Preface

"A considerable amount of time has passed since steam engines, steamers, and railway engines have appeared. God saw that people's life is difficult and boring. Then he decided to help them once more and gave them signals as a present. The devil saw this and created noises, which he mixed up with the signals. People could not use noises and asked scientists to spare them these troubles. People created methods and technologies of filtration and suppression of noises. However, signals were distorted and many valuable and concealed mysteries were lost together with the noise."

This book shows that characteristics of signals and noises at the output of sensors change continuously for both technical and biological objects at the origin of a defect. The known classical conditions are not satisfied. The time for a decision about the problem increases.

For these reasons, in some cases the detection of defects in information systems turns out to be overdue. Sometimes it results in catastrophic consequences.

Taking into account these and other features of the initial stage of the origin of the defect, several technologies are suggested. These technologies allow one to perform the defect monitoring at the beginning of the defect's origin by extracting information from the noise. By duplicating and combining the above technological advantages, a necessary degree of reliability to the results is reached. These technologies are proved theoretically, and the opportunity of their application for solving problems of noise monitoring at the beginning of the defect's origin in oil-gas extraction, in construction, in power engineering, in transport, in seismology, in aviation, in medicine, etc., is shown on numerous examples. In addition, they allow one to improve the results of mathematical modeling, recognition, identification, control, etc., and they can find wide application in solving numerous problems where processing and analysis of signals are required in various fields of science and technology.

The monograph is intended for teachers, post-graduate students, and university students of all professions except the humanities, and also for experts of power engineering, computer science, automation, physics, biology, geophysics, oil-gas extraction, transport, aviation, control, medicine, etc.

At present, the number of failures in power stations, sea objects of oil-gas extraction and communication, main oil-gas pipelines, petrochemical complexes, large-capacity tankers, airlines, etc., remains unjustifiably high because of mistakes in information systems in spite of the repeated increase of reliability of the element base. This often occurs by the fault of traditional information technologies, as they provide defect monitoring after the defect takes its salient character.

Reasons for a defect's origin in various objects such as living organisms and equipment are the subject of research of corresponding science directions. However, considering these problems in view of obtaining information and methods of analysis, monitoring, and diagnostics, one can notice that the problems have much in common with one another [1–14]. In many cases, the signals describing the current state and technical conditions are obtained from the sensors installed on the corresponding objects. The similar or nearly the same information technologies are used in different areas for analyzing these signals as the information carriers [1, 4, 7, 11, 14]. These technologies are realized on the same modern computers. Taking all this into account, the IT specialist does not consider the differences in solving the problem of monitoring the state of these objects despite the wide areas of specific features of each object [1, 4, 11].

However, the process of the origin and evolution of a defect before it takes its salient character has unique features for each object depending on its physical, biological, mechanical, chemical, and other properties [1, 11, 13]. These features also depend on the performed functions, the exploitation modes, etc. Due to these special features, the time period from the origin of the defect of a signal to the time it takes its salient character is unique for each object. It takes a short time for some objects. Others take considerably longer. However, despite all these differences, the information represented as the defect component of a signal has the common property to change continuously for all objects at this period. It becomes stable only after the defect takes its salient character. Thus, the reliability of results for solving the problem of monitoring the early defect's origin depends on used information technologies of analyzing the signal received as the output of the corresponding sensor of objects. As usual, these signals are accompanied by noise. That makes it difficult and sometimes impossible to solve the problem of monitoring the defect at its early origin.

In general, in solving all sorts of questions by signal processing, it is possible to get more or less acceptable results by means of known information technologies only in the case of satisfying classical conditions, i.e., analyzed signals are stationary, they follow the normal distribution law, the correlation between the noise and the legitimate signal is equal to zero, the noise is represented as "white noise," etc. Even in this case, the obtained

results do not always provide enough reliability because noise from real signals differs from "white noise" and its variances and spectral components change in time.

At the same time, in many cases the above-mentioned conditions are not satisfied at all, and it is not always possible to get reliable results and form adequate solutions for the situations that arise in corresponding information systems [15–42]. For this reason, the number of failures of various objects in oil-gas extraction, petroleum chemistry, power engineering, aviation, etc., with catastrophic human, economic, and ecological consequences does not decrease, in spite of the fact that the reliability of both the element base and the equipment in information systems has increased lately. In this connection, filtration methods are often used in traditional technologies for eliminating the noise influence on the results of solved problems.

They provide good results when the filter spectrum and noise spectrum coincide. Simultaneously, for many real processes and particularly in the period of a defect's origin, the noise spectrum and variance widely vary in time. For these reasons, the "filter" spectrum range has to be widened to eliminate the noise influence on the result of signal processing. And this distorts the useful signal much more. In addition, quite often the noise arises as a result of operation of controlled objects. Thus, the noise becomes the data carrier, and this information is erased because of filtration. And in these cases the important and in some cases the only valuable information of the early defect origin is lost [42–54].

Thus, in traditional technologies, the specificity of noise influence of real signals on the desired result is not sufficiently taken into account. These technologies do not have the opportunity to extract information contained in the noise of analyzed signals.

So problems of creating technologies of noise analysis and increasing the reliability of solved problems by signal processing of results are of great importance on the contemporary stage of "signal processing" development [1, 58, 60, 61].

No doubt filtration and traditional information technology allow one to solve numerous necessary problems. However, at the same time, it is necessary and advisable to have alternative technologies possessing the property of extracting the information contained in the noise of noisy signals.

The significance of this work is also connected with the possibility of using the noise as a data carrier for creating technologies of detecting the initial stage of changes to objects. It is now that their time has come. The economy of computer resources was considered as an essential dignity of information technologies in the past for many years. But now, because of their enormous resources, one can create more effective technologies at the expense of the complication of computational process [1].

Unlike traditional technologies, where at the expense of signal noise filtration, the volume of extracted information decreases, in the suggested alternative technology due to analyzing noise as a data carrier, this disadvantage is eliminated. This significant difference opens a wide opportunity for expanding the range of solved problems on the basis of analysis of noisy signals.

