1 Introduction

Genetic algorithms today constitute a family of effective global optimization methods used to solve difficult real-life problems which arise in science and technology. Despite their computational complexity, they have the ability to explore huge data sets and allow us to study exceptionally problematic cases in which the objective functions are irregular and multimodal, and where information about the extrema location is unobtainable in other ways.

They belong to the class of iterative stochastic optimization strategies that, during each step, produce and evaluate the set of admissible points from the search domain, called the random sample or population. As opposed to the Monte Carlo strategies, in which the population is sampled according to the uniform probability distribution over the search domain, genetic algorithms modify the probability distribution at each step.

Mechanisms which adopt sampling probability distribution are transposed from biology. They are based mainly on genetic code mutation and crossover, as well as on selection among living individuals. Such mechanisms have been tested by solving multimodal problems in nature, which is confirmed in particular by the many species of animals and plants that are well fitted to different ecological niches. They direct the search process, making it more effective than a completely random one (search with a uniform sampling distribution). Moreover, well-tuned genetic-based operations do not decrease the exploration ability of the whole admissible set, which is vital in the global optimization process.

The features described above allow us to regard genetic algorithms as a new class of artificial intelligence methods which introduce heuristics, well tested in other fields, to the classical scheme of stochastic global search.

Right from the beginning, genetic algorithms have aroused the interest of users and scientists, who try to apply them when solving engineering problems (optimal design, stuff deposit investigation and defectoscopy etc.) as well as to explain the essence of their complex behavior.

At least ten large international conferences are devoted to the theory and applications of genetic optimization. The most important and well established

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seems to be FOGA[1] (Foundation of Genetic Algorithms), GECCO[2] (Genetic and Evolutionary Computation), PPSN[3] (Parallel Problem Solving from Nature) and CEC[4] (IEEE Congress on Evolutionary Computation). Almost all conferences in artificial intelligence, optimization, distributed processing, CAD/CAE and various branches of technology contain sessions or workshops that gather contributions showing specialized applications of genetic computing and their theoretical motivations. Many important events were organized by national associations, in particular the National Conference in Genetic Algorithms and Global Optimization KAEiOG[5] has taken place in Poland annually since 1996.

There are also several scientific journals devoted solely to genetic algorithm theory and applications. It is worth highlighting *IEEE Transactions on Evolutionary Computation*[6] and *Evolutionary Computation*[7] among them.

Besides the many research papers cited in this book, we would like to draw the reader's attention to monographs that try to comprehend results from various branches of genetic computation and trace new directions in research and applications. The pioneering book in this area, entitled *Adaptation in Natural and Artificial Systems*, was written by Holland [85] in 1975. The author defined binary genetic operations and related them to the real modifications and inheritance of genetic code. He also tried to deliver a formal description and quantitative evaluation of the artificial genetic process by formulating a popular schemata theorem. Important bibliographical items that show the number and variety of evolutionary optimization techniques, as well as more formal descriptions of algorithms, are the books of Goldberg 1989, [74], Michalewicz 1992, [110] and Koza 1992, [99]. Due to its intentions and large scope, the monograph of Bäck, Fogel and Michalewicz from 1997, [15] as well as its compressed and improved version [10, 11], is impressive. An exceptional book which discusses parallel models and implementations of genetic computation was written by Cantú-Paz 2000, [45]. Another title of this type, published by Osyczka 2002, [124] summarizes the genetic algorithm applications to multi-criteria design and optimization.

One well-known book, written in 1999 by Vose, [193] delivers the most important results concerning formal analysis of the so-called *Simple Genetic Algorithm* (SGA). His approach is based on SGA modeling as the Markov chain, whose trajectories are located in the space of states common for the class of algorithms with different population cardinality. The main results characterize the SGA asymptotic behavior by the number of iterations (genetic epochs) tending to infinity, as well as for an infinitely growing population size.

[1] http://www.sigevo.org/foga-2007/
[2] http://www.sigevo.org/gecco-2006/
[3] http://ls11-www.cs.uni-dortmund.de/PPSN/
[4] http://www.cec2007.org/
[5] http://kaeiog.elka.pw.edu.pl/
[6] http://ieee-cis.org/pubs/tec/
[7] http://www.mitpressjournals.org/loi/evco?cookieSet=1

The next important contributions in this area are the chapters written by Rudolph [143, 144, 145], Rudolph and Beyer [27], which are integral parts of three books, edited by Bäck, Fogel and Michalewicz [15, 10, 11]. They analyze the particular type of convergence of genetic algorithms with real number, phenotypic encoding.

There are many distinguished books, which have been printed recently, dealing with genetic algorithm theory. Spears 2000, [178] discusses the role of mutation and recombination, in algorithms with the binary genetic universum, in detail. Beyer 2001, [26] presents an exhaustive analysis of the evolution strategy progress using strong regularity assumptions with respect to fitness. Langdon and Poli 2002, [102] extend some theoretical results which come from binary schemata theory to the case of genetic programming with the genetic universum as a space of graphs. Reeves and Rowe 2003, [134] present a critical view of the various approaches for studying genetic algorithm theory and discuss the perspectives in this area.

Finally it worth mentioning two Polish books. The first one, written by Arabas [5], delivers the author's original approach and comments to selected genetic techniques, preceded by a broad mathematical description concerning single- and multi-criteria optimization problems. The second one [149] marks the beginnings of this book.

