1 Introduction

1.1 Issues of Vagueness

Some people, like 6′ 7″ Gina Biggerly, are just plain tall. Other people, like 4′ 7″ Tina Littleton, are just as plainly not tall. But now consider Mary Middleford, who is 5′ 7″. Is she tall? Well, kind of, but not really—certainly not as clearly as Gina is tall. If Mary Middleford is kind of but not really tall, is the sentence *Mary Middleford is tall* true? No. Nor is the sentence false. The sentence *Mary Middleford is tall* is neither true nor false. This is a counterexample to the **Principle of Bivalence**, which states that every declarative sentence is either true, like the sentence *Gina Biggerly is tall*, or false, like the sentence *Tina Littleton is tall* (*bivalence* means having two values).[1] The counterexample arises because the predicate *tall* is **vague**: in addition to the people to whom the predicate (clearly) applies or (clearly) fails to apply, there are people like Mary Middleford to whom the predicate neither clearly applies nor clearly fails to apply. Thus the predicate is true of some people, false of some other people, and neither true nor false of yet others. We call the latter people (or, perhaps more strictly, their heights) ***borderline*** or ***fringe*** cases of tallness.

Vague predicates contrast with **precise** ones, which admit of no borderline cases in their domain of application. The predicates that mathematicians typically use to classify numbers are precise. For example, the predicate *even* has no borderline cases in the domain of positive integers. It is true of the positive integers that are multiples of 2 and false of all other positive integers. Consequently, for any positive integer *n* the statement *n is even* is either true or false: *1 is even* is false; *2 is even* is true; *3 is even* is false; *4 is even* is true; and so on, for every positive integer. Thus, *even* is a precise predicate. (We hasten to acknowledge that there are also vague predicates that are applicable to positive integers, e.g., *large*.)

Classical logic, the standard logic that is taught in philosophy and mathematics departments, assumes the Principle of Bivalence: every sentence is assumed to be either true or false. Vagueness thus presents a challenge to classical logic, for sentences containing vague predicates can fail to be true or false and therefore such

[1] We will italicize sentences and terms in our text when we are mentioning, that is (in the standard logical vocabulary), talking about them. An alternative convention that we do not use in this text is to place quotation marks around mentioned sentences and terms. We also italicize for emphasis; the distinction should be clear from the context.

sentences cannot be adequately represented in classical logic. "All traditional logic," wrote the philosopher Bertrand Russell, "habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial existence."[2] **Fuzzy logic**, the ultimate subject of this text, was developed to accommodate sentences containing vague predicates (as well as other vague parts of speech). One of the defining characteristics of fuzzy logic is that it admits truth-values other than *true* and *false*; in fact it admits infinitely many truth-values. Fuzzy logic does not assume the Principle of Bivalence.

Some will say, *Why bother? Logic is the study of reasoning, and good reasoning—whether it be in the sciences or in the humanities—exclusively involves precise terms. So we are justified in pursuing classical logic alone, tossing aside as don't-cares any sentences that contain vague expressions*. But as Bertrand Russell pointed out in 1923, vagueness is the norm rather than the exception in much of our discourse. Max Black concluded in 1937 that vagueness must therefore be addressed in an adequate logic for studying natural language discourse, whether that discourse occurs in scientific endeavors or in everyday casual conversations:

> Deviations from the logical or mathematical standards of precision are all pervasive in symbolism; [and] to label them as subjective aberrations sets an impassable gulf between formal laws and experience and leaves the *usefulness* of the formal sciences an insoluble mystery. . . . [W]ith the provision of an adequate symbolism [that is, a formal system] the need is removed for regarding vagueness as a defect of language. The ideal standard of precision which those have in mind who use vagueness as a term of reproach . . . is the standard of scientific precision. But the indeterminacy which is characteristic of vagueness is present also in all scientific measurement. . . . Vagueness is a feature of scientific as of other discourse.[3]

And vague predicates do abound both within and outside academic discourse: *hot*, *round*, *red*, *audible*, *rich*, and so on. After sitting in my mug for a while, my previously hot coffee becomes a borderline case of *hot*; a couple of days before or after full moon the moon may be a borderline case of *round*; as we move away from red in the color spectrum toward either orange or purple we get borderline cases of *red*; slowly turning the dial on our stereo we can move from loud (also a vague term) music to borderline cases of *audible*; and wealthy people may once have dwelled in the borderline of *rich*. Even in the realm of numbers, which we take as the epitome of precision, we have noted that we may speak of *large* (and *small*) ones, employing predicates as vague as *hot* and *round*.