2 Position-Binary Technology of Monitoring Defect at Its Origin

2.1 Specific Properties of Periodic Effect Objects

It's known that in most cases the spectral methods are used for the experimental analysis of the cyclical (periodic) processes [12, 41]. For example, the objects of the back-and-forth motion equipment, the objects of the rotating equipment, those of the biological processes, etc. are cyclical. As a rule, the signals obtained from many cyclical objects have the complicated spasmodic leaping form and are accompanied by significant noise. At present, spectral methods and algorithms are commonly used in the experimental research of such signals [12, 37, 61]. But they are not effective enough for these objects in some cases [37]. Thus, in many cases it is necessary to use the large number of harmonic components of the corresponding amplitudes and frequencies for the appropriate description of spasmodic and leaping signals. That essentially complicates the analysis and use of the obtained results for solving the corresponding problems [37, 41]. That is why, in solving the problem of monitoring the defect origin, there is a need for methods and algorithms allowing one to (1) increase the reliability of the obtained results in comparison with the spectral method and (2) decrease the quantity of the spectrum components of the considered class of the objects [37, 41].

Let us consider the difficulties of using the spectral method for the analysis of the signals obtained from the considered objects in more detail.

It is known that when using the algorithms of this method for description of the periodic signals $X(t)$ of the bounded spectrum, the periodic signals $X(t)$ are represented as the sum of the harmonic components by means of the following expression:

$$X(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right). \qquad (2.1)$$

In Eq. (2.1), a_n, b_n are the amplitudes of the sinusoids and the cosinusoids with the frequency $n\omega$, which are assumed to be the informative indicators in solving the problem of monitoring the origin of the defect. It is known that for providing the accuracy of the signal restoration $X(t)$, the following inequality is required:

$$\sum_{i=1}^{n} \lambda_i^2 \leq S, \qquad (2.2)$$

where λ_i^2 are the squares of deviations between the sum of the right-hand side of Eq. (2.1) and samples of signal $X(t)$ at the moments of sampling t_0, t_1, ..., t_i, ..., t_n with the sampling step Δt; S is the permissible value of the mean-root-square deviation.

The spasmodic leaping signals providing Eq. (2.2) lead to increasing the number of harmonic components, and that correspondingly complicates processing the experimental data. In addition, when the measured information consists of the sum of the useful signal $X(t)$ and the noise $\varepsilon(t)$, condition (2.2) takes place depending to a certain extent on the value of the noise $\varepsilon(t)$. In the existing methods, the influence of the noise is neglected in Eq. (2.1), and the error caused by the noise $\varepsilon(t)$ is assumed to be equal to zero. But for the many cyclical processes, the influence of the noise on the accuracy of the restoration of the initial signal $X(t)$ can be considerable and must be taken into account.

If we take into consideration that in the defect origin the spectrum of the signal continuously changes, the difficulties of using the technology of the spectral analysis in solving the problem of monitoring the defect origin will be clear. That it is why it is necessary to create the new spectral technologies, taking into account the specificity of the signals obtained from the periodic objects. One of the possible variants of these technologies is offered in the next section.

2.2 Position-Binary Technology of Analyzing Noisy Signals Obtained as Outputs of Sensors of Technical Objects

As stated earlier, at present the algorithms of the spectral and correlation analyses are mainly used for the analysis of the periodic processes [12]. But their application in solving the problems of monitoring the defect origin does not provide a reliable result at the origin of a defect. In this connection, the principles and algorithms allowing one to detect the defect

at its origin are of both theoretical and practical interest. The availability of using the position-binary technology for this purpose is offered ahead. In that case, noisy signals are analyzed through the corresponding position-binary impulse signals (PBIS).

In practice, when measuring the signals $X(t)$, there is the minimum value of the increase, which can be provided by the used instrument depending on its resolving capacity. We will denote that minimum value of the increase as Δx. So in measuring the signal, the number of its discrete values is equal to

$$m = X / \Delta x + 1. \tag{2.3}$$

In the process of the analog-digital conversion of the periodic signal $X(t)$, its amplitude quantization takes place for each sampling step Δt, i.e., the range of its possible changes is divided into the m sampling intervals and the value of the signal belonging to the m th sampling interval is related to the center of the sampling interval $m\Delta x$ when the following inequality takes place:

$$m\Delta x - \Delta x / 2 \leq X(t) \leq m\Delta x + \Delta x / 2. \tag{2.4}$$

In this case, the values of the binary codes of the corresponding digits q_k of samples x_i of signal $X(i\Delta t)$ with sampling step Δt are determined on the basis of the following algorithm [12, 14, 37, 41]:

$$q_k(i\Delta t) = \begin{cases} 1 & \text{for} \quad x_{\text{rem}(k)}(i\Delta t) \geq \Delta x 2^k \ ; \\ 0 & \text{for} \quad x_{\text{rem}(k)}(i\Delta t) < \Delta x 2^k \ ; \end{cases} \tag{2.5}$$

$$x_{\text{rem}(k)}(i\Delta t) = x_k(i\Delta t) - \left[q_{k+1}(i\Delta t) + q_{k+2}(i\Delta t) + ... + q_{(n-1)}(i\Delta t) \right],$$

where

$$X(i\Delta t) > 2^n, \ x_{\text{rem}(n-1)}(i\Delta t) = X(i\Delta t),$$

$$n \geq \log \frac{x_{\max}}{\Delta x}, \ k = n-1, n-2, ..., 1, 0.$$

First, according to this algorithm, at each step of sampling Δt, the equality $x_{\text{rem}(n-1)}(i\Delta t) = X(i\Delta t)$ is accepted. Also, according to condition (2.5), the signals $q_k(i\Delta t)$ as a code 1 or 0 are formed by iteration. In this case, in the first step $X(i\Delta t)$ is compared to the value $2^{n-1}\Delta x$. According

to (2.5), at $X(i\Delta t) \geq 2^{n-1}\Delta x$, the value $q_{n-1}(i\Delta t)$ is equated to unit; according to the difference $X(i\Delta t) - 2^{n-1}\Delta x = x_{\text{rem}(n-2)}$, the value of the remainder $x_{\text{rem}(n-2)}$ is determined. When $X(i\Delta t) \geq 2^{n-1}\Delta x$, the value $q_{n-1}(i\Delta t)$ is set to zero and the difference remains constant. At the next iteration, the same takes place. As a result, during the cycle T_c with sampling step Δt, the signal $X(i\Delta t)$ is decomposed into the signals $q_k(i\Delta t)$ having the value 1 or 0 and whose weight depends on their positions. At the same time, the codes do not change in time when the value of the initial signal $X(i\Delta t)$ does the same at the process of sampling. Here and ahead, we will name these signals the position-binary impulse signals (PBIS). The position-binary technology is the series of the procedures of processing based on the decomposition of the continuous signal by the PBIS.

