’s own activities; it is transcendent, and it is accessible by perception and contemplation. The French epistemologist René Thom (1974) says that according to this conception, mathematical structures are not only independent of humans, but people also have only an incomplete and fragmentary notion of them. In this view the task of school education consists of the teacher presenting the world of mathematical ideas with maximum clarity and assists the pupil in mastering the principles of abstract thought. The metaphors of light and perception used by Plato, where the pupil’s mind stands for the ‘eye of the soul’, are still embedded in much mathematical pedagogical discourse. This implicit epistemological conception is the foundation of the so-called traditional education which focuses mainly on exposition followed by exercises, but can also be seen in exploratory tasks which are designed to lead to ‘discovery’ of mathematical ‘truths’ in the same way as a telescope guides discovery of planets. Memorisation and application of procedures are required to accumulate and enact such truths.

Another influential conception of mathematics may be described as ‘*terrestrial*’. It does not presuppose the existence of transcendent autonomous mathematical entities. Mathematical knowledge is seen only to reflect the structure of the natural and perhaps even the social world. The mathematician does not contemplate independent abstract entities; on the

contrary, he abstracts the ideal, mathematical, structure of the world from the world itself. Again, mathematics exists outside of the individual, yet as a structure that he has to extract, not in the form of independent ideas. It is not transcendent but immanent. This implicit epistemological conception is the foundation of reformist education, i.e. pedagogy which endeavours to make the child discover mathematics above all (or only) by manipulation of particular mathematical ‘objects’. Great emphasis is therefore put on the ‘use’ of mathematics in various practical situations. The child is thus shown that (a) mathematics is useful, i.e. can serve a purpose in practical life and that (b) mathematical concepts, laws and structures exist, have a rationality of their own and that it is important to learn to operate with this rationality as the authorities can do. In this case, doing mathematics means to rediscover that which is already given. Yet, this time, analytic manipulation, rather than perception, is both the method of discovery and also the method of using what has been understood.

The third conception of mathematics can be described as ‘*instrumental*’ – mathematical knowledge represents tools which serve the solution of problem situations. Mathematics does not pre-exist either in the skies or hidden in the world around us. To do mathematics is not to discover but to create. The main conclusion is that mathematics is a historical creation by particular people under certain conditions, by people who themselves sought answers to particular problems. In this conception, mathematical activity consists in the generation of particular instrumental operations and, at the same time, in the establishment of a certain field of operations, of their interconnected network. (Note that here I use ‘instrumental’ in a broader sense than that used by Skemp (1976). His use was limited to the use of given tools, where mine implies a need to create, adapt, and interconnect a toolkit, this including relational understandings.)

This epistemological conception is the basis of education which relies methodologically on the belief that learning is the result of a successful demonstration that mathematical knowledge serves as a tool in the solution of initial problem situations. Such situations need not always be concrete and based on everyday experience. It is assumed that learning is related to the pupil’s invention of a concept or of a rule which makes it possible to find a solution for such a situation. At the same time, the child does not have complete freedom to create *any* thing, for the situations in question need to have a potential for the creation of mathematical instruments, need to display inner normativity, or need to display constraints on the activity that can be performed in the situation. The child cannot therefore simply play or disrespect the limits of the situation.

Furthermore, as Charlot points out (1991b), the metaphor of light and vision related to perception (‘to see a solution’, ‘to clarify the assignment’)

leads us to fairly unproductive schemes of interpretation. For instance, to explain why some ‘see the light’ and others do not it is normal to assume gifts and talents, be it in biological terms: ‘he’s got a genius for it’ or ‘she is a mathematics prodigy’; or socio-cultural: ‘he lacks the cultural capital of the abstract code’. The perception metaphor of light and vision gives us no mechanism to deal with those who ‘do not see’. It is a positive feature of more active metaphors of learning and, above all, the instrumental conception of mathematics, that learning is related to mental work or activity, compared to the deterministic interpretations of ‘talents’ and ‘capital’. The activity metaphor includes both the activity of mathematicians in history in particular situations which they had to resolve, and the activity of the child during the learning process.

