Preface

During the last three decades the mathematical foundations of quantum mechanics, related to the theory of quantum measurement, have undergone profound changes. A broader comprehension of these changes is now developing.

It is well understood that quantum mechanics is not a merely dynamical theory; supplied with the statistical interpretation, it provides a new kind of probabilistic model, radically different from the classical one. The statistical structure of quantum mechanics is thus a subject deserving special investigation, to a great extent different from the standard contents of textbook quantum mechanics.

The first systematic investigation of the probabilistic structure of quantum theory originated in the well-known monograph of J. von Neumann "Mathematical Foundations of Quantum Mechanics" (1932). A year later another famous book appeared – A. N. Kolmogorov's treatise on the mathematical foundations of probability theory. The role and the impact of these books were significantly different. Kolmogorov completed the long period of creation of a conceptual basis for probability theory, providing it with classical clarity, definiteness and transparency. The book of von Neumann was written soon after the birth of quantum physics and was one of the first attempts to understand its mathematical structure in connection with its statistical interpretation. It raised a number of fundamental issues, not all of which could be given a conceptually satisfactory solution at that time, and it served as a source of inspiration or a starting point for subsequent investigations.

In the 1930s the interests of physicists shifted to quantum field theory and high-energy physics, while the basics of quantum theory were left in a rather unexplored state. This was a natural process of extensive development; intensive exploration required instruments which had only been prepared at that time. The birth of quantum mechanics stimulated the development of an adequate mathematical apparatus – the operator theory – which acquired its modern shape by the 1960s.

By the same time, the emergence of applied quantum physics, such as quantum optics, quantum electronics, optical communications, as well as the development of high-precision experimental techniques, has put the issue of consistent quantitative quantum statistical theory in a more practical setting.

Such a theory was created in the 1970s–80s as a far-reaching logical extension of the statistical interpretation, resting upon the mathematical foundation of modern functional analysis. Rephrasing the well-known definition of probability theory[1], one may say that it is a theory of operators in a Hilbert space given a soul by the statistical interpretation of quantum mechanics. Its mathematical essence is diverse aspects of positivity and tensor product structures in operator algebras (having their roots, correspondingly, in the fundamental probabilistic properties of positivity and independence). Key notions are, in particular, *resolution of identity* (positive operator valued measure) and *completely positive map*, generalizing, correspondingly, spectral measure and unitary evolution of the standard quantum mechanics.

The subject of this book is a survey of basic principles and results of this theory. In our presentation we adhere to a pragmatic attitude, reducing to a minimum discussions of both axiomatic and epistemological issues of quantum mechanics; instead we concentrate on the correct formulation and solution of quite a few concrete problems (briefly reviewed in the Introduction), which appeared unsolvable or even untreatable in the standard framework. Chapters 3–5 can be considered as a complement to and an extension of the author's monograph "Probabilistic and Statistical Aspects of Quantum Theory", in the direction of problems involving state changes and measurement dynamics[2]. However, unlike that book, they give a concise survey rather than a detailed presentation of the relevant topics: there are more motivations than proofs, which the interested reader can find in the references. A full exposition of the material considered would require much more space and more advanced mathematics.

We also would like to mention that a finite dimensional version of this generalized statistical framework appears to be an adequate background for the recent investigations in quantum information and computing, but these important issues require separate treatment and were only partially touched upon here.

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[1] "Probability theory is a measure theory – with a soul" (M. Kac).

[2] This text is the outcome of an iterative process, the first approximation for which was the author's survey [119], translated into English by Geraldine and Robin Hudsons. The LaTeX files were created with the invaluable help of J. Plumbaum (Phillips-Universität Marburg) and S.V. Klimenko (IHEP, Protvino).