According to algorithm (2.5), the width of the PBIS is proportionate to quantity Δt when $q_k(i\Delta t)$ remains constant. Depending on the form of $X(i\Delta t)$, the same signal $q_k(i\Delta t)$ can change its value several times during one cycle after the corresponding time intervals. It is clear that if the condition of the object is constant, the combinations of the time intervals T_{k1_1}, T_{k0_1}, T_{k1_2}, T_{k0_2}, … of the PBIS at each cycle are constants, and they are repeated. Otherwise, they also change. Let us note that here T_{k1_1}, T_{k0_1}, T_{k1_2}, T_{k0_2}, … correspond to the intervals when the condition $q_k(i\Delta t) = 2^k (\Delta x = 1)$ takes place; T_{k0_1}, T_{k0_2}, … correspond to the intervals when the condition $q_k(i\Delta t) = 2^k (\Delta x = 0)$ takes place.

For example, let us suppose that the cycle time of the analyzed signal is equal to 15 microseconds and the sampling step is equal to 1 microsecond, i.e., $T_c = 15$ mcs, $\Delta t = 1$ mcs. Let us assume that PBIS $q_3(i\Delta t)$ takes the following states for one cycle: 000111100110000. In this case, the parameters of signal $q_3(i\Delta t)$ are represented as follows: 3,0; 4,1; 2,0; 2,1; 4,0. It means that during the cycle, the width of unit and zero states of signal $q_3(i\Delta t)$ corresponds to the following time intervals: 3 mcs-0; 4 mcs-1; 2 mcs-0; 2 mcs-1; 4 mcs-0. It is obvious that in each cycle the sum of all PBIS is equal to the initial signal

$$X(i\Delta t) \approx q_{n-1}(i\Delta t) + q_{n-2}(i\Delta t) + \cdots + q_1(i\Delta t) + q_0(i\Delta t) = X^*(i\Delta t). \quad (2.6)$$

Each $q_k(i\Delta t)$ can be considered as the individual signal because we can assume the sequence of time intervals when $q_k(i\Delta t)$ are in the unit and zero state to be impulse-width signals. At the same time, for the cyclic objects these PBIS $q_{k_j}(i\Delta t)$ are the periodic rectangular impulses having the period T_c with unit T_1 and zero T_0 half-periods correspondingly.

We must note that the representation of the centered signals by PBIS differs only that in this case the initial signal is represented as the sum of the positive and the negative PBIS $q_k(t)$. At the same time, the signals $X(t)$ and $y(t)$ are represented as bipolar periodic PBIS, and their sum is also equal to the initial signal $X(i\Delta t)$.

At the representation of the initial signal $X(i\Delta t)$ as the sum $q_k(i\Delta t)$ at time t_i, the difference between the real value of the initial signal $X(t)$ and the sum of PBIS is

$$X(i\Delta t) - X^*(i\Delta t) = \lambda(i\Delta t). \tag{2.7}$$

Taking into account Eq. (2.4), we have

$$\lambda(i\Delta t) \leq \pm\Delta x / 2 .$$

If we assume that in forming the signals $q_k(i\Delta t)$, the value of the error $\lambda(i\Delta t)$ is under the equiprobable distribution law [30], we obtain

$$P\left[\lambda_i < \frac{\Delta x}{2}\right] \approx P\left[\lambda_i > \frac{\Delta x}{2}\right], \tag{2.8}$$

where P is the sign of probability.

Thus, according to (2.7) and (2.8), the sum of the squares of deviations λ_i at t_0, t_1, …, t_i, … is close to zero. Inequality (2.2) can then be represented as follows:

$$\sum_{i=1}^{n} \lambda^2(i\Delta t) \leq \Delta x .$$

According to this inequality, at the representation of the signal $X(t)$ as the sum of PBIS, the mean-square deviation is not greater than the value Δx, and that shows the possibility of restoration of the signal with high accuracy. For example, in solving the problems of monitoring, if the change of the object condition leads to the change of the corresponding components of the signal by a value greater than Δx, the corresponding parameters $q_k(i\Delta t)$ will be affected. Thus, the difference from the similar parameters will be detected at the initial stage of the defect origin in the process of forming the parameters as the combination of the corresponding time intervals of the signals $q_{n-1}(i\Delta t)$, $q_{n-2}(i\Delta t)$, …, $q_0(i\Delta t)$ of the corresponding cycle. This allows one to form and provide information about changing the condition of the controlled object. So the position-binary technology opens real opportunities for detecting the defect origin, which usually precedes major failures and emergency situations.

It is obvious that the position-binary technology can also be used for the stochastic objects. In this case, the process of solving the problem of monitoring the defect origin is also greatly simplified in comparison with the spectral technologies, and its adequacy thus improves.

It is connected with the fact that the algorithms of the processing $q_k(i\Delta t)$ in practice are realized quite easily, because each position-random function has only two values. In this case, the analysis of the random process by the signals of the PBIS is similar to the analysis of the cyclic processes. The difference is that in this case the observation period of the random process T is selected according to the principles of the correlation analysis.

As follows, the average frequency $\langle f_k \rangle$ and the period $\langle T_k \rangle$ can be determined for both periodic and stochastic objects for each PBIS. It is intuitively understood that for random and periodical noisy signals $g(t)$, the average value of zero and unit half-periods of the position signals $q_k(i\Delta t)$ can be determined by the following formula for a sufficiently long observation period:

$$\langle T_{q_k} \rangle = \langle T_{1q_k} \rangle + \langle T_{0q_k} \rangle, \tag{2.9}$$

where

$$\langle T_{1q_k} \rangle = \frac{1}{\gamma} \sum_{j=1}^{\gamma} T_{1q_{kj}}, \quad \langle T_{0q} \rangle = \frac{1}{\gamma} \sum_{j=1}^{\gamma} T_{0q_{kj}}. \tag{2.10}$$

Here γ is the number of unit and zero half-periods of the PBIS for the observation period; and j is the number of the q_kth position of the PBIS.