In recent decades, the changing views of mathematics, and subsequently of the teaching/learning of the discipline, have led to the dominant Platonic epistemology being increasingly complemented by play-oriented active methods and, occasionally, by instrumental or constructivist conception using situations close to everyday life. The idea that to learn mathematics means ‘to be doing it’, i.e. to create, produce, and make mathematical concepts and procedures as tools for the resolution of tasks and problem situations, is now generally recognized. However, this acceptance is mainly in the discourse of didacticians and mathematicians. In schools, the application of this notion can be hesitant and often fails.

Rather than attribute such failure only to the inability of teachers to use these methods effectively, this should lead us to consider whether learning mathematical terms and techniques, which is still the aim of most education systems and assessment regimes, in problem situations that are modelled after everyday experience is the most efficient procedure. If the goal of education is to introduce learners to formalized mathematics, should it not rather be the task of teachers to underline the specificity of formalized mathematics as opposed to the situated methods which arise in everyday mathematics? Whatever else schools are for, one goal of school socialization in the cognitive domain in general is to initiate the child into an intellectual activity and to contribute through this to the development of mental functions, mental structures, and hence to the development of the child’s personality in ways which would not be possible without school.

Furthermore, we are led to consider whether ‘activity’ or rather, cognitive activity, should not deserve a more differentiated analysis than that suggested by the three conceptions above. The history of relative failure of teaching suggests that learning mathematics is a complexly compounded activity which may encompass both the memorizing of definitions and routines, as in the traditional view, and also the difficult formulations of hypotheses in problematic situations, as in the reform view.

In my search for answers, I rely above all on the cultural-psychological tradition of Vygotsky, and the activity theory of Leontiev.

3. LEARNING CLOSE TO PRACTICAL CONTEXTS AND SITUATIONS

After years of domination by an individual-psychological approach to cognition and learning, the last few decades have seen renewed interest in the socio-cultural character of human cognition and of mental development in general. Much of this renewed interest has been due to, and reflected in, the availability of works in many languages influenced by Vygotsky.

This emphasis has risen remarkably in prominence since the 1980's. It draws on earlier inspirations: the unachieved work of Vygotsky from 1925-1934, followed by the work of Luria and Leontiev. Unfortunately, these were published relatively late and translated into foreign languages only from the 1980's onwards, remaining virtually unknown till then. In the 1970's and 1980's cultural anthropological research and theoretical work in the field of intercultural psychology began to focus on the influence of formal schooling on the mental development and ways of thinking of people within traditional cultures in Africa and other parts of the developing world. Cole, Gay, Glick, and Sharp, (1971); Scribner and Cole (1981); Lave (1977, 1988) and above all Cole (1996), an admirer and indirectly the pupil of Luria, are particularly identified with this shift in focus.

This led to a boom of literature about so-called situated or distributed learning. Significantly, this translates into French as 'learning in context' (*apprentissage en contexte*), a somehow inaccurate expression, but one which makes explicit reference to an important dimension of situated learning, that of context, which in turn reminds us of the other necessary term of the relation, 'text', making salient the 'text – context' relationship.

This turn towards situated learning, towards forms of cognition and learning in practical situations (of Lave's Liberian tailors; of seafarers in the Pacific), towards learning in practice (e.g. everyday arithmetic in the research conducted by Scribner (1986) and by Rogoff, (1990)) led to a full appreciation of cognition as a set of cultural practices. At the same time, it may have led to the overestimation of this form of learning at the expense of the importance and function of school forms of cognition and learning. In conjunction with the reviving educational reformism and a return to student-centredness, this, in my perception of post-communist countries during the 1990's, led to the overall negation of the developmental significance of school forms of cognition. Situated learning in contexts of practical life of the individual was placed on a pedestal, almost as a model for learning at

school, in some educational discourse. Active and reformist (student-centred) conceptions of teaching/learning are strongly nurtured by this conception.

I shall now show that there is a substantial difference between situated learning, i.e. learning in the extra-curricular, everyday (e.g. family) context, and school learning, which was described by Vygotsky in *Thought and Language* (1976) as learning 'scientific' concepts.