But even if vagueness weren't pervasive, there are other reasons for developing logics that can handle vague statements. It is an interesting and informative exercise to see what adjustments can and need to be made to classical logic when

2 Russell (1923), p. 88.
3 Black (1937), p. 429. Black's article is a gem, with its appreciation of the pervasiveness and usefulness of vague terms and its attempt to formalize foundations for a logic that includes vague terms.

the Principle of Bivalence is dropped, and to explore ways of addressing the logical challenges posed by vagueness. Consider the classical **Law of Excluded Middle**, the claim that every sentence of the form *A or not A* is true. In classical logic, where precision is the norm, the Law of Excluded Middle is taken as a given. But while we may agree that the two sentences *Either Gina Biggerly is tall or she isn't* and *Either Tina Littleton is tall or she isn't* are both true, indeed on purely logical grounds, we may balk when it comes to Mary Middleford. *Either Mary Middleford is tall or she isn't* doesn't seem to be true, precisely because it's not true that she's tall, and it's also not true that she's not tall.

Not surprisingly, there is a close connection between the Principle of Bivalence and the Law of Excluded Middle. Negation, expressed by *not*, forms a true sentence from a false one and a false sentence from a true one. So if every sentence is either true or false (Principle of Bivalence)—then for any sentence *A*, either *A* is true or *not A* is true (the latter arising when *A* is false). And if either *A* is true or *not A* is true then the sentence *either A or not A* is also true—and this is the Law of Excluded Middle.[4]

In addition to challenging fundamental principles of classical logic, vagueness leads to a family of paradoxes known as the *Sorites paradoxes*. We'll illustrate with a Sorites paradox using the predicate *tall*. As we noted, Gina Biggerly is tall. That is the first premise of the Sorites paradox. Moreover, it is clear that $1/8''$ can't make or break tallness; specifically, someone who is $1/8''$ less tall than a tall person is also tall. That is the second premise. But then it follows that 4′ 7″ Tina Littleton is also tall! For using the two premises we may reason as follows. Since Gina Biggerly is tall, it follows from the second premise that anyone whose height is $1/8''$ less than Gina Biggerly's is also tall; that is, that anyone who is $6'\ 6^7/_8''$ is tall. But then, using the second premise again, we may conclude that anyone who is $6'\ 6^6/_8''$ is tall, and again that anyone who is $6'\ 6^5/_8''$ is tall, and so on, eventually leading us to the conclusion that Tina Litteleton, along with everyone else who is 4′ 7″, is tall.[5]

Sorites is the Greek word for "heap," and in a heap version of the paradox we have the premises that a large pile of sand—say, one that is 4′ deep—is a heap and that if you remove one grain of sand from a heap what is left is also a heap. Iterated reasoning eventually results in the conclusion that even a single grain of sand is a heap! (In fact, it looks like *no* grains of sand will also count as a heap.) The general pattern of a **Sorites paradox**, given a vague term T, is:

Premise 1 x is T (where x is something of which T is clearly true).
Premise 2 Some type of small change to a thing that is T results in something that is also T.

[4] It *is* possible to retain the Law of Excluded Middle while rejecting bivalence; this is the case for *supervaluational* logics. For references see footnote 1 to Chapter 5.

[5] Indeed, we may replace $1/8''$ with $^1/_{1000}''$ and conclude that *everyone* whose height is 6′ 7″ or less is tall. In fact, Joseph Goguen (1968–1969) pointed out that we can arrive at an even stronger conclusion: Certainly anyone who is $^1/_{1000}''$ *taller* than a tall person is also tall. So we can conclude that *everyone* is tall, given the existence of one tall person.

Conclusion y is T (where y is something of which T is clearly false, but which you can get to from a long chain of small changes of the sort in Premise 2 beginning with x).