It was shown [14] that for a sufficiently long observation period T, the estimates of the periods $\langle T_k \rangle$ of the PBIS become nonrandom values. Thus, using them can greatly simplify solving the problems of monitoring the defect, which are traditionally solved by means of the estimations of statistical and spectral characteristics of the random processes.

2.3 Opportunities of Using Position-Binary Technology for Monitoring Technical Conditions of Industrial Objects

As stated earlier, the description of the random process can be represented by means of the corresponding frequency characteristics PBIS by using the position-binary technology of the analysis [12]. The experiments connected with the frequency properties of the PBIS show that they give the

opportunity to solve the problems of diagnostics and monitoring and are significantly simpler than traditional algorithms of the correlation and the spectral analysis [13]. In practice, they are realized more easily because each position-random function has only two values. At the same time, the average frequency $\bar{f}_k$ and the period $\bar{T}_k$ determined by means of the PBIS are nonrandom values. Due to the simplicity of their determination, solving the problems of monitoring, which traditionally are solved by means of the estimations of the statistical or the spectral characteristics of the random processes, are greatly simplified. For example, the signal $X(t)$ can be represented as the combination of PBIS $q_k(i\Delta t)$ in solving the problems of the diagnostics of the technical conditions of the stochastic objects. It is obvious that changing the conditions of the object leads to changing the combination of their average frequencies $\bar{f}_{q_0}$, $\bar{f}_{q_1}$, ..., $\bar{f}_{q_m}$. If W is the set of all possible failure states of the object, it is easy to solve the problem of diagnostics and monitoring by means of the combination or the set of combinations of the frequencies of the PBIS for each failure state of the object.

It is possible to give examples of various technical problems that can be solved be means of the PBIS. For example, voice verification can be realized by forming the combinations $q_{1_э}(i\Delta t)$, ..., $q_{m_э}(i\Delta t)$ for each word by means of quite simple software and hardware.

Let us consider the use of position-binary technology for the diagnostics of the cyclically worked objects on the example of the diagnostics of the depth-pump equipment of the oil well [45]. The signal obtained from the force sensor of the depth-pump equipment characterizing its technical condition is represented in Fig. 2.1(a).

At the normal condition of the equipment, the curve is a trapezoid [curve 1, Fig. 2.1(a)] for the period T_C, amplitude U_1, and constant U_0. For the sake of simplicity, let us suppose that $U = 9mV$ and the quantization step by amplitude is $\Delta x = 1mV$. In this case, $k \geq \log_2 9 = 4p$, i.e., $k = 4$ binary digits q_3, q_2, q_1, q_0 are required for sampling the initial signal by the amplitude.

The initial signal is broken down into the sequence of the PBIS in Fig. 2.1(b). As the figure shows, the frequency of the 1s and 0s in positions and the width of the unit signals and pauses for the given values Δx and Δt are correspondingly determined by the amplitude value of the initial signal. So, for example, at time t_1, $q_3 = 0$, $q_2 = 1$, $q_1 = 1$, $q_0 = 0$, i.e., the signal amplitude is determined by the four-digit binary code 0110 corresponding to 6 megavolts, etc.
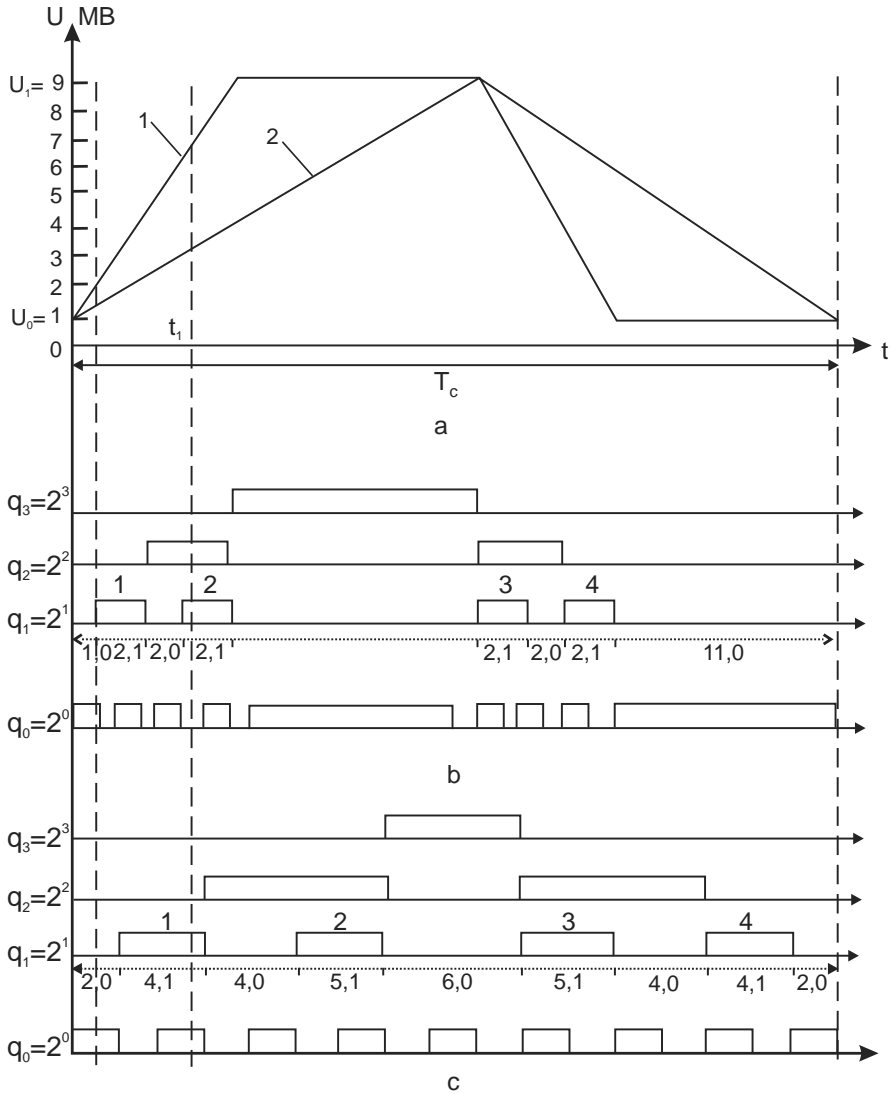
Fig. 2.1. (a) The diagram of the signal of the force sensor, (b) the PBIS for the normal state of the depth-pump equipment, (c) and that for the failure state "Plunger sticking."