Analyses of situated learning had the power to re-orient those educational conceptions which were still under a strong influence of the individual-cognitivist tradition. What do these analyses stand for? The pivotal idea is that learning, apprehending knowledge, can only be construed in a 'situation', and is dependent on the pupil's participation in social and material contexts, the person and his/her world being mutually constitutive. This idea underlies, according to Moro (2002), the following theories: learning as *apprenticeship* associated with the works of Lave (1977, 1988) and Lave and Wenger (1991); learning as *guided participation* associated with the theoretical work by Rogoff (1990) and learning in the person-tool(s) system usually described as distributed learning, associated with the names of Hutchins studying pilots in a cockpit or subway dispatchers in work, (1995, 1990) and Resnick (1987).

All theories of situated learning redirect our attention towards the analysis of the situations in which learning takes place. Each in its own way puts the emphasis on one or other of the elements of Vygotsky's cultural-historical approach towards psychological functions, namely the prime importance of social activities; the inter-psychological nature of psychological functions; the key importance of mediation and the role of the adult-expert; and the formative effect of the artefact-tool. Thanks to these theories, Leontiev's concept of activity and the importance of the unit of analysis in examining psychological phenomena become prominent. What, then, is the problem? Why not merely arrange the theories of apprenticeship, guided participation, and learning in person-tool systems within the socially-mediated approach to learning and make use of them at school, seen as a social situation?

First of all, it is necessary to note that these theories:

(1) localize the dynamic of learning almost predominantly into the world of everyday experience and neglect the importance of activities provided and made necessary by the school, i.e. of activities directed specifically at reflection and abstraction. Thus, they hinder investigations into the differences and tension between an item of knowledge in its everyday form and one which is formalized, and therefore bypass the decisive moment of the cognitive and personal development of the individual.

(2) overestimate the formative influence of artefacts and situational configurations on mental functions, as if these were embodied in tools. This is because they fail to distinguish between the capacity to operate in context on the basis of the tool, and the mental work of an individual transforming particular psychological functions.

(3) fail to dispel the impression that in their psychology of situations the psyche in fact belongs to situations, thus only mechanically transposing mental *gestalts* which are originally localized in the minds of individuals into the situations.

These objections need to be overcome to fully appreciate the role of school in mathematical learning.

4. LEARNING IN THE SCHOOL CONTEXT

The theory of learning in everyday practical contexts differs significantly from the approach of the Vygotskian school in its conception of the unit of analysis and in its conception of mediation. Along with Leontiev, in using the term *unit of analysis* I refer to the isolation of units of enquiry which enable the objectification of psychological facts in their inter- and intra-psychological dimensions. ‘Participation in apprenticeship’ can help grasp activities in the socio-cultural framework and can substitute for the mechanical understanding of internalization; however, the nature of the *intra*-individual activity itself largely escapes it. On the other hand, mediation is considered by Lave and Rogoff above all as communication between individuals, and the prospective zone of proximal development as a communicative-relational network. The cognitive activity itself, that is the apprehension of knowledge *qua* apprehension of norms of activities with a given item of knowledge, is left aside. Similarly, Hutchins’ treatment of ‘tool’ in the pilot’s cockpit is admittedly instrumental and mediating; however, it is not what Vygotsky meant by a psychological tool, since it cannot demonstrate how permanent transformation of psychological functions and the development of the individual come about. For Vygotsky, the use of psychological tools transforms the psychological and intellectual functions of individuals. Finally, many users of situated cognition theory are insensitive to the fact that learning at school is also learning in a context with its own specificities, a context which represents a community of practices largely derived from a concept of scientific knowledge. A comparison with extra-curricular contexts makes it evident that the objective of school is epistemic. It aims at the transformation of modes of thinking, of experiencing, and of the self. This requires a clear conception of the relations between spontaneous learning (the kind of learning we do, and what it is we

learn, in everyday contexts) and education, formal learning and development. What are, in Vygotsky's terms, the main differences between apprehending spontaneous concepts and those which are scientific (acquired mainly at school)?

4.1 Utilitarian vs. epistemic attitude to the world and to language

We will be helped by thinking about the classic comparison between the apprehension of spoken language and language learning at school with the support of writing.

Formalized learning can start where spontaneous learning in contexts of everyday life reaches a limit of what is possible. Spontaneous learning stands on *instrumental* usage, knowing how to say something; for example, knowing how the notion 'brother' works, or who is a particular brother, to make oneself understood. Bourdieu (1996) says that in practical action the word used fits the situation. Formalized learning paves the way for *reflection* and builds on it. This leads to knowing why something can/cannot be said in a particular way; what is essential about the structures of 'kinship' and why a 'sister' is the same as a brother according to the law of language, even if this is sheer nonsense in the context of everyday usage.

