Preface

The main objective of this monograph is the study of a class of stochastic differential systems having unbounded coefficients, both in finite and in infinite dimension. We focus our attention on the regularity properties of the solutions and hence on the smoothing effect of the corresponding transition semigroups in the space of bounded and uniformly continuous functions. As an application of these results, we study the associated Kolmogorov equations, the large-time behaviour of the solutions and some stochastic optimal control problems together with the corresponding Hamilton-Jacobi-Bellman equations.

In the literature there exists a large number of works (mostly in finite dimension) dealing with these arguments in the case of bounded Lipschitz-continuous coefficients and some of them concern the case of coefficients having linear growth. Few papers concern the case of non-Lipschitz coefficients, but they are mainly related to the study of the existence and the uniqueness of solutions for the stochastic system. Actually, the study of any further properties of those systems, such as their regularizing properties or their ergodicity, seems not to be developed widely enough. With these notes we try to cover this gap.

This work is structured in two parts, which are fairly independent of each other and which may be read separately. The first one is devoted to the study of stochastic ordinary differential equations. The second one is devoted to the study of stochastic partial differential systems of reaction-diffusion type. In both cases we start from the stochastic problem, we show that it admits a unique solution and then introduce the corresponding transition semigroup. We analyze the smoothing properties of such a semigroup and this allows us to proceed with the applications to the study of the optimal regularity of the corresponding PDE's, of the ergodicity of the system and of some stochastic optimal control problems.

It is important to stress that the study of the infinite-dimensional case is not a straightforward generalization of the study in finite dimension. In fact, many infinite-dimensional diffusion semigroups that arise e.g. in mathematical physics lack smoothing properties and, in contrast to the finite-dimensional case, there is no general analytic approach to the regularity properties in the infinite-dimensional case. In the present work we show how in some special cases, which are of interest

in the applications, a theory with certain similarities to the finite-dimensional case can be developed, by using mainly stochastic methods.

This book started as a PhD thesis at the Scuola Normale Superiore of Pisa. In the years since I began work on this subject, I found at the Scuola Normale a stimulating environment for my research and had the opportunity there to meet many people who were generous with their help.

My greatest debt of gratitude is to my former supervisor, Giuseppe Da Prato, who has introduced me to the study of SPDE's with his unfailing advice and constant encouragement. It is mainly due to him that in the group of all his students (and former students) there has always been great collaboration and friendship, creating the best conditions for work.

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