Dedicated to the memory of

MOSHE LIVSIC

the founding father of the
characteristic function
in operator theory

Preface

The present book deals with various types of factorization problems for matrix and operator functions. The problems appear in different areas of mathematics and its applications. A unified approach to treat them is developed. The main theorems yield explicit necessary and sufficient conditions for the factorizations to exist and explicit formulas for the corresponding factors. Stability of the factors relative to a small perturbation of the original function is also studied in this book.

The unifying theory developed in the book is based on a geometric approach which has its origins in different fields. A number of initial steps can be found in:

(1) the theory of non-selfadjoint operators, where the study of invariant subspaces of an operator is related to factorization of the characteristic matrix or operator function of the operator involved,

(2) mathematical systems theory and electrical network theory, where a cascade decomposition of an input-output system or a network is related to a factorization of the associated transfer function, and

(3) the factorization theory of matrix polynomials in terms of invariant subspaces of a corresponding linearization.

In all three cases a state space representation of the function to be factored is used, and the factors are expressed in state space form too. We call this approach the *state space method*. It has a large number of applications. For instance, besides the areas referred to above, Wiener-Hopf factorizations of some classes of symbols can also be treated by the state space method.

The present book is the second book which is devoted to the state space factorization theory. The first was published in 1979 as the monograph by H. Bart, I. Gohberg and M.A. Kaashoek, "Minimal factorization of matrix and operator functions," Operator Theory: Advances and Applications **1**, Birkhäuser Verlag. This 1979 book appeared very soon after the first main results were obtained. In fact, some of these results where published in the 1979 book for the first time.

This second book, which is written by the authors of the first book jointly with A.C.M. Ran, consists of four parts. Parts I, II and IV contain a substantial selection from the first book, in a reorganized and updated form. Part III, which covers more than a quarter of the book, is entirely new. This third part is devoted to the theory of factorization into degree one factors and its connection to the combinatorial problem of job scheduling in operations research. It also contains Maple procedures to calculate degree one factorizations. In contrast to the other parts, this third part is completely finite-dimensional and can be considered as a new advanced chapter of Linear Algebra and its Applications. Almost each chapter in this book offers new elements and in many cases new sections, taking into account a number of new results in state space factorization theory and its applications that have appeared in the period of 25 years after publication of the first book. On the other hand in the present book there is less emphasis on Wiener-Hopf integral equation and its applications than in the first book. However these topics are not entirely absent but, for instance, the applications to transport do not appear in this book. The text is largely self-contained, and will be of interest to experts and students in Mathematics, Sciences and Engineering.

The authors are in the process of writing another book, also devoted to the state space approach to factorization. There the emphasis will be on canonical factorization and symmetric factorization with applications to different classes of convolution equations. For the latter we have in mind the transport equation, singular integral equations, equations with symbols analytic in a strip, and equations involving factorization of non-proper rational matrix functions. Furthermore, a large part of this third book will deal with factorization of matrix functions satisfying various symmetries. A main theme will be the effect on factorization of these symmetries and how the symmetries can be used in effective way to get state space formulas for the factors. Applications to H-infinity control theory, which have been developed in the eighties and nineties, will also be included.

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