Although formalized learning of decontextualized 'scientific' knowledge makes use of spontaneous learning (and indeed is based on it), the important thing is that it transforms the substance of the knowledge thus acquired. Due to formal learning and its tendency to decontextualize, the child is brought to reflect upon and realize the specificities of language, and to the necessary generalization of linguistic phenomena. By means of this new attitude towards language, the child's attitude towards the world changes into one which is epistemic and not merely practical. This in turn opens new horizons in other domains of knowledge.

Olson and Torrance (1983) introduce another striking criterion. In their view, both the *context* and the *text* are available to people in their practical attitude to the world. But the situation of spontaneous learning forces them to give priority to information from the context, that is to rely on what is most probable in the given context. Olson and Torrance cite the following example:

They observe that according to classical Piagetian tests children up to 8 years of age understand instructions contextually (and proceed in their thoughts on the basis of such understanding). The critique of these tests features the classical example of a logical 'sub-class/class' relation (there are 9 flowers in the picture, 6 of them tulips and 3 roses). The question is: 'Are there more tulips or more flowers in the picture?' Children answer on the

basis of comparing the sub-class 'tulips' with the sub-class 'roses' and conclude that there are more tulips than flowers. Olson points out that children answer not on the basis of text but depending on the context, i.e. on their everyday experience and act as is common in such contexts. We usually compare sets of the same kind or level. For example in everyday life, we might ask if there are more girls than boys in a class, but, as Brossard reminds us (2004), we rarely ask if there are more girls than pupils in a class. The child is thus guided by the context and not by the linguistic contents of the question and its logical structure, i.e. the 'text'. To follow the text, the child must undergo another type of learning than the more or less 'spontaneous' reaction recorded by Piaget.

This observation is especially important in mathematics, where questions designed to elicit techniques or applications of particular facts are often phrased in ways which relate very little to any well known context, even when a pseudo-everyday context is being cited. But the problem goes much deeper than that. At school, meanings and interpretations are not merely practised; writers and readers engage in reflection on meanings themselves. The processes of learning written knowledge are thus the decisive factor in the change of ways of thinking. Olson (1994) cites a Vygotskian distinction to that effect, namely that, thanks to writing, we have moved from thinking about things to thinking about the representations of things. Vygotsky himself says that spontaneous notions are generalizations about things, while scientific concepts are generalizations of these generalizations. This is what Vygotsky, in *Thought and Language*, describes as the key effect of school teaching/learning (1976). School education brings about (a) a rupture in and (b) the 'intellectualisation' of mental functions.

What does this rupture consist of? The aim of spontaneous everyday learning is to deal with a practical situation in life. The child that enters school has thus already mastered some knowledge, say in arithmetic. This is proto-arithmetic knowledge: he/she can divide marbles into two even parts, knows how many people there are in the family, can compare his/her own age to that of a sibling, can add and subtract from the number of objects and so on. At school, this spontaneous knowledge serves as a basis for the child to develop real operations of addition and subtraction with the help of a teacher; the child constructs (abstracts, rather than extracts) numerical properties of empirical objects. Whether we deal with marbles, apples or books is of no importance; in any situation, it is true that $2 + 1 = 3$ and $3 - 2 = 1$. The child performs a decontextualization based on generalization as an empirical abstraction of the concept of quantity. According to Vygotsky, this is the above-mentioned generalization of a lower order, a 'generalization about things'. Yet, arithmetic operations do not lie in (are not immanent to) the empirical situation, they are not additional properties of objects (besides

colour or size, say). They are necessary non-empirical operations that the child must perform and these become the object of its learning at school.

However, an important breaking point occurs when, by virtue of these operations, the child discovers the properties of the decimal system (Brossard, 2004). At a certain stage of development, and depending on the school curriculum, the child begins to understand how the decimal system fulfils its purpose and how it works, and may even understand that it is also possible to count using other numerical systems (binary or other systems). From then on, the child understands the decimal system as a *particular instance* of other possible numerical systems, and that therefore it must have certain properties which, when understood, generate the whole system. This is a generalization of a higher order. This is the generalization of generalizations, a generalization based only on the relations between numerical entities.

