Chapter 2

Signals in the time and frequency domain

From a physical point of view signals are oscillations or waves. They are imprinted with certain information by changing according to a certain pattern.

Only electrical or electromagnetic signals are used in information technology. They have incomparable advantages compared with other forms of signals – e.g. acoustic signals.

Electric signals ....

- spread at (almost) the speed of light,
- can be directed by means of cables to where they are needed,
- can be transmitted around the world and even into space by means of aerials through the atmosphere and vacuum without cables,
- are unrivalled in the way they can be received, processed and transmitted accurately and interference-proof,
- use hardly any energy compared with other electrical and mechanical systems,
- are processed by the tiniest of chips which can all be manufactured very cheaply (fully automated production in large series),
- when used properly they do not pollute the environment and are not a health hazard.

If a signal contains information then there must be an infinite number of different signals as there is an infinite variety of information.

If one wanted to know everything about all signals and how they react to processes or systems, a course of study would inevitably tend to be infinitely long too. Since this is not possible it is necessary to look for a way of describing all signals according to a unified pattern.

The FOURIER Principle

The FOURIER Principle makes it possible to regard all signals as composed of the same unified "components". Simple experiments with DASY*Lab* or with a signal generator ("function generator"), an oscilloscope, a loudspeaker with a built-in amplifier and – most important ! – your sense of hearing, lead to the insight which the French mathematician, natural scientist and advisor to Napoleon discovered mathematically almost two hundred years ago.

Illustration 21: ***Jean Baptiste FOURIER (1768-1830)***

Fourier is regarded as one of the founders of mathematical physics. He developed the foundations of the mathematical theory of heat conduction and made important contributions to the theory of partial differential equations. He could not have dreamt of the importance that "his" FOURIER transformation would have in natural sciences and technology.

Periodic oscillations

These experiments are to be carried out with various periodic oscillations.

*Periodic oscillation*s are oscillations which are repeated over and over again in the same way after a specific period length T. Theoretically – i.e. seen in an idealised way – they last for an infinite period of time in the past, the present and the future. In practical terms this is never the case but it simplifies the approach.

In the case of many practical applications – for instance, in quartz clocks and other clock pulse generators ("timers") or in the case of network AC voltage the length of signal is so great that it almost corresponds to the ideal "infinitely long". The precision of measurement of time depends largely on how precisely periodic the reference voltage was and is and how periodic it stays.

Although it is very important for many applications, periodic oscillations are not typical signals. They hardly provide new information as their future course can be precisely predicted. The greater the uncertainty about the development of the signal at the next moment, the greater the information *may be* that is contained in it. The more we know what message will be conveyed by a source the less the uncertainty and therefore the information value. Information often seems associated more with knowledge than with the idea of uncertainty.

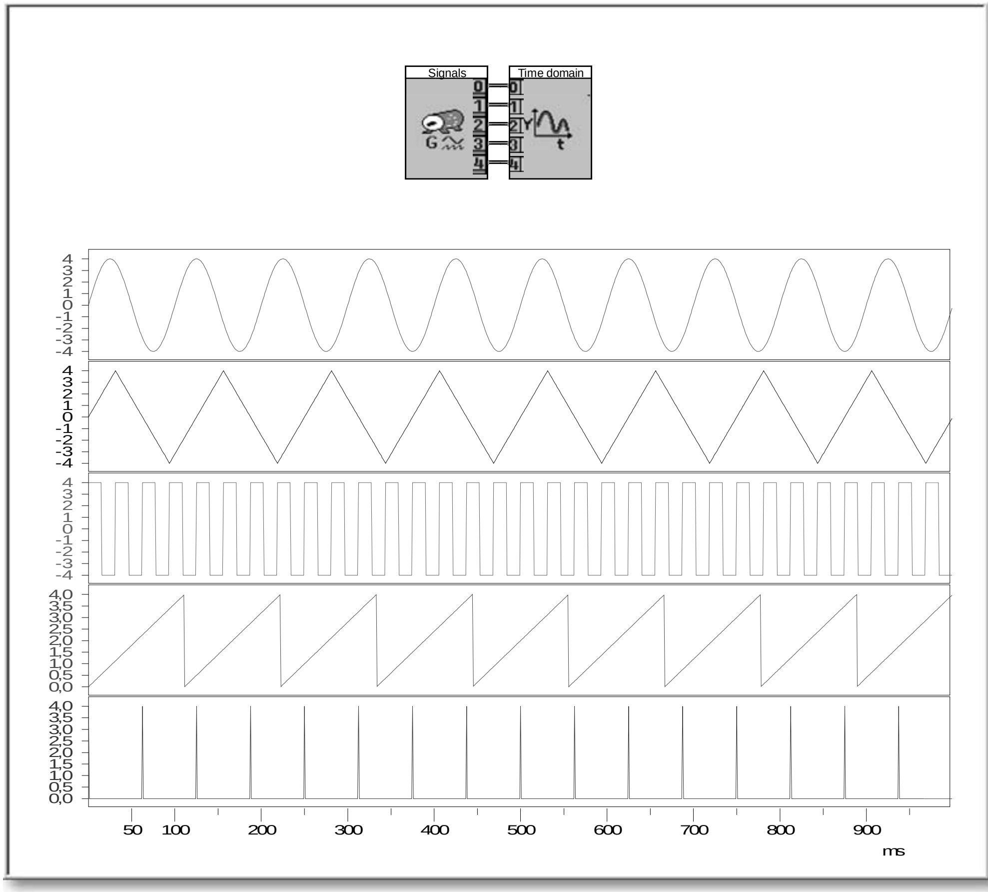

Illustration 22: ***Important periodic signals***

Here you see five important forms of periodic signals, from the top to the bottom: sine, triangle, rectangle, saw tooth and (extremely short!) pulses. From a theoretical point of view periodic signals are of infinite duration, that is, they extend far into the past and future beyond the illustrated segment. Try to determine the period length and frequency of the individual signals.

Surprisingly however, we must say that language and music are not conceivable without "near periodic" oscillations inspite of what has just been said. Periodic oscillations are easier to describe in their behaviour and that is why we are dealing with them at the beginning of this book.

Our ear as a FOURIER-analyzer

By means of very simple experiments it is possible to establish fundamental common features of different oscillation and signal forms. Simple instruments to be found in almost any collection of teaching aids are adequate for this purpose.

A function generator is able to produce different periodic AC voltages. It represents the source of the signal. The signal can be heard over the loudspeaker and can be seen on the screen of the oscilloscope or computer.

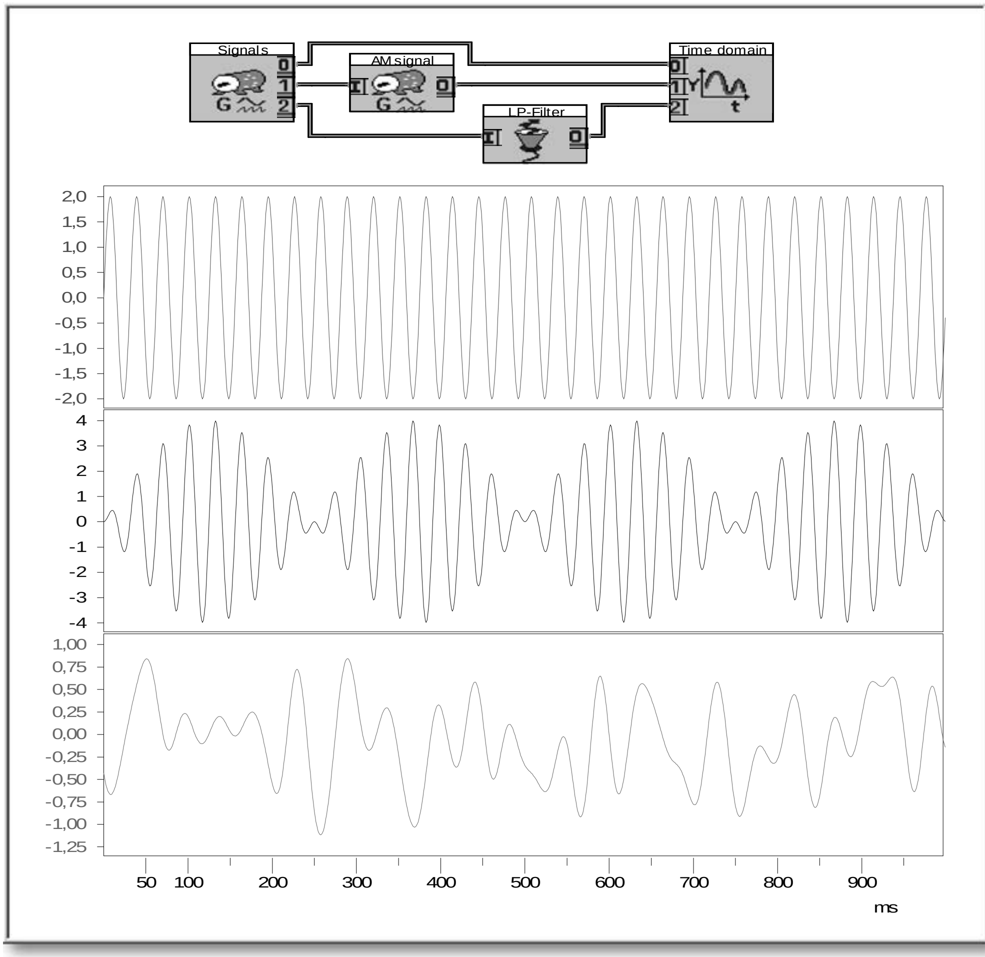

Illustration 23: ***Signal and information***

A generator module produces in the first instance three different signals the lower two of which are subsequently "manipulated". The information value of the above signals increases from the top to the bottom. The signal above is a sine whose course can be predicted exactly. After a time there is therefore no new information. The middle signal is a modulated sine signal, the amplitude follows a certain sinusoidal pattern. Finally the signal bottom right has a rather "random" course (it is filtered noise). It can be least well predicted but contains, for example, all the information about the special characteristics of the filter.

As an example first choose a periodic sawtooth voltage with the period length T = 10 ms (Frequency f = 200Hz). If one listens carefully several tones of different frequency can be heard. The higher the tone the weaker they seem in this case. If one listens longer one finds that the second lowest tone is exactly one octave higher than the lowest, i.e. twice as high as the base tone.

In the case of all the other periodic forms of signal there are several tones to be heard simultaneously. The triangle signal in Illustration 22 sounds soft and round, very similar to a recorder note. The "saw tooth" sounds much sharper, more like the tone of a violin. In this signal there are more stronger high tones (overtones) than in the "triangle". Apparently the overtones contribute to the sharpness of the tone.

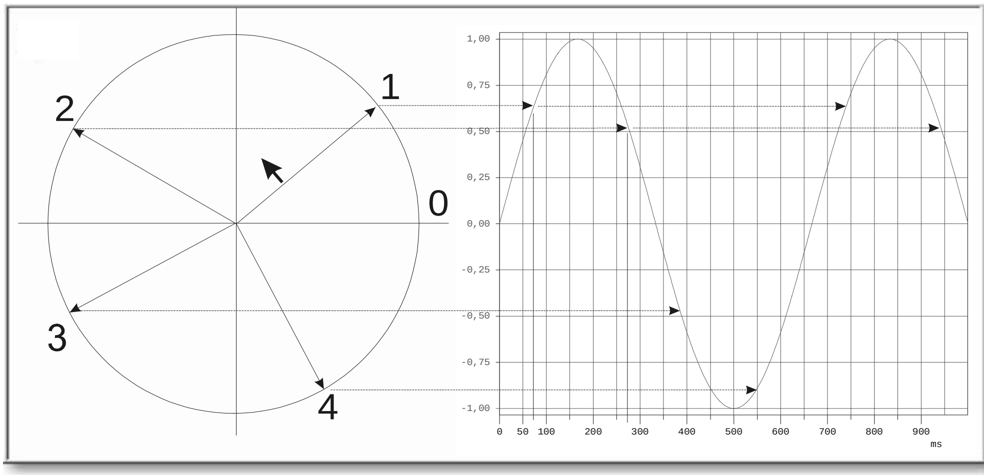

Illustration 24: ***Geometric model for the way in which a sinusoidal signal arises***

Let a pointer rotate uniformly in anti-clockwise direction, beginning in the diagram at 0. When for example the numbers express time values in ms the pointer is in position 1 after 70 ms, after 550ms in position 4 etc. The period length (of 0 to 6.28) is T = 666ms, i.e. the pointer turns 1.5 times per second. Only the projection of the pointer on to the vertical axis can be measured physically. The visible/ measurable sine course results from the pointer projections at any given moment. It should be noted that the (periodic) sinusoidal signal existed before 0 and continues to exist after 1000 ms as it lasts for an infinite length of time in theory! Only a tiny time segment can be represented, here slightly more than the period length T.