For any vague predicate, a Sorites paradox can be formed.[6] Why are these called *paradoxes*? It is because they appear to be valid (truth-preserving) arguments with true premises, and that means that the conclusions should also be true; but the conclusions are clearly false. Sorites paradoxes are an additional motivation for developing logics to handle vague terms and statements—logics that do not lead us to the paradoxical conclusions of the Sorites paradoxes.[7]

There is a further troubling feature of the Sorites paradoxes. An obvious way out of these paradoxes in classical logic is to deny the truth of the second premise, which is sometimes called the Principle of Charity premise. For the *tall* version of the Sorites paradox just given, the classical logician can simply deny the claim that ⅛″ can't make or break tallness. The paradox dissolves, because a valid argument with false premises need not have a true conclusion. But here's the trouble: when we deny a claim, we accept its negation. This means accepting the negation of the claim that ⅛″ can't make or break tallness, namely, accepting that ⅛″ *does* (at some point) make a difference. But that can't be right since it entails that there is some pair of heights that differ by ⅛″, such that one is tall and the other is not. But where would that pair be? Is it, perhaps, the pair 6′ 2″ and 6′ 1⅞″, so that 6′ 2″ is tall but 6′ 1⅞″ isn't? To see how very unacceptable this is, change the second premise to one that states that $^1/_{1000}$″ doesn't make a difference. The conclusion, that Tina Littleton is tall, still follows. But if we deny the second premise we are saying that $^1/_{1000}$″ *does* make a difference, that there is some pair of heights differing by $^1/_{1000}$″ such that one is tall and the other isn't. That's ludicrous!

Some react to Sorites paradoxes as if they are jokes. They are not. The same type of reasoning with vague concepts, because it is so seductive, can be very dangerous in the world we live in. Consider the population of a country that has a reasonable living standard, including diet and housing, for all. Should we worry about population growth? Of course we should, because at some point population may

[6] This may seem contentious for the following reason. Some terms exhibit what I shall call *multi-dimensional vagueness*. *Tall* exhibits one-dimensional vagueness insofar as tallness is a function of a single measure, height. The Sorites argument depends on small adjustments in that single measure. Other terms' vagueness turns on several factors. Max Black (1937) asks us to consider the word *chair*. There is a multiplicity of characteristics involved in being a chair, including being made of suitably solid material, being of a suitable size, having a suitable horizontal plane for a seat, and having a suitable number of legs (a stool is not a chair). In this respect *chair* exemplifies multidimensional vagueness. The reader is asked to consider whether Sorites arguments can always be constructed for terms that exhibit multidimensional vagueness, as claimed in the text, or whether they arise mainly in the case of one-dimensional vagueness.

[7] Some theoreticians, most recently in the school of paraconsistent logics, choose to embrace Sorites paradoxes by concluding that their conclusions are indeed both true and false. See, for example, Hyde (1997) and Beall and Colyvan (2001).

outgrow the sustenance that we can provide. Now, it seems reasonable to say that if the population currently has an acceptable living standard, then if the population increases by .01 percent the living standard will still be acceptable. It may also seem reasonable to say this for *any* population increase of .01 percent, but clearly this will eventually lead to an unsustainable situation.

1.2 Vagueness Defined

Max Black defines the ***vagueness*** of a term as

> the existence of objects [in the term's field of application] concerning which it is intrinsically impossible to say either that the [term] in question does, or does not, apply.[8]

The field of application of a term is the set of those things that are the *sort* of thing that the term applies to. The field of application of the term *tall* includes people and buildings, and it excludes integers and colors. People and buildings are the sort of thing that the term applies to, the sort of thing that *could* be tall. Integers and colors are *not* the sort of thing that could be tall. On the other hand, the field of application of the term *even* includes integers and excludes people, colors, and buildings.

It is *intrinsically impossible* to say that Mary Middleford is tall or that Mary Middleford is not tall. *Intrinsically impossible* means that it is not simply a matter of ignorance—we can know exactly what Mary's height is and still find it impossible to say either that *tall* does or that *tall* does not apply to her. This contrasts with cases where our inability is simply a reflection of ignorance. For example, is the author's brother Barrie Bergmann tall? You probably can't say, because you have no idea what his height is. But Barrie is not in the fringe of this predicate—he is 6′ $3^1/_2$″ and clearly tall. Your inability was not an *intrinsic* impossibility, as it is in the case of Mary Middleford. We call those objects within a term's field of application concerning which it is intrinsically impossible to say that the term does or does not apply ***borderline*** cases, and we call the collection of borderline cases the ***fringe*** of the term. Mary is in the fringe of the term *tall*; Gina, Tina, and Barrie are not.