4.2 Unreflected, or not consciously developed vs. planned and conscious procedure

The mastery of systems of higher generalizations makes it possible to make a relatively permanent developmental shift away from *particular tasks and situations* and, at the same time, to *realize* not only some particular forms of knowledge but also to realize an awareness of *one's own mental processes* and of oneself. This is exactly what Vygotsky calls the 'intellectualisation' of mental functions: it sets in when the mental function becomes dependent on the idea (concept) or is subordinate to it.

The example of intellectualisation of memory and of the relation between thought and memory is well known. A small child thinks by remembering. His/her representations of things and of ways of handling them are not conscious and organized systematically around a certain idea or concept. An older child or a teenager already remembers and recollects by (and thanks to) thought. The intellectualisation of memory consists in the organization of knowledge for the purpose of remembrance. The child thus increasingly works consciously and deliberately on his/her own memory processes. From a certain point on, the relationship between memory and intellect gets reversed. The introduction of conscious and planned (volitional) relations of the child towards his/her own mental processes is what cultural psychologists perceive as the criterion of a higher level of development.

It is valid universally that the emergence or discovery of the *relations of a higher generality* between concepts is the critical point (motor) of mental development. Situated perspectives cannot explain, nor do they have need of, intellectual relations at this level except by recognizing them as discursive patterns.

A remarkable geographical metaphor of Vygotsky's makes it possible to describe the concept as a geographical point at the longitude and latitude intersection, as Brossard (2004) points out. The 'longitude' of the concept determines its place on the meridian leading from the most concrete to the most general meaning. The 'latitude' of the concept then represents the point which it takes in relation to other concepts of equal 'longitude' (of equal generality) but relating to other points of reality. The combination of both key characteristics of the concept determines the extent of its generality. It is given not only by the concrete/abstract scale but also by the richness of connections to other concepts of the given conceptual network which form the domain in question.

Let us demonstrate this global metaphor of intellectualisation in relation to connections between arithmetic and algebra. The result of the operational development so far, e.g. the operations of addition and subtraction, becomes the 'source' of new processes, algebraic operations with variables and unknown quantities. The performance of a thought operation (to define, compare, factor out, divide etc.) presupposes the establishment of relations between various concepts within the corresponding conceptual system. A six year old child cannot 'define' an operator or a straight line, for instance, because the terms that he/she masters are not in relations of sufficient richness to other concepts. If, however, the child masters operations of the decimal system, an infinite number of means to express a concept, for instance of the number 'four', are available to him/her ($2 + 2$; $8 - 4$; $16 \div 4$ etc.). The concept of a higher order of generalization thus represents a point which makes possible several ways forward within the entire 'global' system.

School education plays a decisive part in this process of transformation of mental functioning. When learning 'close to everyday life' the child observes, discovers, considers, argues, and so on (Brossard says, that he/she 'coincides with the significations he/she practices' (2004)). It is due to school education that, along with all of this, the child also focuses his/her attention on mental processes which he/she performs when observing, discovering, considering etc. The child works on 'pure meanings' which are the main object of his/her reflection. Thus, the ability to define the number 'four' in several different ways involving various operators and their combinations necessarily places comparative reflection, the analysis of one's own attention and memory, knowledge about one's efficiency, and so on at the forefront. Such processes would never come about if the child were struggling with ignorance and the absence of automatic fluency with the elementary operator.

However, the child is very unlikely to reach this reflective activity 'spontaneously'. It requires a teacher, a plan, a logic of the curriculum and of

the teaching process, a programme which is at first only external to the child. Mathematical concepts of a higher level of generality are especially distinguished by the necessity to introduce them from the outside; these are 'top-down' conceptualizations. Intellectualisation stands on an increasing subordination of individual operations to the higher organizational principle (with the two characteristics expounded in the above-mentioned geographical metaphor). From this point of view, mathematics represents activities in which, with a growing generality of a concept, the motive of the introduction of the concept is always 'external' in respect to the child and his/her 'spontaneous interest'. The 'new' conscious learning at school is guided by the requirements of the curriculum, or by the object of the cognitive activity. The pupil studies the 'programme' to learn a type of thinking whose observance is guaranteed, to some extent, by the institution of the school and the teacher. If we put this in Olson's terms, the 'textual' approach is exercised at school, sometimes with success, sometimes less so, in an approach which is supervised, systematic and planned. School mathematics which is supposed to fulfil its developmental psychological function must provoke that which is seen to be of greatest value: tension between various levels of conceptualization (the development level achieved by the pupil to date vs. the elaborate form of conceptualization constructed in a didactic school situation in co-operation with a teacher). Brossard (2004) talks about the internal motor of development acting alongside the external (socially motivational) motor.