This work delivers a new approach for studying genetic algorithms by modeling them as dynamic systems which transform the probabilistic sampling measures (probability distributions on the admissible set of solutions) in a regular way. This approach allows us to show that genetic algorithms may effectively find the subsets of the search domain rather than isolated points, e.g. the central parts of the basins of attraction of the local minima rather than isolated minimizers. This feature reflects the character of elementary evolutionary mechanisms implemented here that lead to the whole flock (population) surviving by the fast exploration of new regions of feeding rather than care of the single flock member (individual). The attention of the reader will be turned to such kinds of genetic algorithm instances (e.g. two-phase methods, genetic sample clustering, and sensitivity analysis) for which the above features may guarantee that all solutions are found and the stopping rule is verified. It will also be shown that the traditional use of a genetic algorithm to solve local optimization problems may meet obstacles which arise from the inherent features of this group of methods.

We will focus on the ability of genetic algorithms to solve global, continuous optimization problems so that the admissible solutions make the regular subset (with the Lipschitz boundary) of the positive Lesbegue measure in a dense, finite dimensional linear-metric space. We do not consider genetic algorithm instances which can only solve discrete optimization problems. We will also omit such features of the common algorithm instances that are valid only for the discrete search domain.

Detailed definitions of standard continuous global optimization problems are given at the beginning of this book. Problems which involve finding all global extremes as well as the predefined class of local extremes are discussed in Section 2.1. New optimization problems, leading to recognizing and approximating sets which are the central parts of the basins of attraction of local minimizers, are also introduced and discussed in this section. In Sections 2.2, 2.3, a general, abstract scheme of the stochastic, population search and its basic qualitative, asymptotic features are specified. In the light of these formulations, the basic mechanisms of genetic computation, presented in Chapter 3, exhibit their real nature and working directions. Such an approach is perhaps much less mysterious than the traditional one based on biological analogy. It also allows the synthetic presentation of many details common for quite different algorithm classes (e.g. genetic algorithms with the finite set of codes and evolutionary strategies using phenotypic, real number encoding).

All standard genetic operations presented in Chapter 3 lead to the stationary rule of sampling adaptation i.e. the rule does not depend on the genetic epoch in which it is applied. In other words, the probability distribution utilized for sampling is obtained in the same way, taking only the current population into account. The class of genetic algorithms that permit only stationary adaptation rules will be called *self-adaptive genetic algorithms*, because the transition rules of sampling probability distribution are not modified by any external control or by any feedback signals coming from the previous population monitoring. The taxonomy and a short description of *adaptive genetic algorithms* that break the principle of stationary sampling probability transition are given in Chapter 5.

The core of this book, located in Chapter 4, synthesizes the mathematical models of genetic algorithm dynamics and their asymptotic features. The main results presented in this chapter are based on the stochastic model that operates on the space of states whose elements are populations or their unambiguous representations. Genetic algorithms are assumed to be self-adaptive, which implies the uniform Markovian rule of state transition.

Asymptotic results obtained for the Simple Genetic Algorithm (see Section 4.1.2) are based on features of the so-called *genetic operator*, introduced by Vose and his co-workers, sometimes called the SGA heuristics (see e.g. [193]). In the same section, theorems concerning the transformation of sampling measures, and their transport from the space of states of the genetic algorithm to the search domain, are considered. These results motivate the application of genetic algorithms in searching and approximating the central parts of the basins of attractions of the local, isolated minimizers. Such results are also helpful in the analysis of two-phase global optimization strategies which utilize genetic algorithms during the first, exploration phase (see Chapter 6). Section 4.1.3 contains results of the Markov theory of evolutionary algorithms $(\mu + \lambda)$-type with the elitist selection.

Finally, Section 4.2 comprehends asymptotic results obtained for genetic algorithms with very small populations and Section 4.3 delivers some comments which lead to the precise formulation and verification of the schemata theorem for SGA.

An important part of this book, written by Henryk Telega, is located in Chapter 6. It delivers a survey of two-phase stochastic global optimization methods. Such strategies consist of finding the approximation of extreme attractors in the first phase, called *global phase*, and pass to the detailed, local search in each attractor in the second phase, called *local phase*. Probabilistic asymptotic correctness as well as stopping rules of two-phase strategies are also discussed.

A new global phase strategy, called *Clustered Genetic Search* (CGS), which utilizes the genetic sample to recognize the central parts of attractors, is introduced. The advantageous features of genetic algorithms that regularly transform sampling measures to ones which become denser close to the local extrema guarantee the proper definition of such a strategy. In particular, by using theorems formulated in Chapter 4, the probabilistic asymptotic correctness and the stopping rule of CGS are verified.

We have omitted detailed, technical proofs of some cited theorems, remarks and formulas due to the necessary limitation of volume and the assumed engineering profile of this book. Readers are extensively referred to sources in each particular case.

Several important computation examples are placed in Section 2.4 in order to demonstrate the skill of genetic algorithms in solving optimal design problems, formulated as continuous global optimization ones. The second group of tests exhibits characteristic features of the clustered genetic search (CGS) running for a small set of classical multimodal benchmarks (see Chapter 6).

Readers only require a basic mathematical preparation and a maturity at a level typical for the MS courses in science, especially in the area of real valued function analysis, linear algebra as well as probability theory and stochastic processes.

The book may be recommended in particular to readers who have a basic insight into genetic-based computational algorithms and are looking for an explanation of their quantitative features and for their advanced applications and further development. It may be helpful to engineers in solving difficult global optimization problems in technology, economics and the natural sciences. This is especially true in the cases of multimodality, weak regularity of the objective function and large volumes of the search domain. It may also inspire researchers employed in studying stochastic optimization and artificial intelligence.

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