There is a single form of AC voltage which only has one audible tone: the sinusoidal signal! In these experiments it is only a question of time before we begin to feel suspicious. Thus in the "sawtooth" of 100Hz there is an audible sine of 200Hz, 300Hz etc. This means that if we could not see that a periodic sawtooth signal had been made audible our ear would make us think that we were simultaneously hearing a sinusoidal signal of 100 Hz, 200Hz, 300Hz etc.

Preliminary conclusions:

(1) There is only one single oscillation which contains only one tone: the (periodic) sinusoidal signal

(2) All the other (periodic) signals or oscillations – for instance tones and vowels contain several tones.

(3) Our ear tells us

- one tone = one sinusoidal signal
- this means: several tones = several sinusoidal signals
- All periodic signals/oscillations apart from the sine contain several tones

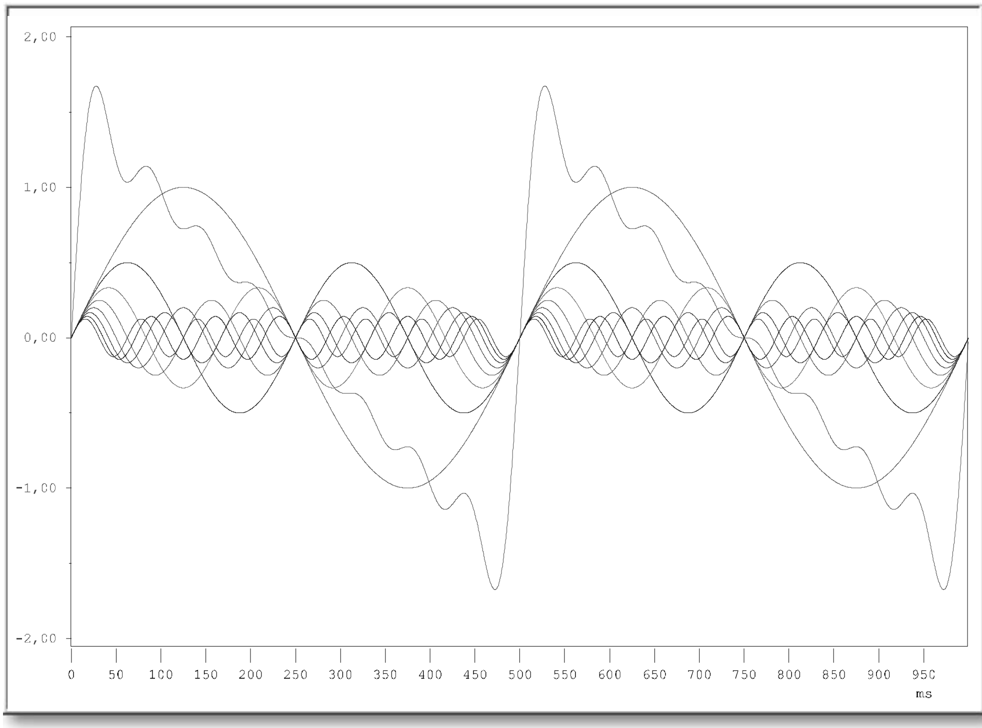

Illustration 25: ***Addition of oscillations/signals from uniform components***

This is the first Illustration of the FOURIER synthesis. Using the example of a periodic sawtooth signal it is shown that sawtooth-like signals arise by adding appropriate sinusoidal signals. Here are the first six of the (theoretically) infinite number of sinusoidal signals which are required to obtain a perfect linear saw-tooth signal with a sudden change. This example will be further investigated in the next few Illustrations. The following can be clearly seen: (a) in some places (there are five visible here) all the sinusoidal functions have the value zero: at those points the "sawtooth" or the sum has the value zero. (b) near the "jump zero position" all the sinusoidal signals on the left and the right point in the same direction, the sum must therefore be greatest here. By contrast, all the sinusoidal signals almost completely eliminate each other near the "flank zero position", so that the sum is very small.

From this the FOURIER Principle results which is fundamental for our purposes.

All oscillations/signals can be understood as consisting of nothing but sinusoidal signals of differing frequency and amplitude.

This has far-reaching consequences for the natural sciences – oscillation and wave physics – , technology and mathematics. As will be shown, the FOURIER Principle holds good for all signals, i.e. also for non-periodic and one-off signals.

The importance of this principle for signal and communications technology is based on its reversal.

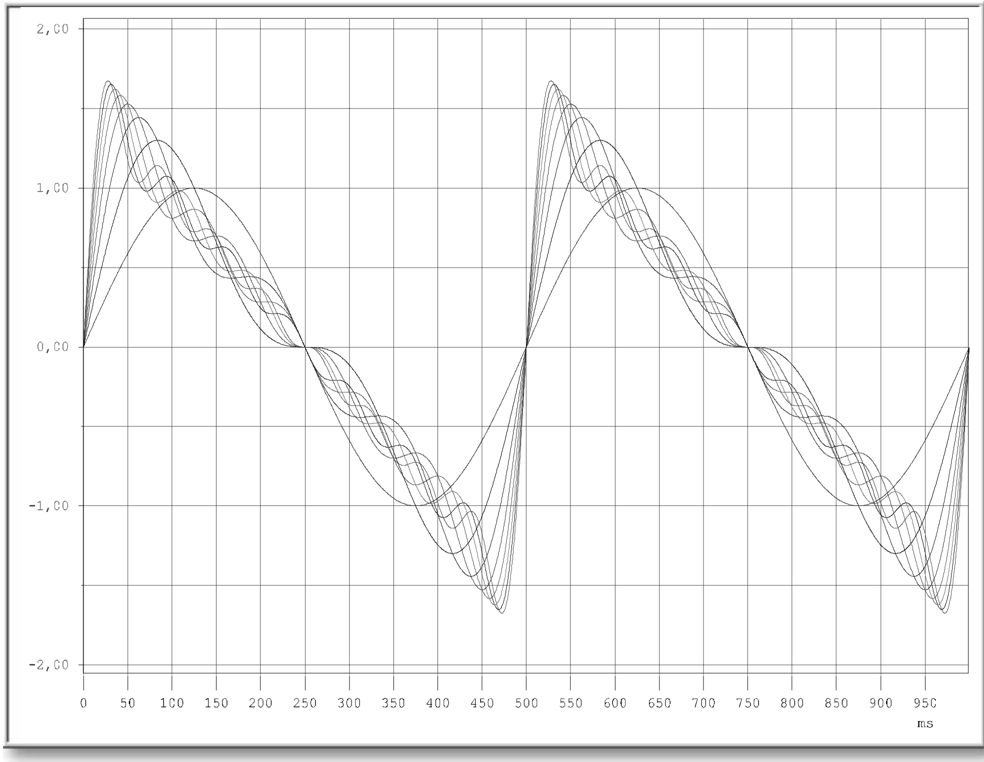

Illustration 26: ***FOURIER synthesis of the sawtooth oscillation***

It is worth looking very carefully at this picture. It shows all the cumulative curves beginning with a sinusoidal oscillation (N = 1) and ending with N = 8. Eight appropriate sinusoidal oscillations can "model" the sawtooth oscillation much more accurately than for example three (N = 3.) Please note – the deviation from the ideal sawtooth signal is apparently greatest where this oscillation changes most rapidly. First find the cumulative curve for N = 6

> *If it is known how a given system reacts to sinusoidal signals of different frequencies it is also clear how it reacts to all other signals because all other signals are made up of nothing but sinusoidal oscillations.*

Suddenly the entire field of communications engineering seems easier to understand because it is enough to to look more closely at the reaction of communications engineering processes and systems to sinusoidal signals of different frequencies.

It is therefore important for us to know everything about sinusoidal signals. As can be seen from Illustration 24 the value of the frequency f results from the angular velocity $\omega = \varphi / t$ of the rotating pointer. If the value of the full angle (equivalent to 360°) is given in rad, $\omega = 2\pi / T$ or $\omega = 2 \pi f$ applies.

In total a sinusdoidal signal has three properties. The most important property is quite definitely the frequency. It determines acoustically the height of the tone.

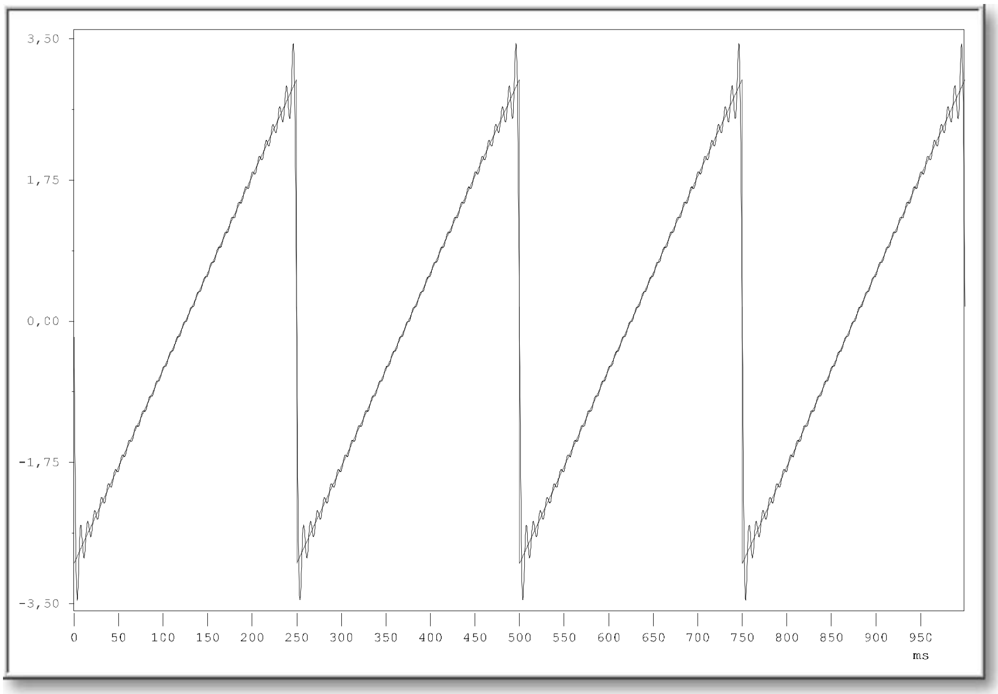

Illustration 27: ***FOURIER synthesis: the more the better!***

Here the first N = 32 sinusoidal signals were added from which a sawtooth signal is composed. At the jump position of the "sawtooth" the deviation is greatest. The cumulative function can never change faster than the sinusoidal signal with the greatest frequency (it is practically visible as "ripple content"). As the "sawtooth" at the jump position can theoretically "change infinitely rapidly", the deviation can only have disappeared when the cumulative function also contains an "infinitely rapidly changing" sinusoidal signal (i.e. $f \rightarrow \propto$). As that doesn't exist, a perfect sawtooth signal cannot exist either. In nature every change takes time!

Terms such as "frequency range" or "frequency response" are well-known. Both concepts are only meaningful in the context of sinusoidal signals:

Frequency range: the frequency range which is audible for human beings lies in a range of roughly 30 to 20,000Hz (20 kHz). This means that our ear (in conjunction with the brain) only hears acoustic sinusoidal signals between 30 and 20,000Hz

Frequency response: if a frequency response for a bass loudspeaker is given as 20 to 2500 Hz this means that the loudspeaker can only transmit acoustic waves which contains sinusoidal waves between 20 and 2500 Hz.

Note: In contrast to the term *frequency range* the term *frequency response* is only used in connection with a system capable of oscillation.