While this is true to some extent in all school subjects, it is especially important in mathematics, since, as the Vygotskian school recognized, an advanced understanding of mathematics is as a system of signs with their own inner logic which cannot be encountered in everyday activity (Volosinov, 1973).

I have repeatedly been using the terms operation, task, activity and so on. Learning in a school context is however characterized by certain specificities which can be better understood with the activity theory model of Leontiev (1978).

4.3 Learning as a relation of operations, tasks and the object of cognitive activity

Leontiev points at the hierarchical and internally differentiated structure of every activity, including the cognitive activity. He understands activity as a fairly molar unit consisting in partial levels represented by tasks or actions. Every task is formed by operations at a subordinate level (1978).

For Leontiev, it is above all the contents of the given activity, i.e. its object, that is crucial. What is also important is whether the cognitive

activity makes sense to the pupil (and what sense it makes). It is of less importance who is setting the task to the child or whether the form of the activity is playful or utility-focused.

The interesting things about this conception of activity are the relations between different levels of the activity and their functions, as demonstrated in the analysis of work-activity by Clot (1999). This points to the necessity of distinguishing between the relations of *efficiency* in practising operations and fulfilling tasks and the relations creating the *sense* of the activity as such. It is also necessary to make sure these relations are mutually interdependent. Examples of these activity levels can be laid out in the table below:

Activity	Object	Function
I. Molar activity: e.g. algebraic transformations	Motives: mastery; aesthetic experience; to be good at mathematics	Encouragement (initiative-provoking): to persist in efforts to overcome obstacles and difficulties arising at level II and III
II. Tasks: the calculation of functions of different types; the solution of a rider/theorem; the solution of a system of equations etc.	Goals: to find the correct solution; to identify the value of the unknown etc.	Orientation: correct input analysis of the task; good 'preparation' of the solution; the layout of steps, their sequence and time allocation etc.
III. Operations: Multiplication; reduction; position record; the discrimination of symbols; managing operations using memory	Means: material tools; symbolic instruments including cognitive processes (memory, attention, arithmetic operations)	Execution: material traces (notes, schemas, auxiliary calculations...); necessary technical support (infrastructure) of the operations

The relation between the quality of operations managed (III) and the quality of the solutions to tasks (II) expresses the efficiency of the cognitive activity/learning, usually in the form of microgenetic improvements. Automatization, the repetition of invariants of an activity, is an exemplar of such microgenetic developments which involves more abbreviated forms of an operation; it opens the way for a higher level of generality of the operational concept used and for the extension of the range of tasks which can be solved in the same domain.

The relation between the nature, frequency, complexity and above all interdependence (articulation) of tasks (II) and the essence of the activity expressed in its object (I) defines the sense of learning.

Learning a mathematical concept is therefore a complexly structured activity which may involve such activities as memorizing definitions, routine practising and consolidation of operations, as well as the difficult formulation of a hypothesis *vis-à-vis* a problem situation. The provision of pertinent tasks complemented only by verbal persuasion and model demonstration, without the elaboration of activities on levels II and III, cannot lead to success, since 'sense' cannot be enforced on the pupil from the outside; the pupil needs to possess tools to elaborate this sense for him or herself. It is the experience of a concept that comes from outside, never its sense. It is impossible to produce meaningful learning without efficient operations (including mental functions: attention, the memory of basic inference) and managed tasks. This efficiency alone, however, cannot ensure that pupils will find meaning in that which they may consider as an illogical chain of unrelated tasks, or even as a purposeless drill of isolated operations. However, fluently performed tasks *may* have a relatively positive effect, for example producing a functional solution of a task situation, but situations which can be resolved this way will fail to contribute to the development of intellectualisation (see above). Thus both efficiency and sense-making have to be thought about when designing and managing pedagogic tasks.