The opposite of *vague* is ***precise***. The term *exactly 6′ 2″ tall* is precise. Given any object in its field of application, the term either does or does not apply, and so there is no intrinsic impossibility in saying whether it does or doesn't. It applies if the object is exactly 6′ 2″ tall and fails to apply otherwise. The term *even* (as applied to positive integers) is also precise. Given any positive integer, the term either applies or fails to apply—it applies if the integer is a multiple of 2 and fails to apply otherwise.

8 Black (1937, p. 430). For the most part we will restrict our attention to terms that are *adjectives* (like *tall*), *common nouns* (like *chair*), and verbs (like to *smile*)—terms that can appear in predicate position in a sentence. Other parts of speech can also be vague; we will return to some of these in Chapter 16.

English speakers also use the word *vague* to describe terms that are not specific about the properties they connote. We will call such terms ***general*** rather than vague. The term *interesting* is general in this sense: what does it mean to say, for example, that a book is interesting? It could mean that the book contains little-known facts, that the book contains compelling arguments, that the style of writing is unusual, and so on. The term *interesting* is not very specific, unlike the term *tall*, which specifically connotes a magnitude of height (albeit underdetermined). Generality is not the source of borderline cases, which are the exclusive domain of vagueness. This is not to say that a general term cannot also be vague—indeed, this is frequently the case. For example, *interesting* is certainly vague as well as general. But it is important for our purposes to distinguish the two categorizations of terms.

Vagueness is also distinct from ambiguity. A term is **ambiguous** if it has two or more distinct meanings or connotations. For example, *light* is ambiguous: it can mean *light in color* or *light in weight*. When I say that my bicycle is light, I can mean either that it has a light color like tan or white or that it weighs very little. Note that my bicycle can be light in both senses, or that it can be light in one sense but not in the other. Indeed, the philosopher W. V. O. Quine proposed the existence of an object to which a term both does and doesn't apply as a test for ambiguity (Quine 1960, Sect. 27). Again, ambiguity is not a source of borderline cases, although an ambiguous term may also be vague in one or more of its several senses. There are objects that are borderline cases of being light in color, as well as objects that are borderline cases of being light in weight.

Finally, vagueness is also distinct from relativity. A term is **relative** if its applicability is determined relative to, and varies with, subclasses of objects in the term's field of application. Vague terms are frequently relative as well. When we say that a woman is tall, we may mean *tall for a woman*—in this case the application is relative to the class of women. In fact, we probably mean more specifically *tall for a certain race or ethnicity of women*. The applicability of the term *tall* thus varies relative to the class to which it is being applied.

1.3 The Problem of the Fringe

As we have seen, a term is vague if there exists a fringe in its field of applicability. Max Black noted another logical problem that arises from borderline cases to which the term neither applies nor fails to apply. Consider the statement that there are objects in a term's fringe:

There are objects that are neither tall nor not tall,

or equivalently

There are objects that are both not tall and not not tall.

The **Principle of Double Negation** states that a doubly negated expression is equivalent to the expression with both of the negations removed—double negations cancel out. So *not not tall* is equivalent to *tall*, and so the statement that an object is not not tall is equivalent to the statement that it is tall. But then, we can equivalently assert that there are objects in the term *tall*'s fringe as

There are objects that are not tall and also tall.

But this is a contradiction and its truth would violate the **Law of Noncontradiction**, which says that no proposition is both true and false, and specifically in this case, that no single object can both have and not have a property.[9] It looks as if the assertion that a term satisfies the criterion for vagueness, that is, the assertion that there are borderline cases, lands us in contradiction! We will call this the ***Problem of the Fringe***; it is another issue that needs to be addressed in an adequate logic for vagueness.

1.4 Preview of the Rest of the Book

This is a text in logic and in the philosophy of logic. We will study a series of logical systems, culminating in fuzzy logic. But we will also discuss ways to assess systems of logic, which lands us squarely in the philosophy of logic. Students who have taken a first course in logic are sometimes surprised to learn that we can question and critically analyze systems of logic. I hope that the issues and problems that have been introduced in this chapter make it clear that we can and will do just that: we will need to analyze systems of logic critically if we are interested in developing a logic that can handle vague statements. (If, on the other hand, we *refuse* to develop such a logic we are also taking a philosophical stand on logical issues—perhaps by insisting that the purpose of logic is to deal only with reasoning about precise claims.)