The other two – also important properties – of a sinusoidal signal are:

- *amplitude* and
- *phase angle*

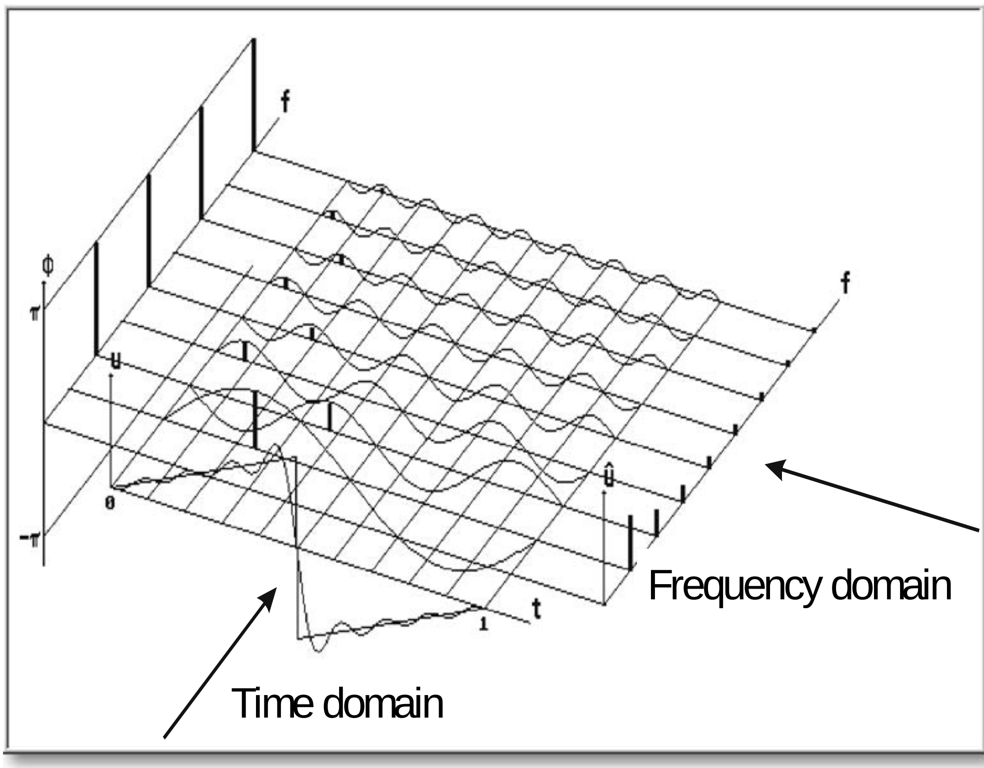

Illustration 28: ***Picture-aided FOURIER transformation***

The Illustration shows in a very graphic way for periodic signals (T = 1) how the path into the frequency range – the FOURIER transformation – arises. The time and frequency domain are two different perspectives of the signal. A "playing field" for the (essential) sinusoidal signals of which the periodic "sawtooth" signal presented here is composed serves as the pictorial "transformation" between the two areas. The time domain results from the addition of all the sine components (harmonics). The frequency domain contains the data of the sinusoidal signals (amplitude and phases) plotted via the frequency f. The frequency spectrum includes the amplitude spectrum (on the right) and the phase spectrum (on the left); both can be read directly on the "playing field". In addition the "cumulative curve" of the first eight sinusoidal signals presented here is also entered. As Illustration 26 and Illustration 27 show: the more sinusoidal signals contained in the spectrum are added, the smaller is the deviation between the cumulative curve and the "sawtooth".

The amplitude – the amount of the maximum value of a sinusoidal signal (is equivalent to the length of the pointer rotating in an anti-clockwise direction in Illustration 24) – is for example in acoustics a measure of volume, in (traditional) physics and engineering quite generally a measure of the average energy contained in the sinusoidal signal.

The phase angle φ of a sinusoidal signal is in the final analysis simply a measure of the displacement in time of a sinusoidal signal compared with another sinusoidal signal or a reference point of time (e.g. t = 0 s).

> As a reminder: The phase angle φ of the rotating pointer is not given in degrees but in "rad" (from radiant: arc of the unit circle (r = l), which belongs to this angle).

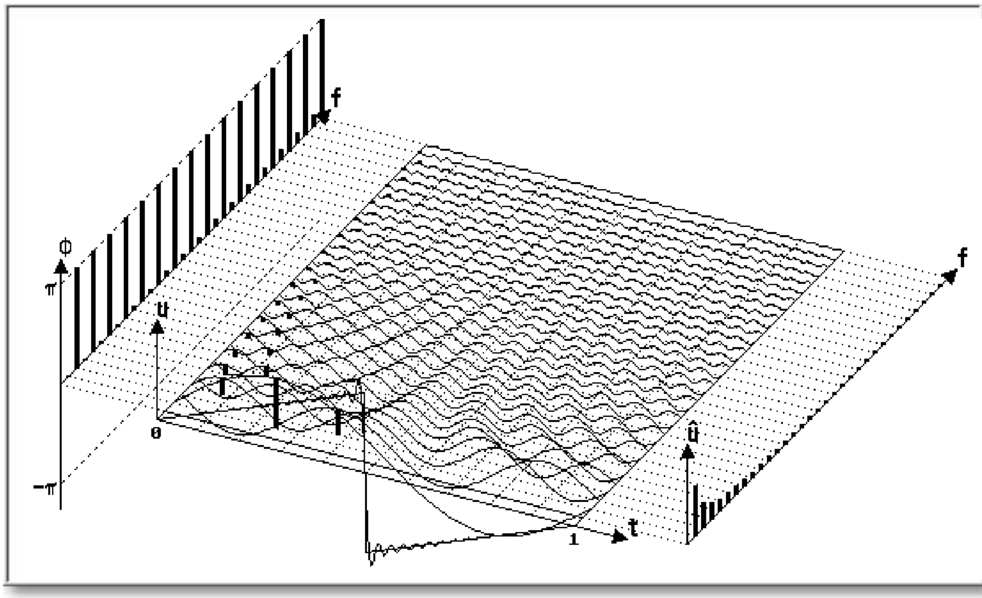

*Illustration 29: **"Playing field" of the sawtooth signal with the first 32 harmonic***

The discrepancy between the sawtooth signal and sum curve is clearly smaller than in Illustration 28. See IIllustration 27.

- Circumference of the unit circle $= 2\ \pi\ 1 = 2\ \pi$ rad
- 360 degrees are equivalent to $2\ \pi$ rad
- 180 degrees are equivalent to π rad
- 1 degree is equivalent to $\pi/180 = 0.01745$ rad
- x degrees are equivalent to $x * 0.01745$ rad
- for example, 57.3 degrees are equivalent to 1 rad

FOURIER -Transformation: from the time domain to the frequency domain and back

As a result of the FOURIER Principle all oscillations or signals are seen from two perspectives, i.e. :

the *time domain* and the

the *frequency domain*

In the *time domain* information is given on the values of a signal at any given time within a certain period of time (time progression of the values at any given moment).

In the *frequency domain* the signal is described by the sinusoidal signals of which it is composed.

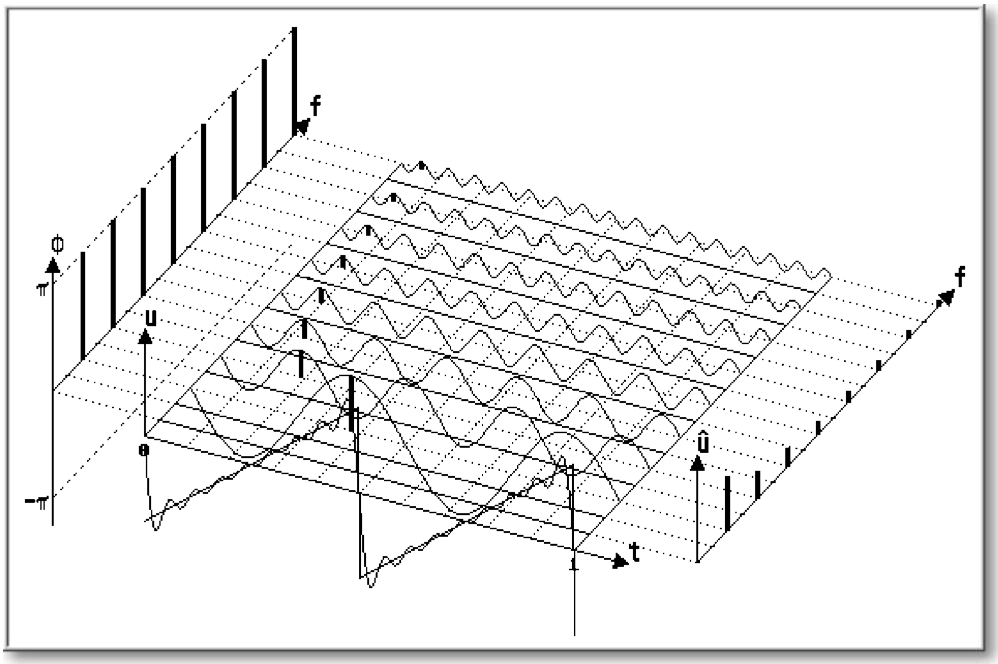

Illustration 30: ***Doubling frequency***

Here the period length of the sawtooth signal is T = 0.5s (or for example 0.5 ms). The frequency of the sawtooth signal is accordingly 2 Hz (or 2 kHz). The distance between the lines in the amplitude and phase spectrum is 2 Hz (or 2 kHz). Note the changed phase spectrum. Although it is an oversimplification it is possible to say: our eyes see the signal in the time domain on the screen of the oscillograph but our ears are clearly on the side of the frequency domain.

As we shall see in the case of many practical problems it is sometimes more useful to consider signals sometimes in the time domain and sometimes in the frequency domain.

Both ways of presenting this are equally valid, i.e. they both contain all the information. However, the information from the time domain occurs in a transformed form in the frequency domain and it takes a certain amount of practice to recognise it.

Apart from the very complicated (analogous) "harmonics analysis" measurement technique there is now a calculating procedure (algorithm) to compute the frequency-based way of presentation – the spectrum – from the time domain of the signal and vice-versa. This method is called the FOURIER transformation. It is one of the most important signal processes in physics and technology.

FOURIER-Transformation (FT):

Method of calculating the (frequency) spectrum of the signal from the progression in time.

Inverse FOURIER Transformation (IFT)

Method of calculating the progression of a signal in time from the spectrum.

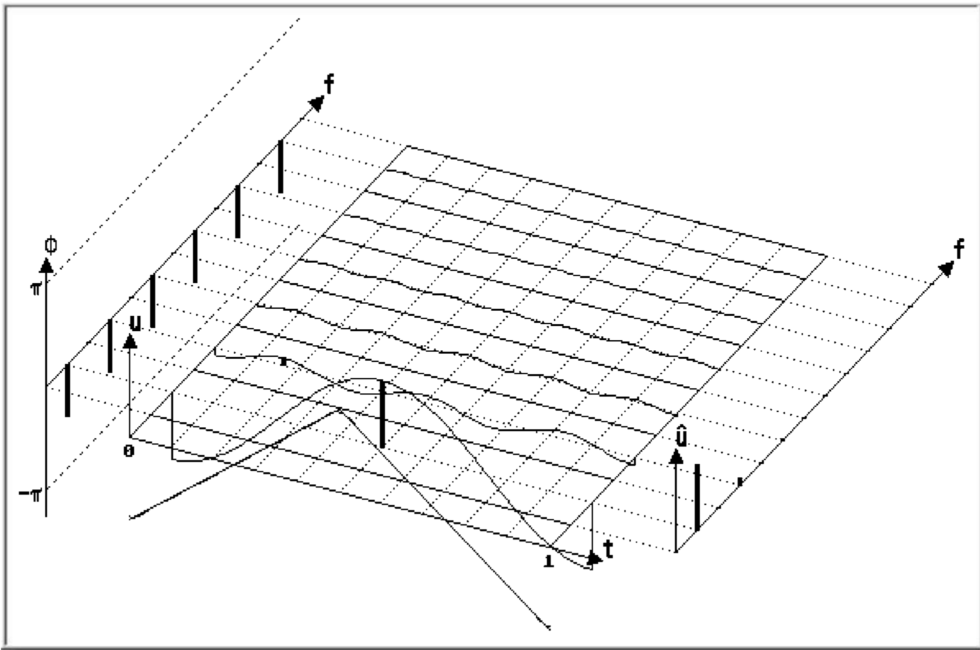

Illustration 31: ***Periodic triangle signal***

The spectrum appears to consist essentially of one sinusoidal signal. This is not surprising in that the triangle signal is similar to the sinusoidal signal. The additional harmonics are responsible for subtle differences (see sum curve). For reasons of symmetry the even-numbered harmonics are completely absent.

