Introduction

Completeness is one of the most important notions in logic and the foundations of mathematics. Many variants of the notion have been defined in literature. We shall concentrate on these variants, and aspects, of completeness which are defined in propositional logic.

Completeness means the possibility of getting all correct and reliable schemata of inference by use of logical methods. The word 'all', seemingly neutral, is here a crucial point of distinction. Assuming the definition as given by E. Post we get, say, a global notion of completeness in which the reliability refers only to syntactic means of logic and outside the correct schemata of inference there are only inconsistent ones. It is impossible, however, to leave aside local aspects of the notion when we want to make it relative to some given or invented notion of truth. Completeness understood in this sense is the adequacy of logic in relation to some semantics, and the change of the logic is accompanied by the change of its semantics. Such completeness was effectively used by J. Łukasiewicz and investigated in general terms by A. Tarski and A. Lindenbaum, which gave strong foundations for research in logic and, in particular, for the notion of consequence operation determined by a logical system.

The choice of logical means, by use of which we intend to represent logical inferences, is also important. Most of the definitions and results in completeness theory were originally developed in terms of propositional logic. Propositional formal systems find many applications in logic and theoretical computer science. Due to the simplicity of the language, one can use in research various methods and results of abstract algebra and lattice theory. Propositional completeness theory is a prerequisite for the other types of completeness theory and its applications.

In this monograph we wish to present a possibly uniform theory of the notion of completeness in its principal version, and to propose its unification and, at the same time, generalization. This is carried out through the definition and analysis of the so-called Γ-completeness (Γ is any set of propositional formulas) which generalizes and systematizes some variety of the notion of completeness for propositional logics — such as Post-completeness, structural completeness and many others. Our approach allows for a more profound view upon some essential properties (e.g., two-valuedness) of propositional systems. For these purposes we shall use, as well, the elementary means of general algebra, the theory of logical matrices, and the theory of consequence operations.

The subject of completeness became a separate area of research in propositional logic in the 1970s. Results of research of many authors, mainly Polish, have been used here. Our exposition is based on a former manuscript [88], 1982. We have tried to include all important results which are indispensable for further work in the area. In addition, we have included some of the more recent results stimulating present research. The book is organized on the following plan. Basic methods and constructions of universal algebra and propositional logic are briefly discussed in Chapter 1. Main results are exposed in the next two chapters; Chapter 2 deals with local, and Chapter 3 with global, aspects of the notion of completeness. In the last chapter and appendices we present some more advanced topics which combine several methods and ideas involved in previous fragments. The terminology and notation employed in our monograph is standard. The set theoretical symbols $\emptyset$, $\in$, $\subseteq$, $\cap$, $\cup$ etc. have their usual meanings. The power set of the set A is denoted by 2^A and $Nc(A)$ is the cardinality of A. We use $\Rightarrow$, $\vee$, $\wedge$, $\neg$,$\Leftrightarrow$, $\forall$, $\exists$ for (meta)logical connectives and quantifiers whereas $\rightarrow$, $+$, $\cdot$, $\sim$, $\equiv$ are reserved for (intra)logical symbols, i.e., symbols of the considered logical systems.

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