

Contents

Preface	vii
1 Predicate Logic	1
1.1 Introduction	1
Biography: Bertrand Russell	6
1.2 Propositional logic	7
1.3 Quantifiers	14
1.4 First-order languages and theories	23
Biography: Euclid	27
1.5 Examples of first-order theories.....	32
1.6 Normal forms and complexity	38
1.7 Other logics	47
2 Axiomatic Set Theory	59
2.1 Introduction	59
2.2 “Naive” set theory	61
Biography: Georg Ferdinand Cantor	62
2.3 Zermelo–Fraenkel set theory	68
Biography: John von Neumann	72
2.4 Ordinals	77
2.5 Cardinals and the cumulative hierarchy	84

xiii

xiv

Contents

3 Recursion Theory and Computability	95
3.1 Introduction	95
Biography: Emil Post	97
3.2 Primitive recursive functions	98
3.3 Turing machines and recursive functions.....	108
Biography: Alan Turing	109
3.4 Undecidability and recursive enumerability	118
3.5 Complexity theory	125
4 Gödel's Incompleteness Theorems	135
4.1 Introduction	135
4.2 The arithmetization of formal theories.....	136
Biography: Kurt Gödel	137
4.3 A potpourri of incompleteness theorems	146
4.4 Strengths and limitations of PA	156
Biography: Frank Ramsey	159
5 Model Theory	165
5.1 Introduction	165
5.2 Basic concepts of model theory	167
5.3 The main theorems of model theory	172
5.4 Preservation theorems	181
5.5 Saturation and complete theories.....	191
Biography: Julia Robinson	198
5.6 Quantifier elimination	205
5.7 Additional topics in model theory	214
6 Contemporary Set Theory	225
6.1 Introduction	225
6.2 The relative consistency of AC and GCH	228
6.3 Forcing and the independence results	234
6.4 Modern set theory and large cardinals	242
6.5 Descriptive set theory	254

Contents	xv
Biography: Nikolai Luzin.....	261
6.6 The axiom of determinacy	267
7 Nonstandard Analysis	279
7.1 Introduction	279
Biography: Archimedes	282
Biography: Leonhard Euler	287
7.2 Nonarchimedean fields	289
7.3 Standard and nonstandard models.....	293
7.4 Nonstandard methods in mathematics	304
8 Constructive Mathematics	317
8.1 Introduction	317
8.2 Brouwer's intuitionism	325
Biography: L. E. J. Brouwer	326
8.3 Bishop's constructive analysis	330
A A Deductive System for First-order Logic	347
B Relations and Orderings	349
C Cardinal Arithmetic	355
D Groups, Rings, and Fields	361
Bibliography	375
Symbols and Notation	381
Index	387