The computer can work out the **FT** and the **IFT** for us. We are here only interested in the results presented graphically. In the interests of a clear Illustration a presentation has been selected in which the time and frequency domain are presented together in a three-dimensional Illustration.

The FOURIER Principle is particularly well illustrated in this form of representation because the essential sinusoidal oscillations which make up a signal are all distributed alongside each other. In this way the FT is practically described graphically. It can be clearly seen how one can change from the time domain to the spectrum and vice versa. This makes it very easy to extrapolate the essential transformation rules.

In addition to the sawtooth signals the cumulative curve of the first 8, 16 or 32 sinusoidal signals (harmonics) is included. There is a discrepancy between the ideal sawtooth and the cumulative curve of the first 8 or 32 harmonics, i.e. the spectrum does not show all the sinusoidal signals of which the (periodic) sawtooth signals consist.

As particularly Illustration 25 shows the following applies for all periodic signals:

All periodic oscillations/signals contain as sinusoidal components all the integer multiples of the base frequency as only these fit into the time frame of the period length T. In the case of periodic signals all the sinusoidal signals contained in them must be repeated after the period length T in the same manner!

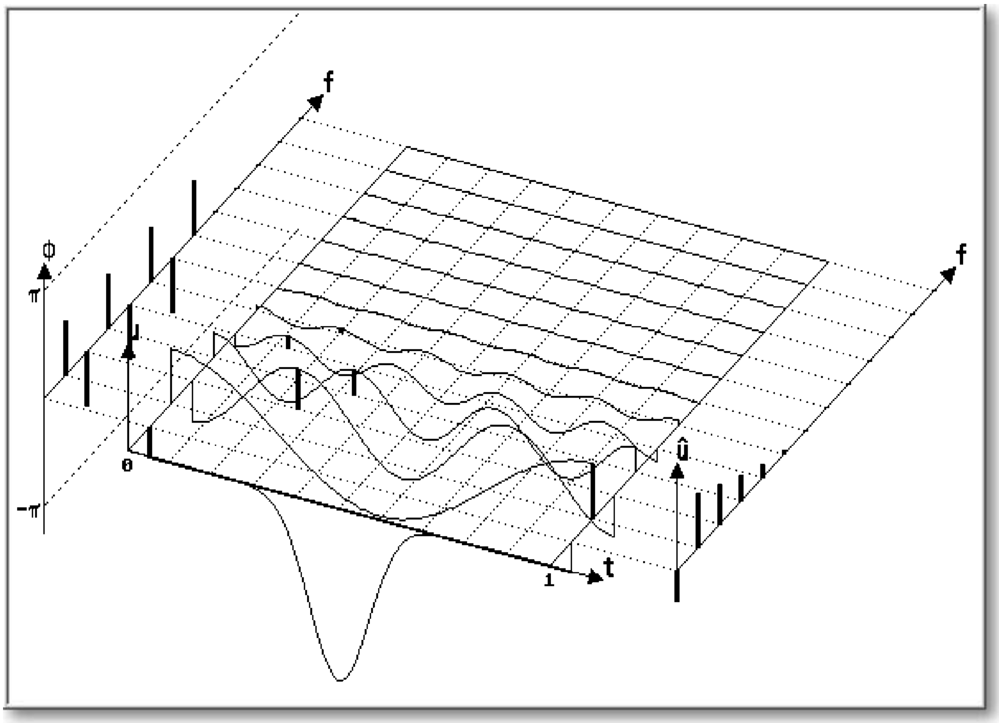

Illustration 32: ***Pulse form without rapid transitions***

Within the (periodic) sequence of GAUSSian pulses each pulse begins and ends gently. For this reason the spectrum cannot contain any high frequencies. This characteristic makes GAUSSian pulses so interesting for many modern applications. We will come across this pulse form frequently.

> *Example:* a periodic sawtooth of 100Hz only contains the sinusoidal components 100 Hz, 200 Hz, 300Hz etc.

The spectrum of periodic oscillations/signals accordingly always consists of lines at equal distances from each other.

> *Periodic signals have **line** spectra!*

The sawtooth and square wave signals contain steps in "an infinitely short space of time" from, for example 1 to -1 or from 0 to 1. In order to be able to model "infinitely rapid transitions" by means of sinusoidal signals, sinusoidal signals of infinitely high frequency would have to be present. Hence it follows:

> *Oscillations/signals with step function (transitions in an infinitely short period of time) contain (theoretically) sinusoidal signals of infinitely high frequency.*

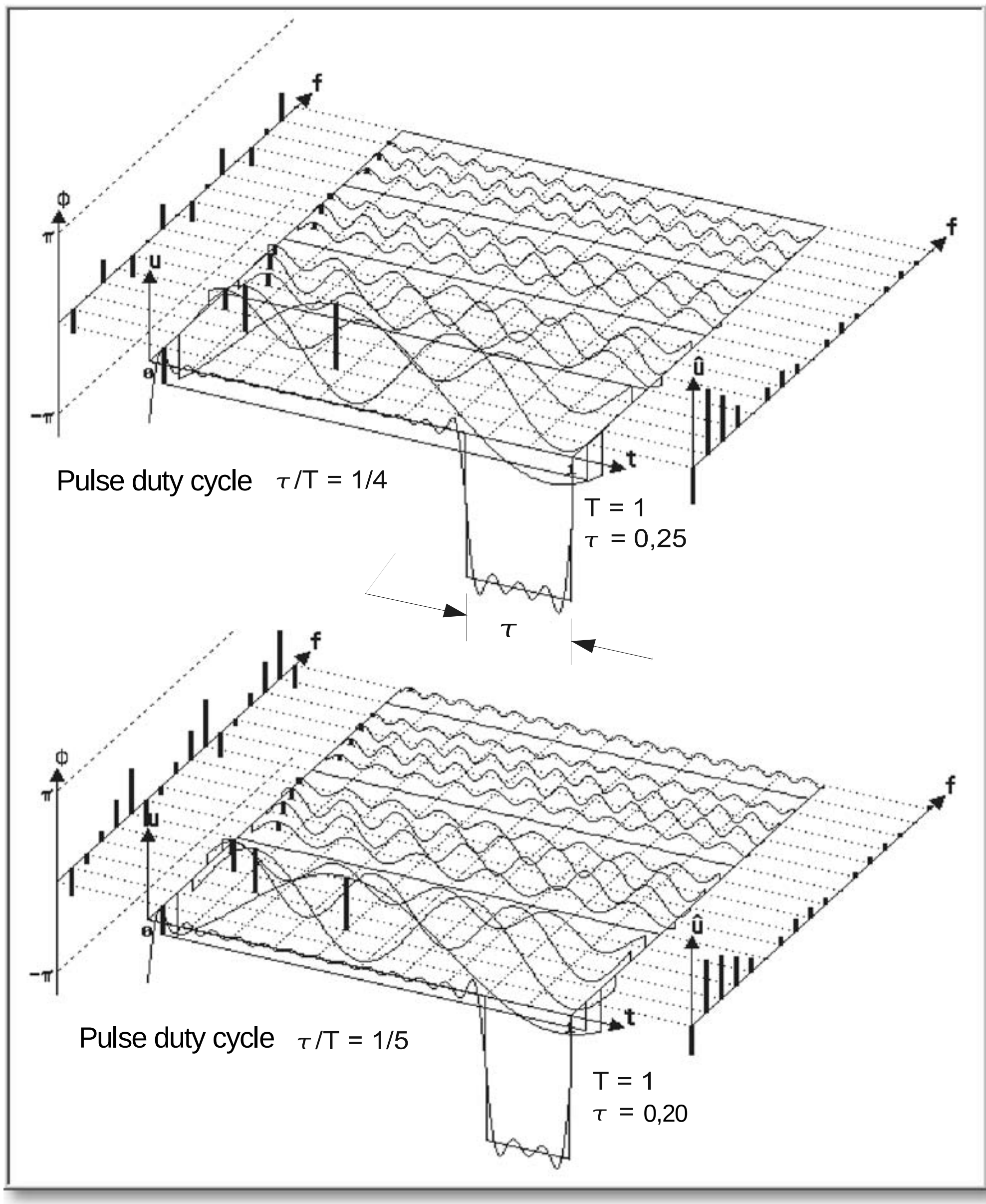

Illustration 33: ***Periodic square wave signals with different pulse duty factors***

This Illustration shows how the information from the time domain is to be found in the frequency domain. The period length T is to be found in the distance I/T of the lines of the frequency spectrum. As in this Illustration T = 1s a line distance of 1Hz results. The pulse duration τ is 1/4 in the upper representation and in the lower 1/5 of the period length T. It is striking that every fourth harmonic above (4 Hz, 8Hz etc, and every fifth harmonic below (5 Hz, 10 Hz etc) has the value 0. The zero position is in each case at the point l/τ. It is also possible to determine the period length T and the pulse duration τ in the frequence domain.

As from a physical point of view there are no sinusoidal signals "of a infinitely high frequency", in nature there cannot be signals with "infinitely rapid transitions".

In nature every change, including steps and transitions, needs time as signals/oscillations are limited as far as frequency is concerned.

As Illustration 26 and Illustration 27 show, the difference between the ideal (periodic) sawtooth and the cumulative curve is greatest where the rapid transitions or steps are present.

The sinusoidal signals of high frequency contained in the spectrum serve as a rule to model rapid transitions.

Thus, it also follows that

Signals which do not exhibit rapid transitions do not contain high frequencies either.

Important periodic oscillations/signals

As a result of the FOURIER Principle it can be taken as a matter of course that the sinusoidal oscillation is the most important periodic "signal".

Triangle and sawtooth signals are two other important examples because they both change in time in a linear fashion. Such signals are used in measuring and control technology (for example, for the horizontal deflection of the electron beam in a picture tube).

They are easy to produce. For example, a capacitor switched into a constant current source is charged linearly.

Their spectra show interesting differences. In the first place the high frequency part of the spectrum of the triangle signal is much smaller, because – in contrast to the sawtooth signal – no rapid steps occur. While in the case of the (periodic) "sawtooth" all the even numbered harmonics are contained in the spectrum, the spectrum of the (periodic) "triangle" shows only odd-numbered harmonics (e.g. 100 Hz, 300 Hz, 500 Hz etc). In other words, the amplitudes of the even-numbered harmonics equal zero.

Why are the even-numbered harmonics not required here?

The answer lies in the greater symmetry of the triangle signal. At first, the sinusoidal signal looks very similar. This is why the spectrum only shows "small adjustments". As Illustration 31 shows, only sinusoidal signals can be used as components which exhibit this symmetry within the period length T and those are the odd-numbered harmonics.

Comparison of signals in the time and frequency domain

As a result of digital technology, but also determined by certain modulation processes, (periodic) square waves or rectangular pulses have a special importance. If they serve the purpose of synchronization or the measurement of time they are aptly called clock signals. Typical digital signals are however not periodic. As they are carriers of (constantly changing) information they are not periodic or only "temporarily" so.

The so-called pulse duty factor, the quotient from the pulse duration τ and the period length T is decisive for the frequency spectrum of (periodic) rectangular pulses. In the case of the symmetrical rectangular signal $\tau/T = 1/2 = 0.5$. In this case there is symmetry as in the case of the (regular) triangle signal and its spectrum therefore contains only the odd-numbered harmonics. (see Illustration 34).

We can obtain a better understanding of these relationships by close examination of the time and frequency domains in the case of different pulse duty factors τ/T (see Illustration 33). In the case of the pulse duty factor 1/4 it is precisely the 4th, the 8th, the 12th harmonic etc which are missing, in the case of the pulse duty factor 1/5 the 5th, the 10th the 15th etc, in the case of the pulse duty factor 1/10 the 10th, 20th, 30th harmonic (see Illustration 35).