Our first task, in Chapters 2 and 3, is to review *classical* (bivalent) propositional logic and classical first-order logic. This will set out a framework for what follows and will serve to introduce notation and terminology that will be used in subsequent chapters. In Chapter 4 we introduce Boolean algebras, systems that capture the "algebraic" structure that classical logic imposes on truth-values. Boolean algebras are not usually covered in introductory symbolic logic courses, so we do not presume that the material in this chapter is a review. We include the topic because, as we will see, algebraic analyses feature prominently in the study of formal fuzzy logic systems.

[9] Actually, the earlier assertion *There are objects that are not tall and also not not tall* already violates the Law of Noncontradiction, but we follow Black in removing the double negation in order to make the point.

In Chapters 5 and 6 we will present several well-known systems of three-valued propositional logic, systems in which the Principle of Bivalence is dropped. Chapters 7 and 8 present first-order versions of the three-valued systems. In Chapter 9 we explore algebraic structures for the three-valued systems. We consider three-valued logical systems as candidates for a logic of vagueness. Some readers may feel satisfied that three-valued systems are adequate to this purpose, while others will not. Whichever is the case, the study of three-valued systems will uncover many principles that generalize very nicely as we turn to fuzzy logic.

Our very brief Chapter 10 introduces two new problems concerning vagueness that arise in three-valued logical systems. These problems will motivate the move from three-valued logic to fuzzy logic, in which formulas can have any one of an infinite number of truth-values.

Finally, Chapters 11 and 12 present fuzzy propositional logic—semantics and derivation systems; Chapter 13 introduces algebras for fuzzy logics; and Chapters 14 and 15 present fuzzy first-order logic. Chapter 16 examines augmenting fuzzy logic to include fuzzy qualifiers (like *very*: *how tall is very tall?*) and fuzzy "linguistic" truth-values (*when is a statement more-or-less true?*), and Chapter 17 addresses issues about defining membership functions (used in fuzzy logic) for vague concepts.

1.5 History and Scope of Fuzzy Logic

Formal infinite-valued logics, which form the basis for formal fuzzy logic, were first studied by the Polish logician Jan Łukasiewicz in the 1920s. Łukasiewicz developed a series of many-valued logical systems, from three-valued to infinite-valued, each generalizing the earlier ones for a greater number of truth-values. Although some of the most widely studied fuzzy logics are based on Łukasiewicz's infinite-valued system, Łukasiewicz's philosophical interest in his systems was not based on vagueness but on indeterminism—we will discuss this in Chapter 5.

In 1965 Lotfi Zadeh published a paper (Zadeh (1965)) outlining a theory of fuzzy sets, sets in which members have varying degrees of membership. Fuzzy sets contrast with classical sets, to which something either (fully) belongs or (fully) doesn't belong. One of Zadeh's examples of a fuzzy set is the set of tall men, so the relationship between vague terms and fuzzy sets was clearly established. We'll talk more about fuzzy sets as we introduce fuzzy logic. Two years after Zadeh's paper on fuzzy sets, Joseph Goguen (1967) generalized Zadeh's concept of fuzzy set, relating it to more general algebraic structures, and Goguen (1968–1969) connected fuzzy sets with infinite-valued logic and presented a formal fuzzy logical analysis of the Sorites arguments. Goguen's second article was the beginning of formal fuzzy logic, also known as *fuzzy logic in the narrow sense*.

In 1979 Jan Pavelka published a three-part article (Pavelka 1979) that provides the full framework for fuzzy logic in the narrow sense. Acknowledging his debt to Goguen, Pavelka developed a (fuzzy) complete and consistent axiomatic system for propositional fuzzy logic with "graded" rules of inference: two-part rules that state that one formula can be derived from others and that define the (minimal) degree of truth for the derived formula based on the degrees of truth of the formulas from which it has been derived. Pavelka's paper contains several important metatheoretic results as well. In 1990 Vilém Novák (1990) extended this work to first-order fuzzy logic.[10] In 1995–1997 Petr Hájek made significant simplications to these systems (Hájek 1995a, 1995b), and in 1998 he introduced an axiomatic system BL (for *basic logic*) that captures the commonalities among the major formal fuzzy logics along with a corresponding type of algebra, the BL-algebra (Hájek 1998a). Since the 1990s, Novák and Hájek have dominated the field of fuzzy logic (in the narrow sense) with several texts and numerous articles, more of which will be cited later.