5. PERFORMANCE IN THE SITUATION VS. DEVELOPMENT

The difference between a *performance in the situation* (performance of a function), consisting of the repetition of invariants of an activity in a variety of situations, on the one hand, and *development* on the other is stressed by French psychologists Béguin and Clot (2004). Spontaneous learning first and foremost pursues efficient performance of a function in a situation whose boundaries are not transcended (such as to calculate correctly a subtraction;

or, more generally, ‘giving correct answers to the questions’). The results of spontaneous learning are often preserved even within learning of scientific concepts at school, and can be resistant to the requirements of scientific knowledge. However, spontaneous concepts represent the basic starting point for the subsequent conceptual work. In Leontievan terms, we have to deal with a situated level of operations (manipulating ‘tools’) and tasks. Their incorporation into a routine is a sort of an organizational condition for the cognitive activity itself (this is especially true of the memory automatism regarding certain algorithms, e.g. arithmetic ones). However, such ‘practical’ learning (in regard to the school context) rarely goes beyond the level of the performance of a function in a situation, such as getting the answer. Hence, there is often no opportunity for a more scientific apprehension of a concept so as to open the way for development. Such learning, learning to perform in a situation, can fail to grasp the object of cognitive activity itself.

On the other hand, effectively mediated learning of the concept paves the way for development of the pupil’s thinking, and hence of identity (as becoming someone who can think at a more abstract level than before). This requires that routine tools be used in a variety of tasks (actions), and that the tool use in various situational contexts enriches their functionality (e.g. basic mathematical operators should be practised in the context of calculus operating with both one-digit and double-digit numbers, in the context of tasks in arithmetic and tasks in geometry). Only such cognitive work, learning, enables a relevant generalization going beyond the limits of particular situation. Only thus could operations in decimal systems become, at least for some, a special particular instance of a more general set of conceptualizations. Learning which releases knowledge from a context without ignoring functionality in particular situations renders development possible: firstly the development of the child’s thinking; then the development of other psychological functions. For example, we memorize better those things the inner logic of which we have understood. Finally the development of the personality of the pupil follows, through developing a feeling of mastery over self and knowledge, and hence becoming harder to manipulate by others, and less likely to fall victim to biased information.

Activity theory shows how education influences the process of intellectualisation and the transformation of mental functions. For this reason, we should be warned against the reduction of the learning activity to mere operations and tasks which are close to the child’s current situation, and to superficial attractiveness and playfulness and for immediate sense. Of mathematics is this especially true, for it has an exceptional potential to contribute to the development of mental functions of the child and his/her personality; not merely to the broadening of his/her knowledge and capability in everyday situations, but also to develop mental functions. There

is simply no ‘immediate’ (non-mediated) connection between mathematical concepts or questions and social problems in the lives of people. It is futile to search for and incorporate this connection artificially into the education of mathematics under the pretext of its becoming more attractive. This connection exists only as highly mediated. Instead, it is a key function of mathematics to contribute to the developmental emancipation of a young person by way of intellectualisation, as I explained above. Through mathematics, students can become able to deal with generalizations of generalizations, and other relations between generalizations, and become more skilled at engaging in other forms of mediated activity in which the questions to be addressed are not those of everyday social existence. What I want to emphasize is that mathematics and its didactics should not lose their developmental-psychological potential by accepting the reductionism of an active, constructivist and problem-situated attitude towards education in the discipline.

In conclusion, I recognize that the situated perspective has much to offer in increasing our understanding of the whole complex picture of classrooms, and the essential differences between learning mathematics in school and learning and using it out of school. But it also alerts us to the limitations of integrating out-of-school methods into the classroom. Such attempts ignore an important part of the essential nature of mathematics as an abstract discipline, and there are many who would say that this is not a problem – such knowledge is not necessary for all students. However, I have tried to point out that one of the powerful purposes of having mathematics as a school subject is that, taught well as a scientifically organized subject, it contributes directly to learners’ development of higher psychological functions, and hence to the development of their personality, and their identity as mature people.

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² Where the text referred to is a translation into Czech the more widely known title of the translated text is cited in brackets.

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