These "gaps" are termed "zero positions of the spectrum" because the amplitudes formally have the value of zero at these positions. Consequently, all the even-numbered harmonics are lacking in the case of the symmetrical rectangular signal with the pulse duty factor 1/2

It can now be seen that the core values of the time domain are "hidden" in the frequency domain:

The inverse ratio of the period length T is equivalent to the distance between the spectral lines in the spectrum. In this connection please again look carefully at Illustration 30. The frequency line distance ($f = 1/T$ equals the base frequency f_1 (1st harmonic).

Example:

T = 20 ns results in a base frequency or a frequency line distance of 50 MHz.

The inverse ratio of the pulse duration τ is equivalent to the distance ΔF_o between the zero positions in the spectrum:

Zero position distance $\Delta F_o = 1/\tau$

This allows one to draw a conclusion about the fundamental and extremely important relationship between the time domain and the frequency domain.

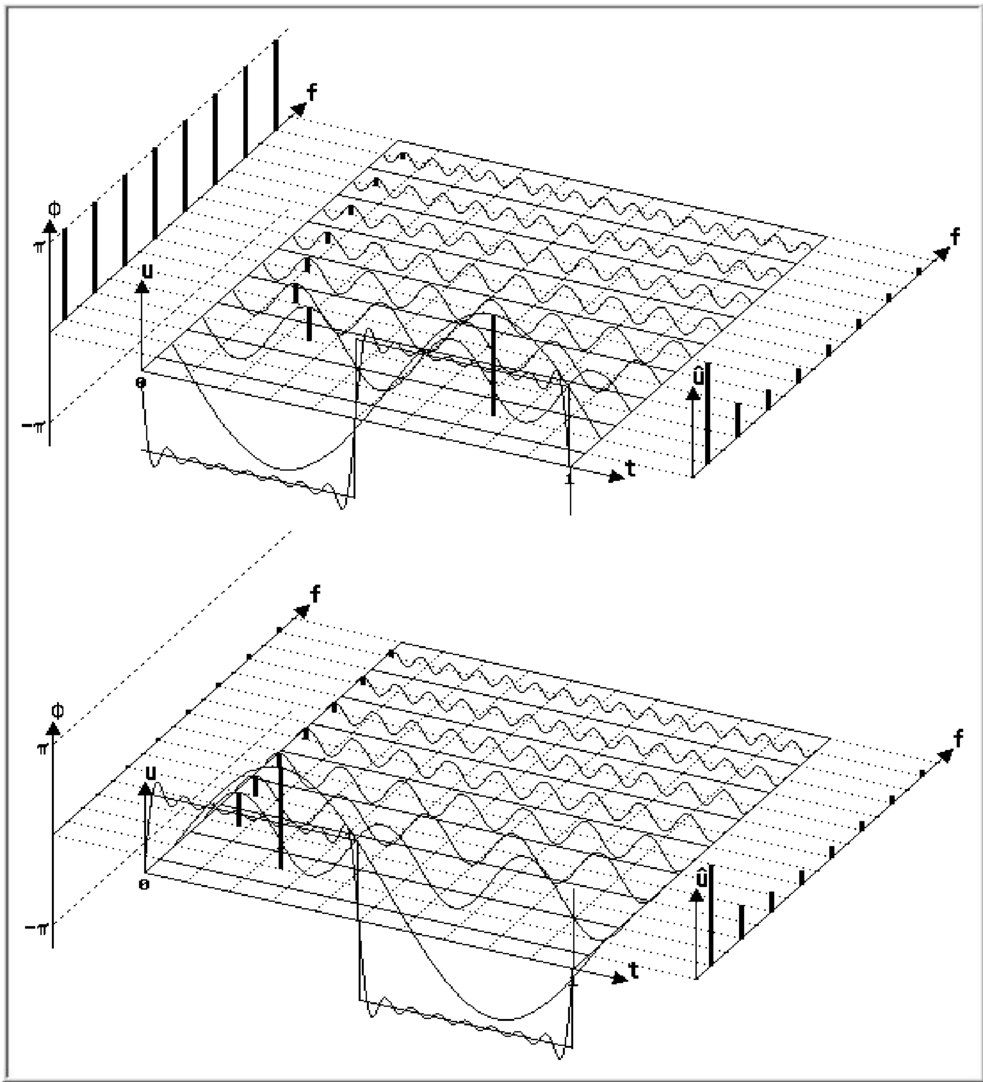

Illustration 34: ***Symmetrical rectangular pulse sequence with varying time reference point t = 0 s***

In both representations it is the same signal. The lower one is staggered compared with the upper one by T/2. Both representations have a different time reference point t = 0 s. A time displacement of T/2 is exactly equivalent to a phase displacement of π. This explains the different phase spectra. On account of τ /T = 1/2 all the even-numbered harmonics are lacking (i.e. the zero positions of the spectrum are 2 Hz, 4Hz etc).

> *All the **large** characteristic time values appear **small** in the frequency domain, all the **small** characteristic time values appear **large** in the frequency domain.*

Example: Compare period length T and pulse duration τ

The confusing phase spectrum

It is also possible to draw an important conclusion with regard to the phase spectrum. As Illustration 34 shows, the same signal can have different phase spectra. The phase spectrum depends on the time reference point t = 0.

By contrast, the *amplitude* spectrum is unaffected by time displacements.

For this reason the phase spectrum is more confusing and much less revealing than the amplitude spectrum. Hence in the following chapters usually only the amplitude spectrum will be demonstrated in the frequency domain.

Note:

- In spite of this, only the two spectral representations together provide all the information on the progression of a signal/oscillation in the time domain. The inverse FOURIER transformation IFT requires the amplitude and phase spectrum to calculate the course of the signal in the time domain.

- The property of our ear (a FOURIER analyzer!) which scarcely perceives changes in the phase spectrum of a signal is a particularly interesting phenomenon. Any important change in the amplitude spectrum is immediately noticed. In this connection you should carry out acoustic experiments with DASY*Lab*.

Interference: nothing to be seen although everything is there.

The (periodic) rectangular pulses in Illustration 33 have a constant (positive or negative) value during the pulse duration τ, but between pulses the value is zero. If we only considered these periods of time $T - \tau$, we might easily think that "there cannot be anything there when the value is zero", i.e. no sinusoidal signals either.

This would be fundamentally erroneous and this can be demonstrated experimentally. In addition, the FOURIER Principle would be wrong (why?). One of the most important principles of oscillation and wave physics is involved here:

(Sinusoidal) oscillations and waves may extinguish or strengthen each other temporarily and locally (waves) by addition.

In wave physics this principle is called *interference*. Its importance for oscillation physics and signal theory is too rarely pointed out.

Let us first off all look at Illustration 33 again. The cumulative curve of the first 16 harmonics has everywhere been – intentionally – included. We see that the sums of the first 16 harmonics between the pulses equal zero only in a very few places (zero crossings), otherwise they deviate a little from zero. Only the sum of an infinite number of harmonics can result in zero. On the sinusoidal "playing field" we see that all the sinusoidal signals of the spectrum remain unchanged during the entire period length T.

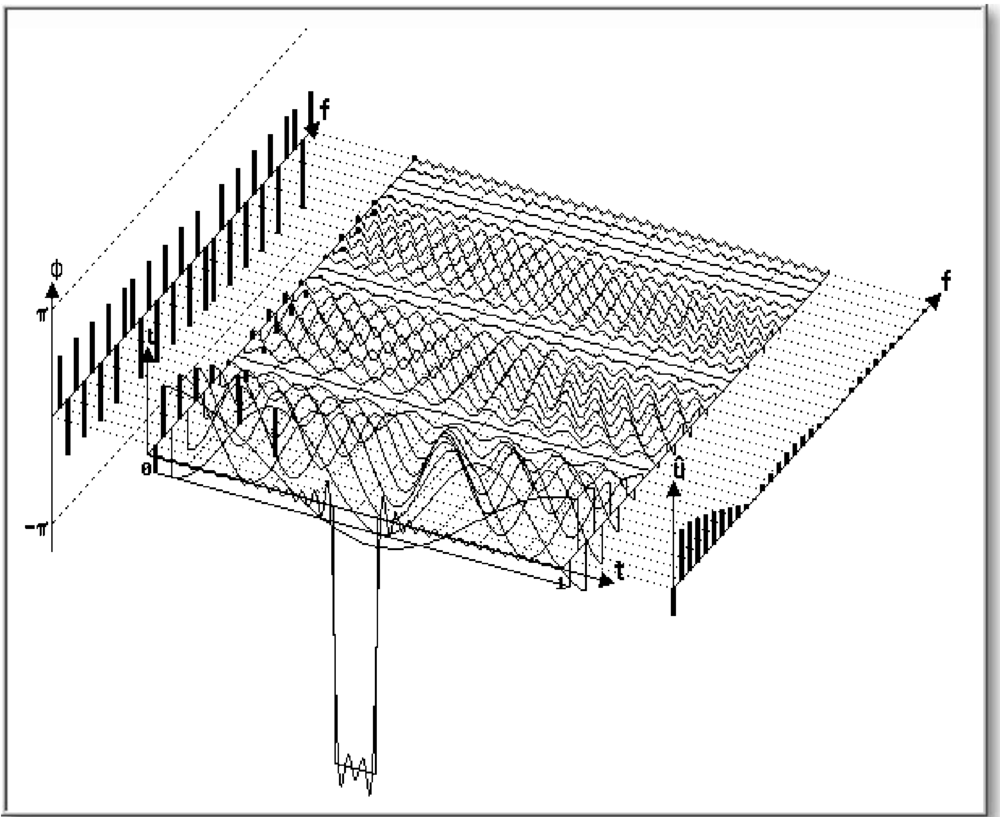

Illustration 35: ***An exact analysis of relationships.***

In this Illustration the important relationships are to be summarised once again and additions made:

- *The pulse duty factor of the (periodic) rectangular pulse sequence is 1/10. The first zero position of the spectrum lies at the 10th harmonic. The first 10 harmonics lie at the position t = 0.5 s in phase so that in the centre all the "amplitudes" add up towards the bottom. At the first and every further zero position a phase step of π rad takes place. This can easily be recognised both in the phase spectrum itself and also on the "playing field" . In the middle all the amplitudes overlay each other at the top and afterwards – from the 20th to the 30th harmonic towards the bottom again etc.*
- *The narrower the pulse becomes, the bigger the deviation between the sum of the first (here N = 32) harmonics and the rectangular pulse appears. The difference between the latter and the cumulative oscillation is biggest where the signal changes most rapidly, for example at or near the pulse flanks.*
- *Where the signal is momentarily equivalent to zero – to the right and left of a pulse – all the (infinite number of) sinusoidal signals add up to zero; they are present but are eliminated by interference. If one "filters" out the first N = 32 harmonics from all the others this results in the "round" cumulative oscillation as represented; it is no longer equivalent to zero to the right and left of the pulse. The ripple content of the cumulative oscillation is equal to the highest frequency contained.*

Even when the value of signals is equal to zero over a time domain Δt, they nevertheless contain sinusoidal oscillations during this time. Strictly speaking, "infinitely" high frequencies must also be contained because otherwise only "round" signal progressions would result. The "smoothing out effect" is the result of high and very high frequencies.

In Illustration 35 we see a value ("offset") in the amplitude spectrum at the position f = 0. On the "the playing field" this value is entered as a constant function ("zero frequency"). If we were to remove this value –U – for instance by means of a capacitor – the previous zero field would no longer be zero but +U. Thus the following holds true:

If a signal contains a constant part during a period of time Δt the spectrum must theoretically contain "infinitely high" frequencies.

In IIllustration 35 there is a (periodic) rectangular pulse with the pulse duty factor 1/10 in the time and frequency domain. The (first) zero position in the spectrum is therefore at the 10th harmonic.

The first zero position of the spectrum is displaced further and further to the right in Illustration 36 the smaller the pulse duty factor selected (e.g. 1/100). If the pulse duty factor approaches zero we have a (periodic) delta pulse sequence whereby the pulse duration approaches zero.

Opposites which have a great deal in common: sine and δ–pulse

Such needle pulses are called δ-pulses (delta-pulses) in the specialised theoretical literature. After the sinusoidal signal the δ-pulse is the most important form of oscillation or time function.

The following factors support this:

- In digital signal processing (DSP) number-strings are processed at regular time intervals (clock pulse frequency). These strings pictorially represent a sequence of pulses of a certain magnitude. Number 17 could for instance be equivalent to a needle pulse magnitude of 17. More details will be given later in the chapters on digital signal processing.