In this text we are strictly concerned with fuzzy logic in the narrow sense. But when many speak of fuzzy logic they often have in mind either fuzzy set theory or fuzzy logic *in the broad sense*. Needless to say, although fuzzy set theory is *used* in fuzzy logic, it is a distinct discipline. Fuzzy logic *in the broad sense* originated in a 1975 article in which Zadeh proposed to develop *fuzzy logic* as "a logic whose distinguishing features are (i) fuzzy truth-values expressed in linguistic terms, e.g., *true*, *very true*, *more or less true*, *rather true*, *not true*, *false*, *not very true and not very false*, etc.; (ii) imprecise truth tables; and (iii) rules of inference whose validity is approximate rather than exact" (Zadeh 1975, p. 407). It is a stretch to call what has developed here a *logic*, at least in the sense in which logicians use that word.

We'll take a brief look at Zadeh's linguistic truth-values at the end of this text, since they may be used to answer at least one philosophical objection to fuzzy logic. The approximate rules to which Zadeh alludes generate reasoning such as the following (Zadeh's example):

a is small
a and b are approximately equal
Therefore, b is more or less small.

As is evident, the logic behind these rules allows us to conclude that if two objects are "approximately" equal and one has a certain property, then the other object "more or less" has that property. The rules used in computational systems based on Zadeh's fuzzy logic *in the broad sense* are like rules of thumb, are stated in English, and are quite useful in contexts such as expert systems. A typical rule for a fuzzy expert system looks like

IF temperature is high AND humidity is low THEN garden is dry

where *temperature* and *humidity* are given as data and *high*, *low*, and *dry* are measures based on fuzzy sets. Zadeh has also called his version of fuzzy logic *linguistic logic*, and perhaps that would be a more appropriate name for this general area of

[10] This and further work of Novák's appears in Novák, Perfilieva, and Močkoř (1999).

research.[11] Ruspini, Bonissone, and Predrycz (1998) is a good introduction to fuzzy logic *in the broad* sense.

Finally, we note that certain technologies advertise the use of "fuzzy logic." Fuzzy logic rice cookers have been around for a decade or so, cookers that "[do] what a real cook does, using [their] senses and intuition when [they are] cooking rice, watching and intervening when necessary to turn heat up or down, and reacting to the kind of rice in the pot, the volume and the time needed" (Wu 2003, p. E1). And there are fuzzy logic washing machines, fuzzy logic blood pressure monitors, fuzzy logic automatic transmission systems in automobiles, and so forth. The "fuzzy logic" in these cases is the circuit logic built into microchips designed to handle fuzzy measurements. For more on fuzzy technologies see Hirota (1993).

1.6 Tall People

Visit the Web site http://members.shaw.ca/harbord/heights.html. This is fun and will get you thinking about what *tall* means.

1.7 Exercises

SECTION 1.2

1 In his article "Vagueness," Max Black claimed that all terms whose application involves use of the senses are vague. For example, we use color words like *green* and shape words like *round* to describe what we see—and both of these terms are vague. The sea sometimes appears greenish, and this is typically a borderline case of *green*—not really green, but not really not green. While the moon is round when full and not round when in one of its quarters, phases close to full are borderline cases of *round* for the moon—it's not really round, but also not clearly not round.

Give examples of vague terms whose application involves each of the other senses: one for hearing, one for smell, one for taste, and one for touch. Show that your terms are vague by describing one or more borderline cases—cases of things to which the term does not clearly apply or clearly fail to apply.

2 Show that each of the following terms is vague by giving an example of a borderline case: *young, fun, husband, sport, stale, chair, many, flat, book, sleepy*.

3 Are any of the terms in question 2 also ambiguous? General? Relative? Give examples to support your claims.

[11] Not only would such a term make clear the distinction between formal fuzzy logic originating from Goguen's work and Zadeh's version of fuzzy logic; its use would also make it clear when attacks on "fuzzy logic" by logicians (such as Susan Haack [1979]) are targeting the claim that fuzzy logic "in the broad sense" is logic, rather than work done in formal fuzzy logic.