In the given example, the duration of the initial signal's cycle is $T_c = 36\,\mathrm{s}$ for the step $\Delta t = 1\,\mathrm{s}$ of sampling by time. In this case, for the corresponding positions q_k, for example, for position q_1 the binary

sequence 1.0, 2.1, 2.0, 2.1, 12.0, 2.1, 2.0, 2.1, 11.0, where the first number means the duration of the interval by seconds and the second shows belonging of the interval to unit or zero states, is generated for a cycle. The similar binary sequences are generated for other positions.

During the change of the technical condition of the depth-pump equipment (for example, during the appearance of the "Plunger sticking"-type failure of a pump [45]), curve 1 [Fig. 2.1(a)] becomes similar to curve 2, and, as shown in Fig. 2.1(b) and (c), the positions and the parameters of the durations and pauses of PBIS change according to that. So, for example, the new values q_k: $q_0 = 1$, $q_1 = 1$, $q_2 = 0$, $q_3 = 0$ are generated at time t_1 due to the change of the form of the initial signal, i.e., the binary code 1100 corresponds to the amplitude of the initial signal. At the same time, the new binary sequences are generated at the corresponding positions q_k. In particular, the binary sequence similar to 2.0, 4.1, 4.0, 2.1, 4.0, 4.1, 2.0 is generated for q_1.

It is obvious that other combinations of above-mentioned time intervals are received for other failures. Diagnosing the technical state of depth-pump equipment of the oil wells can be performed by these combinations.

However, despite the obvious advantage of this technology, when solving the problem of monitoring the defect, it does not allow one to detect the beginning of its origin. That explains the necessity of analyzing the noise as the data carrier appears quite often. In turn, this requires determining the sampling step Δt_ε on the basis of the high-frequency spectrum of the noise $\varepsilon(i\Delta t)$. The frequency of noise sampling can be determined by the frequency of the change of state of the lower position-binary impulse, i.e., by the value of its average period $\langle T_{q_0} \rangle$ and the average frequency $\langle f_{q_0} \rangle$ after transforming by the frequency f_u and recording the samples of the analyzed signal $g_i(i\Delta t)$ [1, 2, 58, 59].

It is obvious that if we choose the sampling step Δt_ε based on the spectrum of the noise, it will be sufficiently less than Δt.

As shown in Section 1.5, and according to the model of the signals, the spectrum and the estimates of the corresponding characteristics continuously change on the outputs of the sensors in the process of the origin and the evolution of the defect. That sufficiently affects the accuracy of the received estimates. There is the real opportunity to perform the change of the sampling step in real time, and by this way to increase the reliability of the obtained results due to the simplicity of the realization of expressions (2.9) and (2.10).

2.4 Position-Selective Adaptive Sampling of Noisy Signals

Let us consider the opportunity of determining the sampling step of the initial signal Δt by taking into account the value of the given error ε_0 by means of the frequency properties of the PBIS.

Let us assume that the analyzed signal is processed by analog-digital conversion by the current frequency f_ν and by the certainly small sampling step of the quantization by time Δt. In this case, according to the inequalities $\Delta t_\nu \ll \Delta t$, many of these samples will be repeated due to the following equality:

$$P[X(i\Delta t)] \approx P[X((i+1)\Delta t)].$$

This explains why the values of the binary codes of the samples $X(i\Delta t)$ will also be repeated for each step $X((i+1)\Delta t)$ of quantization in the interval Δt_ν. Due to this, the frequency f_{q_0} of the lower PBIS $q_0(t)$, which can be determined by the following formula:

$$f_{q_0} = \frac{1}{\left\langle T_{q_0} \right\rangle} \tag{2.11}$$

[where $\left\langle T_{q_0} \right\rangle$ is the average value of the period of the signal $q_0(t)$] will be sufficiently less than the current frequency of sampling f_ν. At the same time, the following inequality connecting the current frequency f_ν and the cutoff frequency f_C, found by the sampling theorem, takes place:

$$f_\nu \gg f_C.$$

The value f_{q_0} can be assumed to be constant for all realizations of the same stationary random signal or cyclical signal for present ADC, i.e.,

$$f_{q_0} \approx \text{const} .$$

Thus, if we choose the value f_ν satisfying this condition, the value f_{q_0} can be determined for the analyzed signal. In this case, the following condition takes place between f_{q_0} and the cutoff frequency f_C of the signal $X(i\Delta t)$ found by known methods:

$$f_C \geq f_{q_0} .$$

At the same time, taking into account that each PBIS is formed by obtaining two impulses on the lower digit of ADC, the previous condition can be represented as follows:

$$f_C \geq 2f_{q_0}.$$

On the basis of this condition, the sampling step for the useful signal $X(i\Delta t)$ can be chosen in accordance with the following inequality:

$$\Delta t \leq \frac{1}{2f_{q_0}}.$$

In this case for determining f_{q_0}, it is necessary to determine the average period of the impulses of the lower PBIS $\langle T_{q_0} \rangle$ and the average frequency of their appearances by means of the samples of the analyzed signal after its conversion and recording in memory the frequency f_v by the following expressions:

$$\langle T_{q_0} \rangle = \langle T_{1q_0} \rangle + \langle T_{0q_0} \rangle,$$

$$f_{q_0} = \frac{1}{\langle T_{q_0} \rangle}. \tag{2.12}$$

Then the values $\langle T_{1q_0} \rangle$, $\langle T_{0q_0} \rangle$ can be found in accordance with the following expressions:

$$\langle T_{1q_0} \rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{1q_0 j} \text{ and } \langle T_{0q_0} \rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{0q_0}. \tag{2.13}$$