- Any signal can theoretically be conceived of as being composed of as a continuous sequence of δ–pulses of a certain magnitude following each other. See Illustration 37 in this connection.

- A sinusoidal signal in the time domain results in a "needle function" (δ-function) in the frequency domain (line spectrum). What is more – all periodic oscillations/signals result in line spectra that are equidistant (appearing at the same intervals) delta functions in the frequency domain.

- From a theoretical point of view, the δ-pulse is the ideal test signal for all systems. If a δ-pulse is connected to the input of a system, the system is tested at the same time with all frequencies and, in addition, with the same amplitude. See the following pages, especially Illustration 36.

- The (periodic) δ-pulse contains in the interval $\Delta f = 1/T$ all the (integer multiples) frequencies from zero to infinity always with the same amplitude.

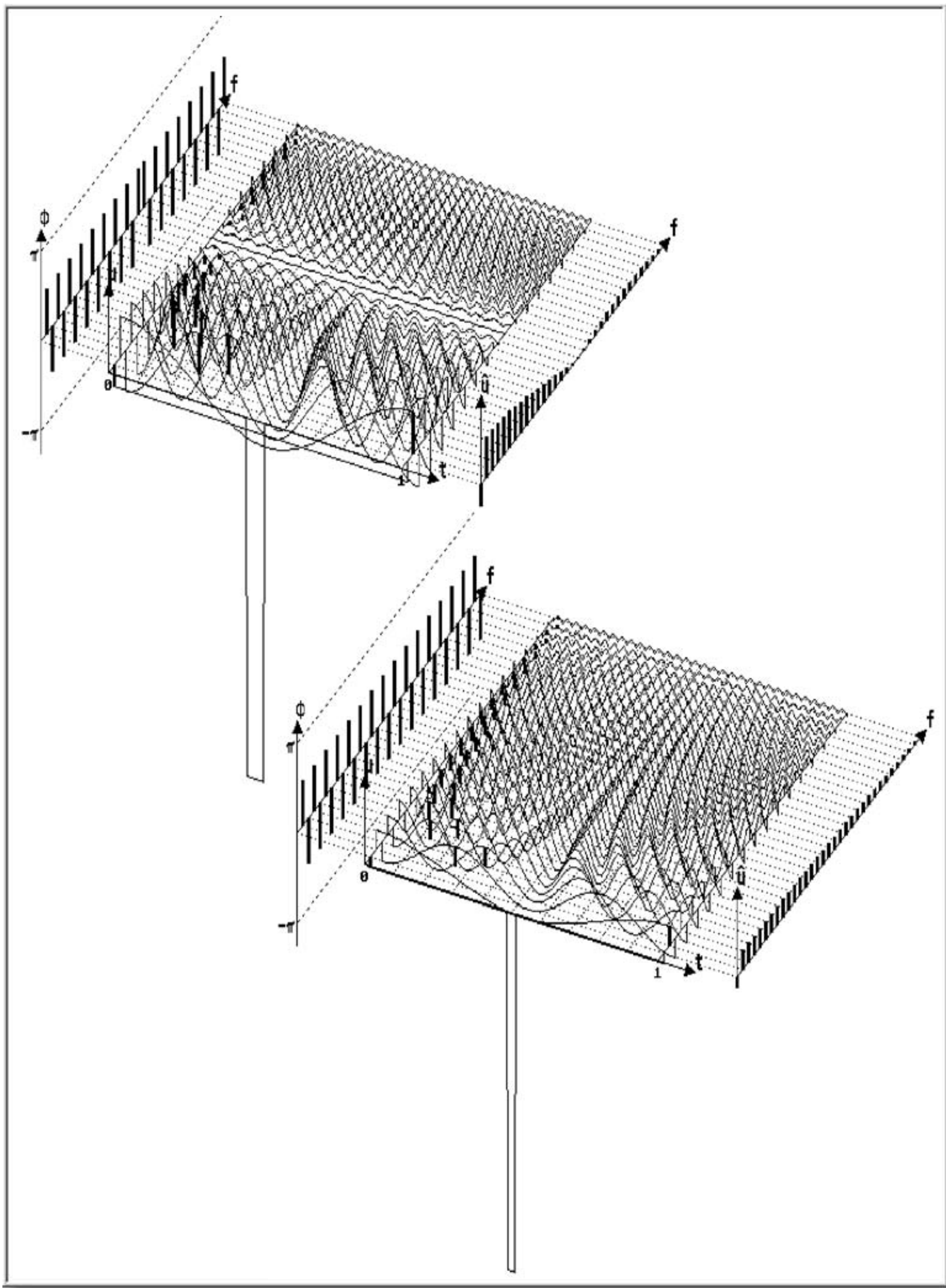

Illustration 36: ***Steps in the direction of a δ–pulse***

The pulse duty factor above is roughly 1/16 above and 1/32 below. Accordingly, the the first zero position above is at N = 16, and below at N = 32. The zero position "moves" towards the right with higher frequencies if the pulse becomes narrower. Below, the lines of the spectrum represented seem to have almost equally large amplitudes. In the case of a "needle" pulse or δ-pulse the width of the pulse tends towards zero, thus the (first) zero position of the spectrum tends toward infinity. Hence, the δ-pulse has an "infinitely wide frequency spectrum"; in addition, all the amplitudes are the same.

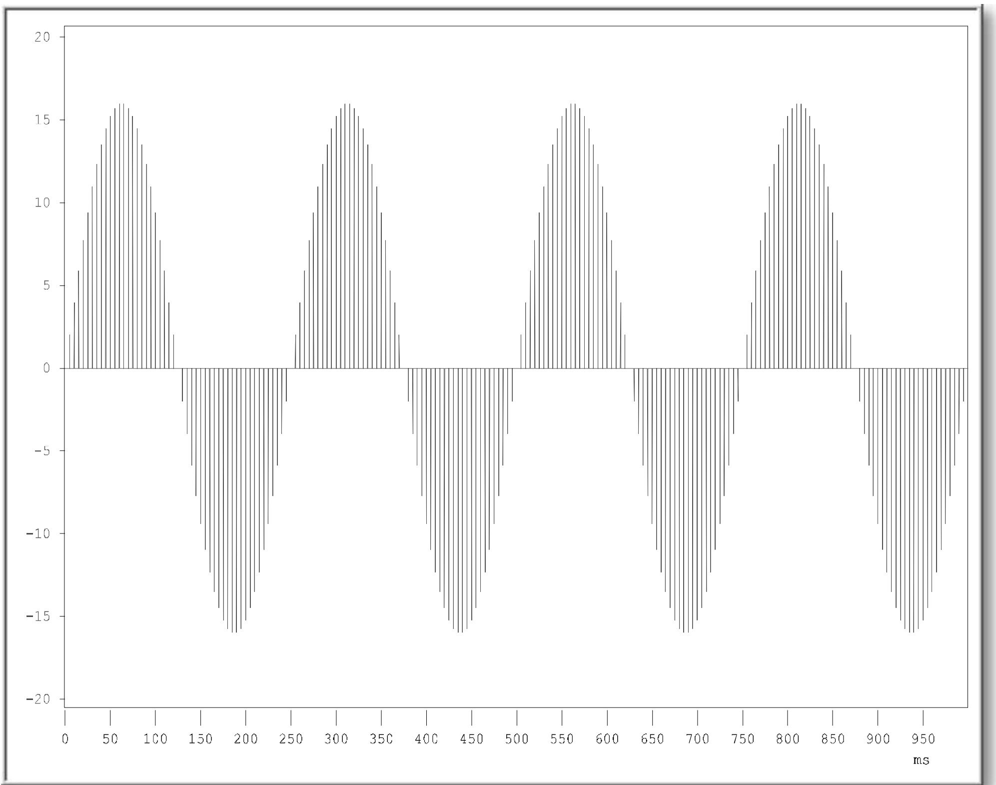

Illustration 37: ***Signal synthesis by means of δ-pulses***

Here a sine wave is "assembled" from δ-pulses of an appropriate magnitude following on each other. This is exactly equivalent to the procedure in "digital signal processing" (DSP). Their signals are equivalent to "strings of numbers" which, seen from a physical point of view, are equivalent to a rapid sequence of measurements of an analog signal; every number gives the "weighted" value of the δ-pulse at a given point of time t.

This strange relationship between sinusoidal and needle functions (Uncertainty Principle) will be looked at more closely and evaluated in the next chapter.

> *Note:*
>
> Certain mathematical subtleties result in the δ-pulse being theoretically given an amplitude tending to infinity. Physically this also makes a certain sense. An "infinitely short" needle pulse cannot have energy unless it were "infinitely high". This is also shown by the spectra of narrow periodic rectangular pulses and the spectra of δ-pulses. The amplitudes of individual sinusoidal signals are very small and hardly visible in the Illustrations, unless we increase the pulse amplitude (to extend beyond the screen of the PC).

For purposes of Illustration we normally choose delta pulses of magnitude "1" in this book.

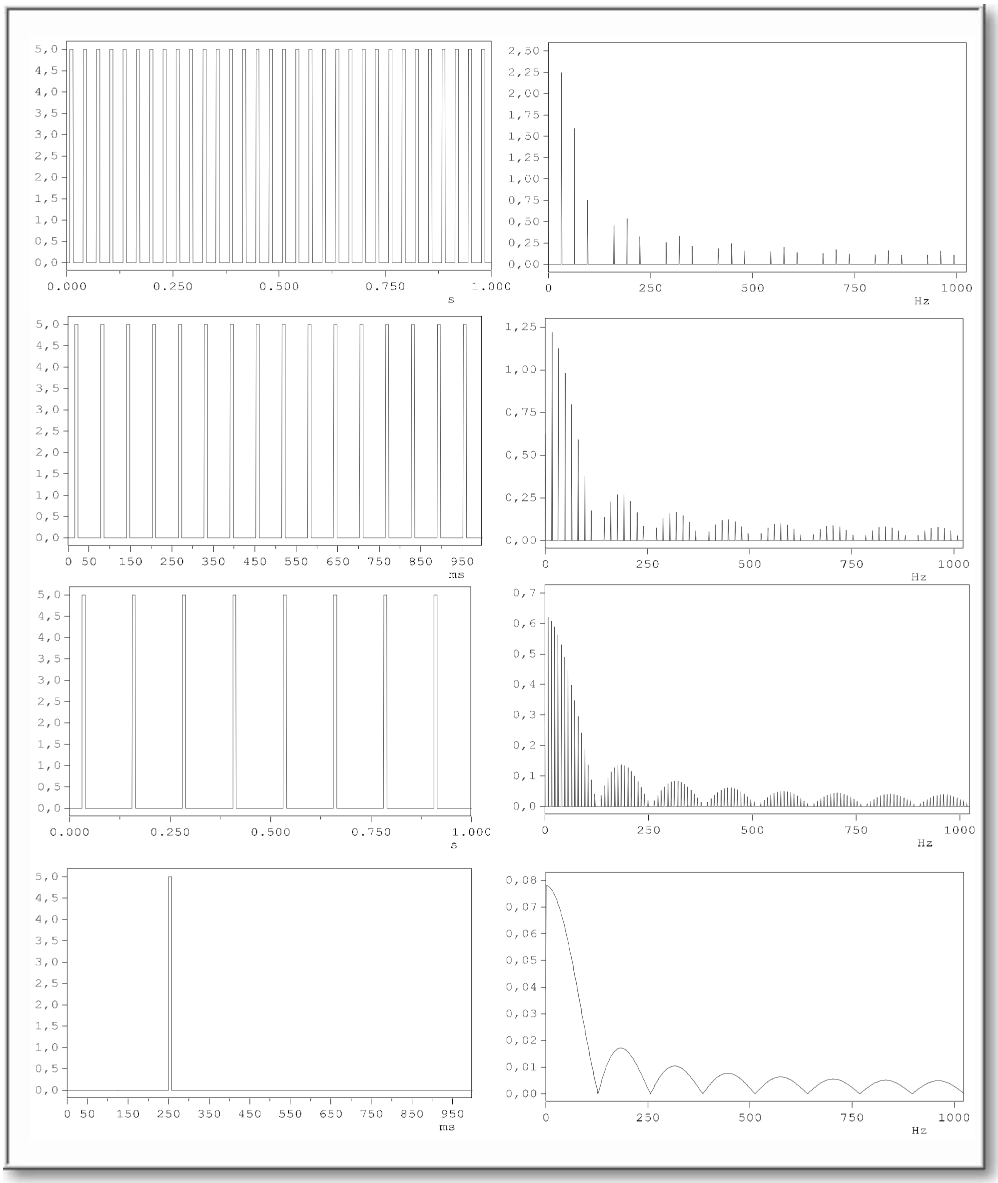

Illustration 38: ***From the periodic signal with a line spectrum to the non-periodic signal with a continuous spectrum.***

On the left in the time domain you see sequences of periodic rectangular pulses from top to bottom. The pulse frequency is halved in each case but the pulse width remains constant. Accordingly the distance between the spectral lines becomes smaller and smaller (T = 1/f), but the position of the zero positions does not change as a result of the constant pulse duration.

