Tracer particles in turbulent superfluid helium

Y. A. Sergeev[1], C. F. Barenghi[2] and D. Kivotides[2]

[1] School of Mechanical and Systems Engineering, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK (contact: yuri.sergeev@ncl.ac.uk)
[2] School of Mathematics, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK

1 Motivations

Much of the interest in the turbulence of superfluid helium is motivated by the simplicity of the vortex structures: the core of a vortex line has atomic thickness, the circulation, $\kappa \approx 10^{-3}\,\mathrm{cm}^2/\mathrm{s}$ is quantized, and the superfluid has zero viscosity. The recent success [1, 2] of implementing the PIV technique in liquid helium has opened up the possibility of visualizing flow patterns. This technique consists of tracking the motion of micron-size inertial particles using lasers. However, the interpretation of PIV data is complicated by the presence in He II of two fluid components: the viscous normal fluid and the actual inviscid superfluid. Our aim is to provide the theoretical understanding of the dynamics of small particles which is necessary to interpret the data.

2 Equations of motion and particle dynamics

We consider first the motion of spherical solid particles under the assumptions that they do not modify the turbulence, trapping of particles by superfluid vortices does not occur, and the particle radius, a_{p} is smaller than the Kolmogorov length in the normal fluid and the intervortex distance in the superfluid. The Lagrangian equations of particle motion are [3]:

$$\frac{d\mathbf{r}_{\mathrm{p}}}{dt} = \mathbf{v}_{\mathrm{p}}, \quad \frac{d\mathbf{v}_{\mathrm{p}}}{dt} = \frac{1}{\tau}(\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{p}}) + \frac{3}{2\rho_0}\left(\rho_{\mathrm{n}}\frac{D\mathbf{v}_{\mathrm{n}}}{Dt} + \rho_{\mathrm{s}}\frac{D\mathbf{v}_{\mathrm{s}}}{Dt}\right) + \frac{\rho_{\mathrm{p}} - \rho}{\rho_0}\mathbf{g}\,, \quad (1)$$

where ρ is the density (subscripts n, s and p refer to the normal fluid, superfluid, and solid particle, respectively), $\rho = \rho_{\mathrm{n}} + \rho_{\mathrm{s}}$, $\rho_0 = \rho_{\mathrm{p}} + \frac{1}{2}\rho$, and $\tau = 2a_{\mathrm{p}}^2\rho_0/(9\mu_{\mathrm{n}})$ is the particle relaxation time. Analysis of Eqs. (1) shows that for neutrally buoyant particles a number of different regimes [3] can be identified: in some regimes the particles trace the normal fluid, in others the superfluid, in others the total mass current. However, an instability of particle trajectories may require modification of these conclusions. We find that in pure superfluid (at $T < 1$ Kelvin), due to the instability and the mismatch of initial velocities of particles and the fluid, small particles cannot be used to visualize the full superfluid velocity field [4]. However, it is possible to obtain information from the observation of particles that are trapped on vortex lines.

In the case of finite temperature superfluid, we analyze collisions of particles with superfluid vortices taking into account normal fluid disturbances induced by the mutual friction between the normal fluid and superfluid. We find that these disturbances can deflect the particle which otherwise could have been trapped by the vortex. We also perform calculations of particle interaction with the superfluid vortex ring propagating against a particulate sheet and show that particle trajectories collapse to the normal-fluid path lines. We propose an experiment in which, by measuring velocities of particles, direct information could be obtained about the normal-fluid velocity [5].

3 Interaction of particles and superfluid vortex lines

Particle trapping on vortex lines can lead to a modification of our results. To address this issue, we analyze a close interaction between a quantized vortex and a particle. Our calculation is self-consistent and takes into account an influence of the particle on the superfluid vortex (including reconnection of the vortex line with the particle surface.) We find that trapping occurs only in the presence of a dissipative force acting on the particle. At finite temperatures such a force is provided by the viscous drag exerted by the normal fluid. However, even at temperatures below 1 K, when the normal fluid is absent, there exists a dissipative force caused by phonon scattering.

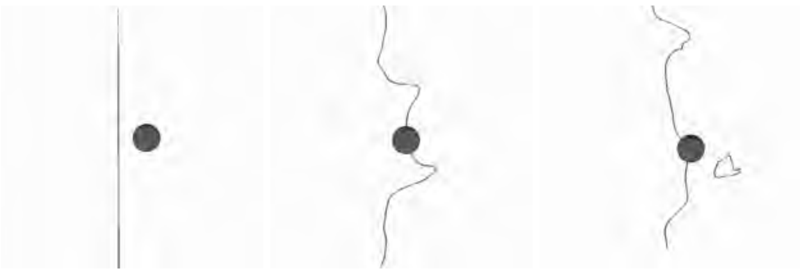

Fig. 1. Trapping of the particle by the initially straight vortex line.

A sequence shown on Fig. 1 illustrates trapping of the neutrally buoyant $1\,\mu$m particle by the initially straight vortex line. Fig. 1 (middle) shows the reconnection of the superfluid vortex with the particle surface, accompanied by excitation of Kelvin waves propagating along the vortex line. Fig. 1 (right) shows the moment of trapping accompanied by the emission of small vortex ring reducing the total energy of the particle-vortex system.

Based on the mechanism of particle-vortex interaction, we develop a theory of particle motion in turbulent thermal counterflow in helium II. In counterflow, the vortex tangle can be dense so that several vortices can be attached to the particle surface. We find that the nonuniform pressure distribution, caused by N vortices attached to the sphere, yields the body force

$$\mathbf{F} = \frac{\rho_{\mathrm{s}}\kappa^2}{4\pi} \ln\frac{a_{\mathrm{p}}}{\xi} \sum_{n=1}^{N} \mathbf{n}_i \,, \tag{2}$$

where $\xi \approx 10^{-8}$ cm is the core radius, and $\mathbf{n}_i$ the unit vector along the ith attached strand. In symmetric configuration (Fig. 2 (a) and (b)) $\mathbf{F} = \mathbf{0}$. As the particle moves through the tangle, the vortex loops will be attached asymmetrically, as in Fig. 2 (c) and (d), causing a net body force. This force enables us to explain the surprising result of a recent experiment [2] that in turbulent counterflow the tracer particles move with about half the speed of the normal fluid. Fig. 2 (right) shows the calculated particle velocity vs the normal fluid velocity. The results compare well with experimental data [2].

This research is funded by EPSRC grant GR/T08876/01.

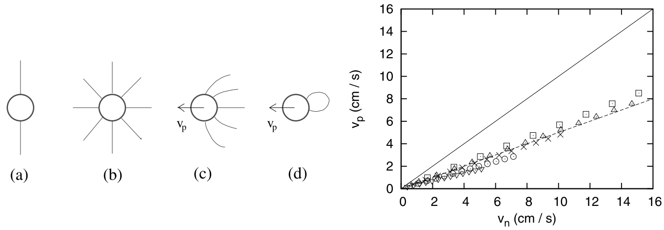

Fig. 2. Left ((a)-(d)): Sphere-vortex configurations. Right: Particle velocity vs the normal fluid velocity; dashed line – calculation based on Eq. (2).

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