To ensure the necessary accuracy of the conversion is provided, it is expedient to choose Δt on the basis of the following condition:

$$\Delta t \leq \frac{1}{(2 \div 5)f_{q_0}}. \tag{2.14}$$

Experimental research has shown that in some cases measuring the time parameters of the lower digits of the PBIS is distorted by the influence of the error. Thus, in cases where the traditional technologies of the signal analysis are used, the sampling step can be determined by the frequency characteristics of the higher PBIS, i.e., by the average values of the duration of their unit T_{k1} and zero T_{k0} half-cycles. They are also determined by averaging out the time intervals in accordance with formulas (2.12), (2.13), i.e.,

$$\left\langle T_k \right\rangle = \left\langle T_{1q_k} \right\rangle + \left\langle T_{0q_k} \right\rangle, \ \left\langle T_{1q_k} \right\rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{1q_{kj}}, \ \left\langle T_{0q_k} \right\rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{1q_{kj}}. \quad (2.15)$$

Taking into account that for the random stationary signals under the normal distribution law the following approximate equality takes place:

$$\left\langle T_{q_0} \right\rangle \approx \frac{1}{2}\left\langle T_{q_1} \right\rangle \approx \frac{1}{2}\left\langle T_{q_2} \right\rangle \approx \dots \approx \frac{1}{2}\left\langle T_{n-1} \right\rangle,$$

it is advisable to determine Δt by means of the average period of the impulses of the higher digits of the PBIS. That allows one to represent the expression

$$\Delta t \geq \frac{1}{2 f_{q_0}}$$

as follows:

$$\Delta t \leq \frac{1}{2 \cdot 2^k f_k}.$$

For example, for the q_1 th PBIS, the previous formula can be represented as

$$\Delta t \leq \frac{1}{2 \cdot 2 f_1}. \quad (2.16)$$

For providing the given error of the formula of the determination Δt, (2.15) and (2.16) can be represented as follows:

$$\Delta t \leq \frac{1}{(2 \div 5) \cdot 2^k f_k}, \quad (2.17)$$

$$\Delta t \leq \frac{1}{(2 \div 5) \cdot 2 f_1}. \quad (2.18)$$

It is obvious that the use of expressions (2.12), (2.14), and (2.16)–(2.18) allows one to sufficiently simplify the determination procedure of the sampling step Δt.

Let us consider the use of these formulas for determination Δt by the above example. Let us assume that the maximum amplitude of the trapezoid

signal is 256 mV, the cycle time is 36 s, the duration of the origin-up protion is 8 s, the duration of the peak is 10 s, the fall time is 8 s, the zero value time is 10 s, and the 8-digital ADC are used for its conversion.

If we suppose that the frequency conversion of ADC is 1 kHz, the state of the first digit of ADC changes no more than 128 times because the amplitude of the signal reaches its maximum possible value of 256 mV for this time. The state of the second digit changes 64 times.

During the cycle, i.e., for 36 s, the state of the first digit changes 512 times, and the state of the second changes 256 times. It is obvious that the average frequency of the first digit is $512{:}36 = 14.2$ Hz and the average frequency of the second digit is $256{:}36 = 7.1$ Hz.

If we use the above-mentioned formulas, for the f_{q_0} th frequency, we get

$$\Delta t = \frac{1}{5 f_{q_0}} = \frac{1}{5 \cdot 14.2} = 0.0125\,\text{s}\,,$$

and for the f_{q_1} th frequency, we get

$$\Delta t = \frac{1}{5 \cdot 2 \cdot 7.1} = 0.0125\,\text{s}\,.$$

So for converting the mentioned signal by means of the 8-digital ADC, it is sufficient to realize the conversion by steps of 0.01 s, which corresponds to a sampling frequency of 100 Hz.

At the same time, the use of the sampling theorem meets various difficulties for the given signal, and the cutoff frequency appears to be more than 1000 Hz.

The given example shows that it is quite easy to determine the necessary sampling frequency taking into account the digit capacity of the ADC by software processing of the files formed as the result of the conversion of the initial signal. Thus here, in contrast to the traditional methods, the meteorological characteristics of the ADC itself are also automatically taken into account for determining the sampling step. So if the 9-digital ACD is used for the conversion of the considered signal, the found sampling frequency is equal to the average frequency of the first digit $1024{:}36 = 28.4$ kHz and the second $512{:}36 = 14.2$ Hz. At the same time, the sampling step Δt is equal to

$$\Delta t = \frac{1}{5 f_{q_0}} = \frac{1}{5 \cdot 28.4} \approx 0.0061\,\text{s}\,,$$

$$\Delta t = \frac{1}{5 \cdot 2 \cdot f_{q_1}} = \frac{1}{5 \cdot 2 \cdot 14.2} \approx 0.0061 \, \text{s} \, .$$

That corresponds to the meteorological characteristics of the ADC, while this specific property of determining the samples of signal $g(i\Delta t)$ is not taken into account in practice during the use of the traditional methods.

So the considered algorithm of the position-selective choice of the sampling frequency is quite simple. At the same time, the meteorological properties of the measuring instruments are also taken into account. Due to this property, the sampling step chosen by this method appears to be close to the sampling step chosen by means of the other most accurate methods. The software determination of the sampling step Δt, according to the above-mentioned algorithm, can be represented as follows:

1. the initial signal $X(i\Delta t)$ is converted in digital form by the super-fluous frequency f_v during the observation period T by means of ADC and the file of its samples is generated;

2. $\langle T_{q_k} \rangle$ is determined by Eq. (2.15):

$$\langle T_{q_k} \rangle = \langle T_{1q_k} \rangle + \langle T_{0q_k} \rangle ;$$

3. f_{q_k} is found by Eq. (2.12):

$$f_{q_k} = \frac{1}{\langle T_{q_k} \rangle} ;$$

4. Δt is determined by the formula

$$\Delta t \leq \frac{1}{5 \cdot 2^k f_{q_k}} \, .$$

It is necessary to perform the analysis of the noise $\varepsilon(i\Delta t)$ of the noisy signals $g(i\Delta t)$ as the data carrier when solving the problem of monitoring of the defect's origin. For this case, Δt_ε can be determined on the basis of the following condition:

$$\Delta t_\varepsilon \leq \frac{1}{5 f_{q_0}} \, .$$

Here we take into consideration that the frequency of the lower PBIS represents the most high-frequency spectrum of the total signal $g(i\Delta t)$.