Finally, in the lower sequence a one-time rectangular pulse is depicted. Theoretically it has the period length $T \to \infty$ *. The spectral lines lie "infinitely close" to each other, the spectrum is continuous and is drawn as a continuous function.*

We have now gone over to the customary (two-dimensional) representation of the time and frequency domains. This results in a much more accurate picture in comparison to the "playground" for sinusoidal signals used up to now.

Non-periodic and one-off signals

In actual fact a periodic oscillation cannot be represented in the time domain on a screen. In order to be absolutely sure of its periodicity, its behaviour in the past, the present and the future would have to be observed. An (idealised) periodic signal repeated itself, repeats itself and will repeat itself in the same way. In the time domain only one or a few periods are shown on the screen.

It is quite a different matter in the frequency domain. If the spectrum consists of lines at regularly spaced intervals, this immediately signals a periodic oscillation. In order to underline this once again – there is at this moment only one (periodic) signal whose spectrum contains precisely one line – the sinusoidal signal.

We shall now look at the non-periodic signals which are more interesting from the communcations technology point of view. As a reminder: all information-bearing oscillations (signals) may have a greater information value the more uncertain their future course is (see Illustration 23).

In the case of periodic signals their future course is absolutely clear.

In order to understand the spectra of non-periodic signals we use a small mental subterfuge. Non-periodic means that the signal does not repeat itself "in the foreseeable future". In Illustration 36 we constantly increase the period length T of a rectangular pulse without changing its pulse duration until it finally tends "towards infinity". This boils down to the sensible idea of not attributing the period length $T \to \infty$ ("T tends towards infinity") to all non-periodic or one-off signals.

If however the period length becomes greater and greater the distance (f = 1/T between the lines in the spectrum gets smaller and smaller until they "fuse". The amplitudes ("end points of lines") no longer form a discrete sequence of lines at regular intervals but now form a continuous function (see Illustration 38).

> *Periodic oscillations/signals have a discrete line spectrum whereas* ***non-periodic*** *oscillations/signals have a* ***continuous*** *spectrum.*

A glance at the spectrum is enough to see what type of oscillation is present – periodic or non-periodic. As is so often the case the dividing line between periodic and non-periodic is not entirely unproblematical. It is occupied by an important class of signals which are termed *near-periodic*.These include language and music, for instance.

One-off signals are, as the word says, non-periodic. However, non-periodic signals which only change within the period of time under consideration, for instance a bang or a glottal stop, are also called non-periodic.

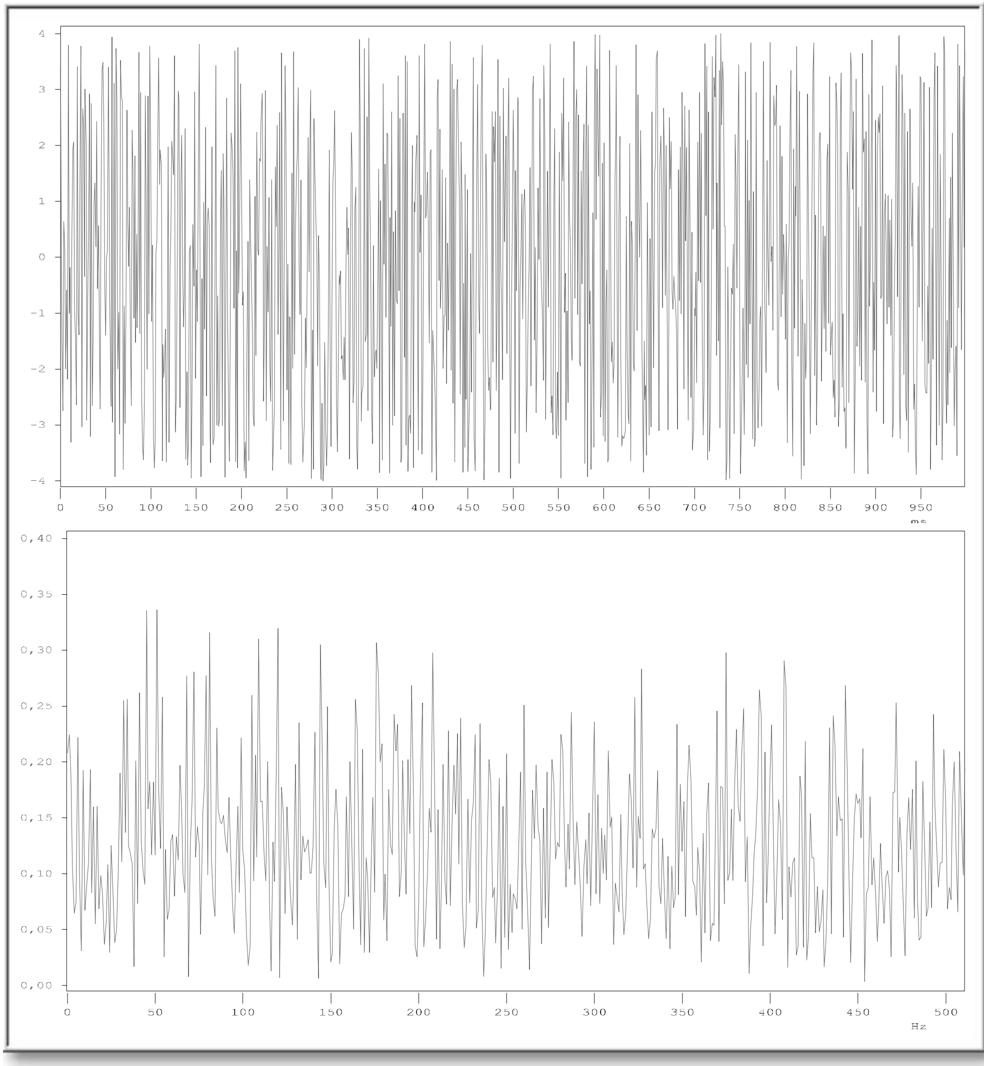

Illustration 39: ***Stochastic noise***

The upper picture shows stochastic noise in the time domain (for 1s) and below this the amplitude spectrum of the above noise. As the time domain develops randomly regularity of the frequency spectrum within the period of time under consideration is not to be expected (otherwise the signal would not be stochastic). In spite of many "irregular lines" it is not a typical line spectrum for otherwise the time domain would have to be periodic!

Pure randomness: stochastic noise

Noise is a typical and extremely important example of a non-periodic oscillation. It has a highly interesting cause, namely a rapid sequence of unpredictable individual events.

In the roar of a waterfall billions of droplets hit the surface of the water in a completely irregular sequence. Every droplet goes "ping" but the overall effect is one of noise. The applause of a huge audience may also sound like noise, unless they clap rhythmically to demand an encore (which simply represents a certain order, regularity or periodicity!)

Electric current in a solid state implies movement of electrons in the metallic crystal grid. The movement of an individual electron from an atom to the neighbouring atom takes place quite randomly.

Even though the movement of electrons mainly points in the direction of the physical current this process has a stochastic – purely random, unpredictable – component. It makes itself heard through noise. There is therefore no pure direct current DC; it is always accompanied by noise. Every electronic component produces noise, that is any resistance or wire. Noise increases with temperature.

Noise and information

Random noise means something like absolute chaos. It contains no "pre-arranged, meaningful pattern" – i.e. no information.

Stochastic noise has no "conserving tendency", i.e. nothing in a given time segment B reminds one of the previous time segment A. In the case of a signal, the next value is predictable at least with a certain degree of probability. If for example you think of a text like this, where the next letter will be an "e" with a certain degree of probability.

Stochastic noise is therefore not a "signal" because it contains no information bearing pattern – i.e. no information.

Everything about stochastic noise within a given time segment is random and unpredictable, i.e. its development in time and its spectrum. Stochastic noise is the "most non-periodic" of all signals!

All signals are for the reasons described always (sometimes more or less or too much) accompanied by noise. But signals which are accompanied by a lot of noise differ from pure stochastic noise in that they display a certain conserving tendency. This is characterised by the pattern which contains the information.

Noise is the biggest enemy of communications technology because it literally "buries" the information of a signal.

One of the most important problems of communications technology is therefore to free signals as far as possible from the accompanying noise or to protect or modulate and code the signals from the outset in such a way that the information can be retrieved without errors in spite of noise in the receiver.

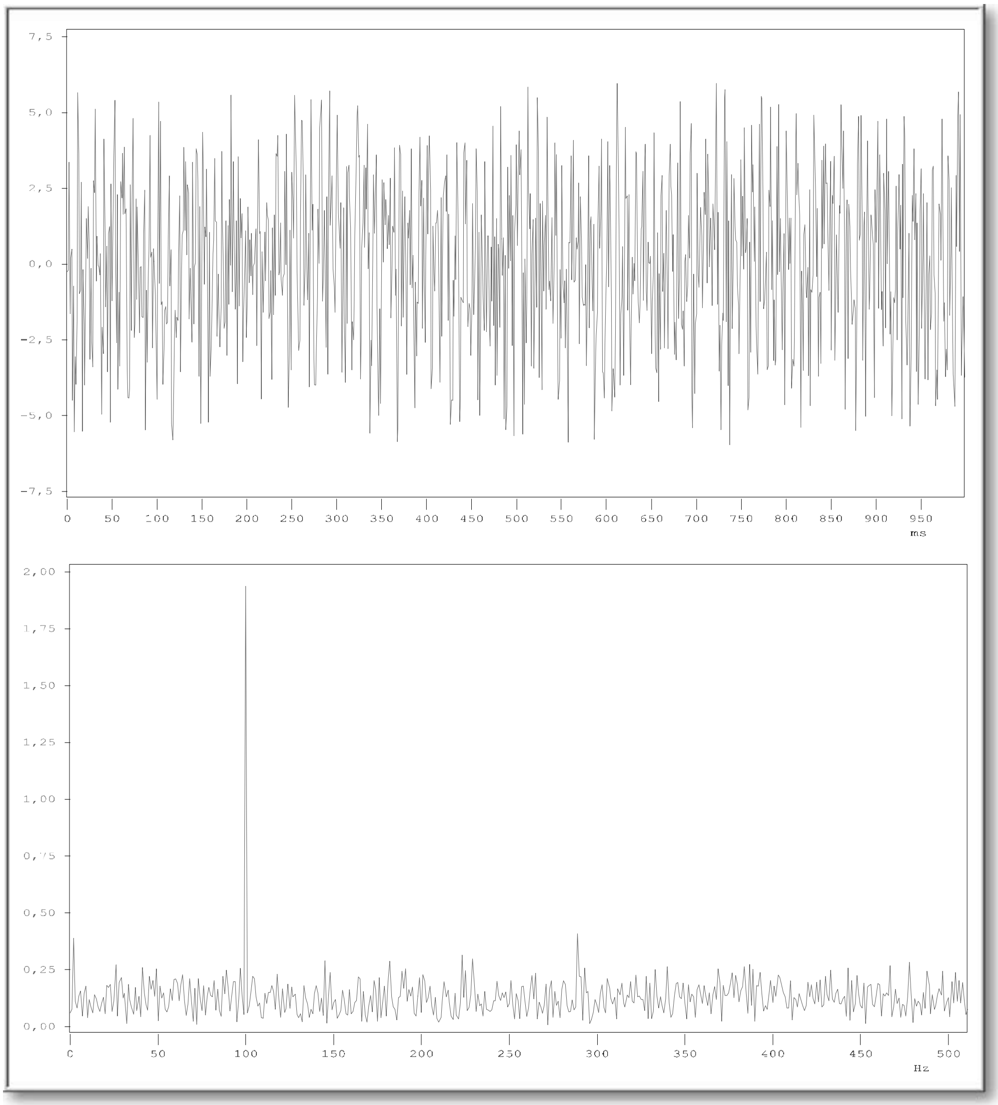

Illustration 40: ***Conserving tendency of a noisy signal***

Both Illustrations – the time domain above, the amplitude spectrum below – describe a noisy signal, that is not pure stochastic noise, which displays a conserving tendency (influenced by the signal). This is shown by the amplitude spectrum below. A line protruding from the irregular continuous spectrum at 100 Hz can clearly be seen. The cause can only be a (periodic) sinusoidal signal of 100 Hz hidden in the noise. It forms the feature which conserves a tendency although it is only vaguely visible in the time domain. It could be "fished out" of the noise by means of a high-quality bandpass filter.

This is in fact the central theme of "information theory". As it presents itself as a theory formulated in purely mathematical terms, we shall not deal with it systematically in this book. On the other hand, *information* is the core term of information and communications technology. For this reason important findings of information theory turn up in many places in this book.

Signals are regularly non-periodic signals. The less their future development can be predicted, the greater their information value may be. Every signal has a "conserving tendency" which is determined by the information-bearing pattern. Stochastic noise is by contrast completely random, has no "conserving tendency" and is therefore not a signal in the true sense.

We should, however, not completely denigrate stochastic noise. Since it has such extreme qualities, i.e. it embodies the purely random, it is highly interesting. As we shall see it has great importance as a test signal for (linear) systems.

Exercises for Chapter 2:

Exercise 1:

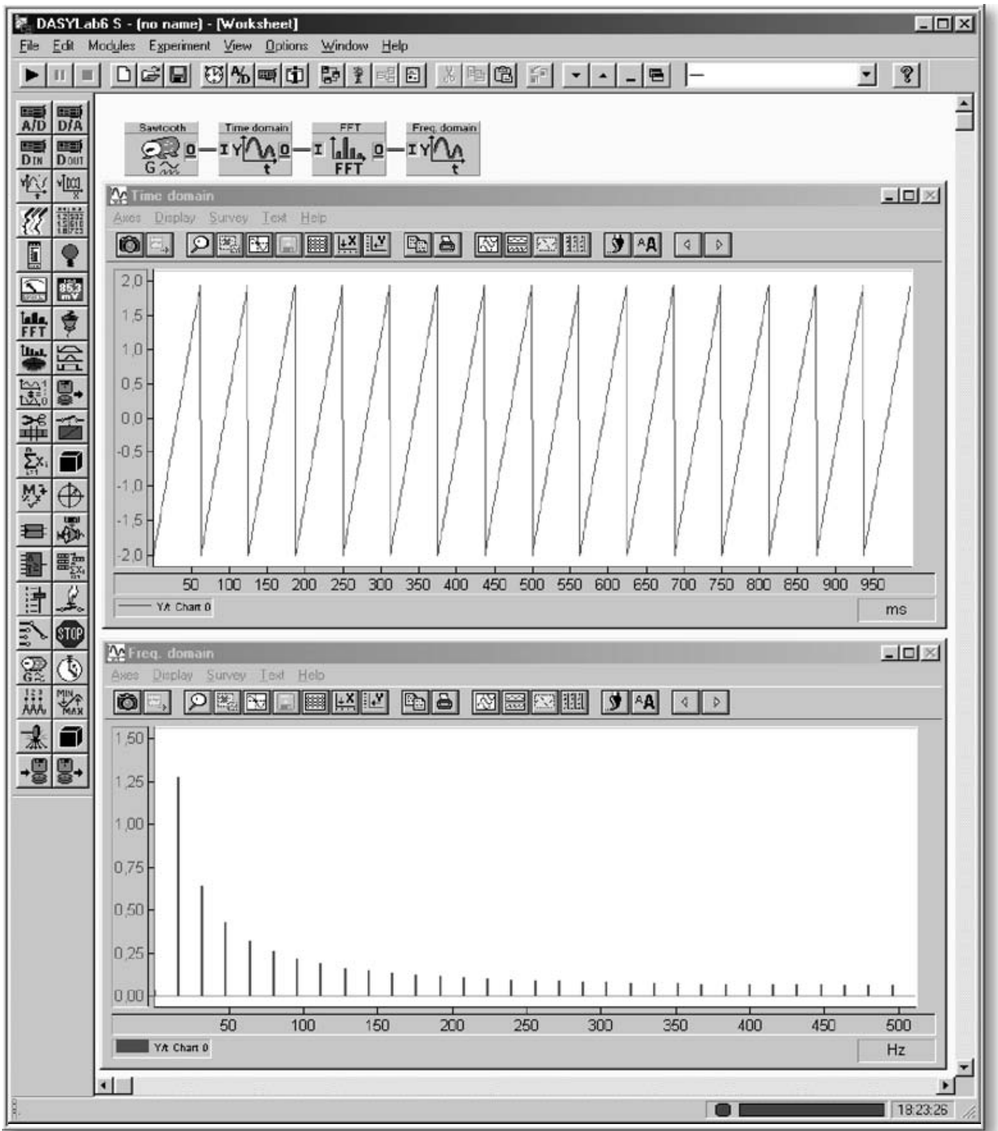

Illustration 41: ***Sawtooth in time and frequency domain***

Here you see the whole DASY*Lab* window displayed. By far the most important circuit for analysis and representation of signals in the time and frequency domain is to be found at the top of the picture.

(c) Create this circuit and visualise – as above – a periodic sawtooth without a direct voltage offset in the time and frequency domain.

(d) Measure the amplitude spectrum by means of the cursor. According to what simple rule do the amplitudes decrease?

(e) Measure the distance between the "lines" in the amplitude spectrum in the same way. In what way does this depend on the period length of the sawtooth?

(f) Expand the circuit as shown in Illustration 22 and display the amplitude spectra of different periodic signals one below the other on a "screen".

Exercise 2:

(a) Create a system using DASY*Lab* which produces the FOURIER synthesis of a sawtooth as in Illustration 25

(b) Create a system using DASY*Lab* which gives you the sum of the first n sinusoidal signals (n = 1,2,3,....9) as in Illustration 27

Exercise 3:

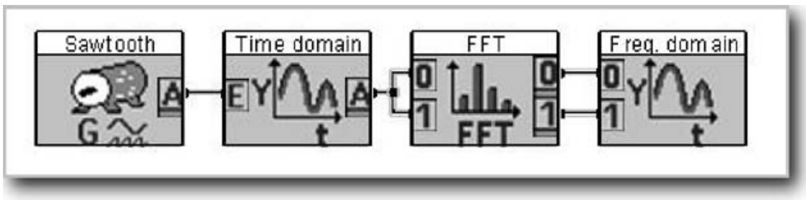

Illustration 42: ***Block diagram: Amplitude and phase spectrum***

(a) Try to represent the amplitude spectrum and the phase spectrum of a sawtooth one directly beneath the other as in Exercise 1. Select amplitude spectrum on channel 0 in the menu of the module "frequency domain" and "phase spectrum" on channel 1. Select "standard setting" (sampling rate and block length = 1024 = 2^{10} in the A/D button of the upper control bar) and a low frequency (f = 1; 2; 4; 8 Hz. What do you discover if you choose a frequency whose value cannot be given as a power of two?

(b) Select the different phase modifications π (180^0), $\pi/2$ (90^0), $\pi/3$ (60^0) and $\pi/4$ (45^0) for the sawtooth in the menu of the generator module and observe the changes in the phase spectrum in each case.

(c) Do the phase spectra from Exercise 2 agree with the 3D representation in Illustration 28 ff.? Note deviations and try to find an explanation for the possible erroneous calculation of the phase spectrum.

(d) Experiment with various settings for the sample rate and block length (A/D button on the upper control bar, but select both values in the same size, e.g. 32, 256, 1024!)

Exercise 4:

Noise constitutes a pure stochastic signal and is therefore "totally non-periodic".

(a) Examine the amplitude and phase spectra of noise. Is the spectrum continuous? Do amplitude and phase spectra display stochastic behaviour?

(b) Examine the amplitude and phase spectrum of lowpass filtered noise (e.g. cutoff frequency 50 Hz, Butterworth filter 6th order). Do both exhibit stochastic behaviour? Is the filtered noise also "completely non-periodic"?

Exercise 5:

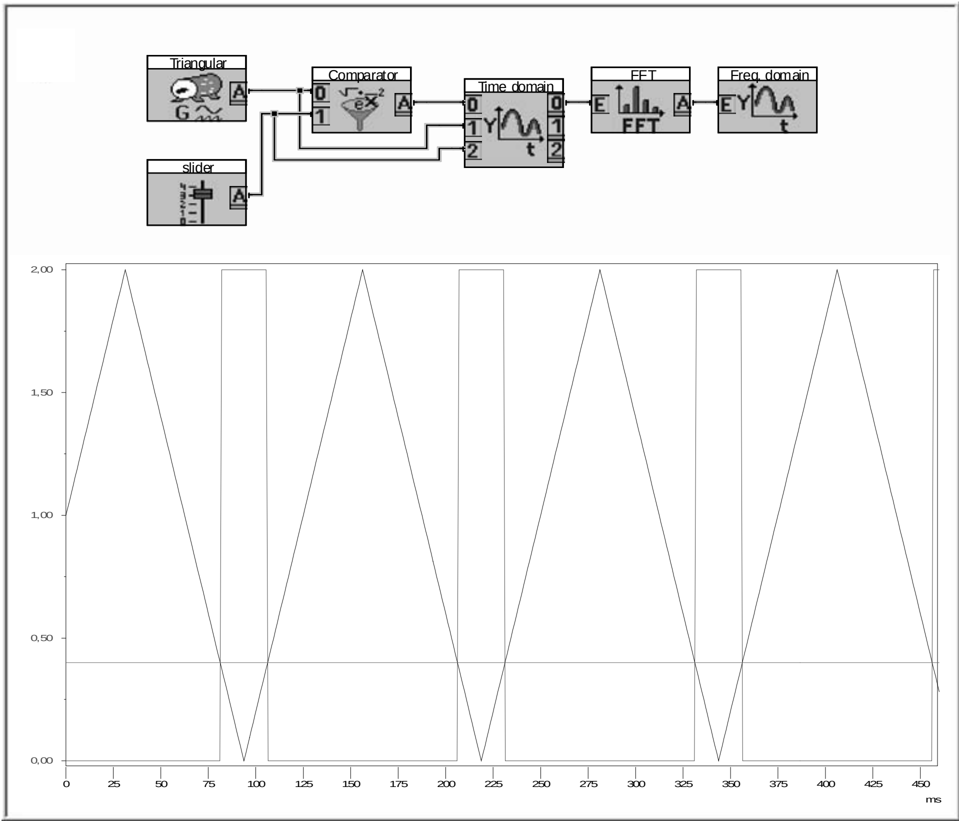

Illustration 43: ***Square wave generator with variable pulse duty factor***

(a) Design a square wave signal generator by means of which the pulse duty factor and the frequency of the periodic rectangular signal can be set as desired. If necessary use the enclosed Illustration to help you.

(b) Interconnect (as above) your square wave signal generator with our standard circuit for the analysis and visualisation of signals in the time and frequency domain.

(c) Examine the amplitude spectrum by keeping the frequency of the square wave signal constant and making the pulse duration τ smaller and smaller. Observe particularly the development of the "zero positions" of the spectrum as shown in Illustration 33 ff.

(d) In the amplitude spectrum usually additional small peaks appear between the expected spectral lines. Experiment on ways of optically avoiding these, for instance by the selection of suitable scanning rates and block lengths (A/D setting in the upper control bar) and signal frequencies and pulse lengths. You will discover their cause in Chapter 10 (Digitalisation).

(e) Try to develop a circuit such as that used for the representation of signals in Illustration 38 – transition from a line spectrum to a continuous spectrum. Only the frequency, not the pulse length should be variable.

Exercise 6:

(a) How could one prove using DASY*Lab* that practically all frequencies – i.e. sinusoidal oscillations- are present in a noise signal. Try experimenting.

(b) How is is possible to ascertain whether a (periodic) signal is contained in an extremely noisy signal?