Taking into account that, according to the model (1.13), the high-frequency spectrum of the total signal $g(i\Delta t)$ continuously changes in the process of the evolution of the defect, it is advisable to perform the determination $\langle T_{1q_0}^m \rangle$, $\langle T_{0q_0}^m \rangle$, $\langle T_{q_0}^m \rangle$ by the expressions

$$T_{1q_0} = \frac{1}{\gamma}\left(\sum_{j=1}^{\gamma} T_{1q_0 j} - T_{1q_{0(j-\gamma)}} + T_{1q_{0v}} \right), \tag{2.19}$$

$$T_{1q_0} = \frac{1}{\gamma}\left(\sum_{j=1}^{\gamma} T_{1q_0 j} - T_{1q_{0v-\gamma}} + T_{1q_{0v}} \right), \tag{2.20}$$

$$T_{q_0} = T_{1q_0} + T_{0q_0}, \tag{2.21}$$

$$f_{q_0} = \frac{1}{5T_{q_0}}, \tag{2.22}$$

where

$$j = 1 \div v, \ v = \gamma + 1 \div v^*,$$

$$v^* = \frac{T_1 + T_2 + T_3 + T_4}{\Delta t_\varepsilon}.$$

It is easy to ensure that the opportunity of the adaptation of the sampling step in accordance with the evolution of the defect appears during the use of the expressions (2.19) and (2.20).

It is clear that Δt_ε changes gradually by the evolution of the defect. The spectra of the noise $\varepsilon(i\Delta t)$ are close to the spectra of the useful signal, and the steps Δt_ε and Δt_ε^m are the same at period T_4.

2.5 Position-Binary Detecting Defect Origin by Using Noise as a Data Carrier

Let us consider the use of the correlation between the defect origin and the value of the noise by the position-binary technology [14, 44–46]. As mentioned earlier, the values of the binary codes of the corresponding digits

q_k of the samples $g(i\Delta t)$ of the signal $g(t)$ at the beginning of each sampling step Δt are assumed to be equal to

$$g_{\text{rem}(n-1)}(i\Delta t) = g(i\Delta t),$$

where

$$g(i\Delta t) > 2^n; \; g_{\text{rem}(n-1)}(i\Delta t) = g(i\Delta t).$$

Then the signals $q_k(i\Delta t)$ are iteratively formed as the code 1 or 0. At the same time, the samples $X(i\Delta t)$ are compared with the value $2^{n-1}\Delta g$ at the first step. The value $q_{n-1}(i\Delta t)$ is taken to be equal to 1 for $g(i\Delta t) \geq 2^{n-1}\Delta g$. And the remainder value $g_{\text{rem}(n-2)}(i\Delta t)$ is determined by the difference

$$g(i\Delta t) - 2^{n-1}\Delta g = g_{\text{rem}(n-2)}(i\Delta t). \tag{2.23}$$

The sequence of these signals $q_k(i\Delta t)$ is the position-binary-impulse signals (PBIS), the sum of which is equal to the initial signal, i.e.,

$$X(i\Delta t) \approx q_{n-1}(i\Delta t) + q_{n-2}(i\Delta t) + \ldots + q_1(i\Delta t) + q_0(i\Delta t) = X^*(i\Delta t). \tag{2.24}$$

They are reflected as the noise $\varepsilon(i\Delta t)$ during the defect origin, and the signal $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ is formed as the output of the sensor. The short-term impulses $q_{\varepsilon k}(i\Delta t)$, the duration of which is many times less than the position signals $q_k(i\Delta t)$, are formed by influence of $\varepsilon(i\Delta t)$ in the representation of $g(i\Delta t)$ by the PBIS. In [19, 20, 22, 23] it is shown that they can be marked out by the following expressions:

$$q_{\varepsilon k}^*(i\Delta t) = \begin{cases} 1, \; if \; \overline{q_k((i-1)\Delta t)} \wedge q_k(i\Delta t) \wedge \overline{q_k((i+1)\Delta t)} \vee q_k((i-1)\Delta t) \wedge \overline{q_k(i\Delta t)} \wedge q_k((i+1)\Delta t), \\ 0, \; if \; q_k((i-1)\Delta t) \wedge q_k(i\Delta t) \wedge \overline{q_k((i+1)\Delta t)} \vee \overline{q_k((i-1)\Delta t)} \wedge q_k(i\Delta t) \wedge q_k((i+1)\Delta t), \\ q_k((i-1)\Delta t) \wedge \overline{q_k(i\Delta t)} \wedge \overline{q_k((i+1)\Delta t)} \vee \overline{q_k((i-1)\Delta t)} \wedge q_k(i\Delta t) \wedge q_k((i+1)\Delta t), \end{cases}$$

$$\tag{2.25}$$

$$q_{\varepsilon k}^*(i\Delta t) = \begin{cases} 1 \; if \; q_k((i-1)\Delta t) = 0, \; q_k(i\Delta t) = 1, \; q_k((i+1)\Delta t) = 0, \\ 0 \; if \; q_k((i-1)\Delta t) = q_k(i\Delta t) = q_k((i+1)\Delta t), \\ -1 \; if \; q_k((i-1)\Delta t) = 1, \; q_k(i\Delta t) = 0, \; q_k((i+1)\Delta t) = 1. \end{cases} \tag{2.26}$$

The position noises $q_{\varepsilon k}(i\Delta t)$ of the noisy signal $X(i\Delta t)$ can be formed and marked out in the coding process of each position signal by formula (2.23) and expressions (2.25) and (2.26). It is obvious that their sum is the approximate value of the samples of the noise, i.e.,

$$\varepsilon^*(i\Delta t) \approx q_{\eta 0}(i\Delta t) + q_{\eta 1}(i\Delta t) + q_{\eta 2}(i\Delta t) + \cdots + q_{\eta k}(i\Delta t)$$

$$+ \cdots + q_{\eta(m-1)}(i\Delta t) = \sum_{k=0}^{m-1} q_{\eta k}(i\Delta t). \tag{2.27}$$

Then the approximate values of the samples of the useful signal $X^*(i\Delta t)$ can be determined by the difference

$$X^*(i\Delta t) \approx g(i\Delta t) - \varepsilon^*(i\Delta t) \approx g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t). \tag{2.28}$$

At the same time, the estimates of the variance, the estimates of the spectral characteristics of the noise, the estimates of the mutually correlation function, and the estimates of the correlation coefficient between the noise and the useful signal can be determined by the following expressions:

$$D_\varepsilon^* = \frac{1}{N} \sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N} \sum_{i=1}^{N} \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right]^2, \tag{2.29}$$

$$R_{x\varepsilon}^*(0) = \frac{1}{N} \sum_{i=1}^{N} \left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right] \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right], \tag{2.30}$$

$$r_{x\varepsilon}^* = \frac{R_{x\varepsilon}(0)}{\sqrt{D_\varepsilon R_{xx}(0)}} = \frac{\displaystyle\sum_{i=1}^{N} \left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right] \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right]}{\sqrt{\displaystyle\sum_{i=1}^{N} \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right]^2 \sum_{i=1}^{N} \left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right]^2}}, \tag{2.31}$$

$$R_{g\varepsilon}^*(0) = \frac{1}{N} \sum_{i=1}^{N} g(i\Delta t)\varepsilon(i\Delta t) \approx \frac{1}{N} \sum_{i=1}^{N} \left[g(i\Delta t) \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right], \tag{2.32}$$

$$r_{g\varepsilon} = \frac{R_{g\varepsilon}(0)}{\sqrt{D_\varepsilon R_{gg}(0)}} \approx \frac{\displaystyle\sum_{i=1}^{N} \left[g(i\Delta t) \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right]}{\sqrt{\displaystyle\sum_{i=1}^{N} \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t) \right]^2 \sum_{i=1}^{N} \dot{g}^2(i\Delta t)}}, \tag{2.33}$$

$$a_{n\varepsilon} = \frac{2}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t) \cos n\omega(i\Delta t) \approx \frac{2}{N} \sum_{i=1}^{N} \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^{*}(i\Delta t) \right] \cos n\omega(i\Delta t), \quad (2.34)$$

$$b_{n\varepsilon} = \frac{2}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t) \sin n\omega(i\Delta t) \approx \frac{2}{N} \sum_{i=1}^{N} \left[\sum_{k=0}^{m-1} q_{\varepsilon k}^{*}(i\Delta t) \right] \sin n\omega(i\Delta t). \quad (2.35)$$

So the process of the defect origin is reflected on the lower position-binary-impulse signals $q_{\varepsilon 0}(i\Delta t)$, $q_{\varepsilon 1}(i\Delta t)$, $q_{\varepsilon 2}(i\Delta t)$, ..., $q_{\varepsilon k}(i\Delta t)$, which can be marked out by the expressions (2.25) and (2.26) and can be used for determination of the estimate of the noise $\varepsilon(i\Delta t)$ by the expressions (2.29)–(2.35). Therefore, monitoring the defect at its origin becomes possible by the obtained estimates D_{ε}^{*}, $R_{x\varepsilon}^{*}(0)$, $R_{g\varepsilon}^{*}(0)$, $r_{x\varepsilon}^{*}$, $r_{g\varepsilon}^{*}$, $a_{n\varepsilon}$, and $b_{n\varepsilon}^{*}$ as a result of the use of the position-binary technology. Thus, the real opportunities of timely detection of the origin of the defects leading to the emergency condition of a diagnosed object appear.

In references [19–23] it is shown that if the condition of an object is stable, then during time T the ratio of the number $N_{\varepsilon k}$ of signals $q_{\varepsilon k}(i\Delta t)$ to the total number N_{qk} of positional-impulse signals $q_{k}(i\Delta t)$

$$K_{q_0} = \frac{N_{\varepsilon_0}}{N_{q_0 k}}, \; K_{q_1} = \frac{N_{\varepsilon_1}}{N_{q_1 k}}, \; ..., \; K_{q_{m-1}} = \frac{N_{\varepsilon_{(m-1)}}}{N_{q_{(m-1)}k}} \qquad (2.36)$$

is the nonrandom value. At the same time, from the beginning of the process of the formation of a defect in all positional signals, the number $N_{\varepsilon k}$ is increased during time T. Hence, since this time, the magnitudes of the coefficients K_{q_0}, K_{q_1}, ..., $K_{q_{m-1}}$ will also vary. Therefore, they are the informative indicators, and they can be used to increase the reliability of monitoring results when solving the problem of detecting the defect's origin.

The performed research shows that solving the problem of monitoring with traditional methods does not give satisfactory results for a great number of the most important objects. At the same time, the use of the correlation between the defect origin and the change of the coefficients K_{q_0}, K_{q_1}, K_{q_2}, ..., and other characteristics of the noise obtained by the position-binary technology give the reliable results. For example, analysis of the signals obtained in the drilling process, in the compressor station operation, etc., shows that such characteristics of the noise as D_{ε}^{*}, $R_{x\varepsilon}^{*}(0)$, $r_{x\varepsilon}^{*}$, K_{q_0}, K_{q_1}, ..., K_{q_n} contain important and useful information, allowing one to detect the process of the defect's origin.

It is obvious that their use opens great possibilities for solving the corresponding problems of monitoring. As another example, it is easy to show the possibility of the use of this technology in medicine. The performed research shows that in many cases the initial stage of the various diseases has no an effect on both the corresponding signals and the estimates of their correlation and spectral characteristics. The beginning of the pathological-physiological processes is simultaneously reflected as the noise in the electrocardiograms, electroencephalograms, and other signals sufficiently early. Their detection and analysis also open great possibilities for monitoring the beginning of various diseases by means of position-binary technology.