CHAPTER 5

Transmission Lines

5.1 Introduction

Lossy transmission lines are the norm on circuit boards, especially when signaling over narrow trace at high frequencies, where skin effect and dielectric losses cause signal distortion. As we'll see in this chapter, at high frequencies the distortion is chiefly caused by unequal attenuation of the signal's harmonics, but phase distortion is the principal cause at lower frequencies. The attenuation is caused by losses due to the series resistance in the conductor and by shunt losses due to the dielectric. The calculations presented in the early sections lump these losses together and involve the use of complex numbers, but simplifications that avoid the use of imaginary numbers and separate out the resistive losses from the dielectric losses are later shown.

Although rectangular waveforms are usually of most interest to the digital circuit designer, the bulk of this chapter focuses on the treatment lossy lines give to sinusoids at single frequencies. This is appropriate because rectangular waves are made up of many single frequency harmonics, and the way each of those harmonics is treated as a pulse travels down a lossy line determines its final wave shape once it arrives at the load. The harmonics reassembly and the effect distorted pulses have on signaling is presented in Chapter 7.

This chapter begins by using ideas from circuit and network theory to analyze a lossy transmission line circuit model. This prepares the way for the discussion in Section 5.4 on traveling waves. The study of traveling waves can become mired in mathematics, but most of that has been sidestepped in this chapter. Instead, as is usual throughout this book, the aim has been to provide enough mathematics to allow an engineer to make hand calculations or to explain results from field-solving or circuit-simulator software. Those wishing a detailed mathematical treatment are referred to the references.

5.2 General Circuit Model of a Lossy Transmission Line

As described in Chapter 4, a TEM transmission line consists of one or more signal lines and a return. When a signal propagates down a transmission line, a time-dependent voltage difference exists between the signal wire and its return, and equal but opposite currents flow along them. The two conductors guide the electric and magnetic fields.

A general circuit model of a TEM transmission line appears in Figure 5.1. As shown, it's made up of many small resistor, inductor, conductance and capacitance (RLGC) segments chained together to represent the entire length of line. Figure 5.1(a) shows the signal and return lines, with each having a resistance (R) and inductance (L).

The signal and return are separated by a dielectric, so a capacitance (C) appears between them. Because the dielectric is not perfect, a shunt loss element (G) appears across the capacitor. The inductance models the energy contained in the magnetic field, while the capacitance models the electric field energy. The series resistance represents the series losses, and the conductance represents the dielectric losses. These elements are smoothly distributed along the length of an actual transmission line, but in the model they appear in lumps representing a small section (Δx) of line. The sections (*lumps*) must be very small (here defined as only a fraction of a wavelength) to give the appearance of a continuous, smooth line.

The resistance and inductance of the return wire may be "folded into" the signal wire, as shown in Figure 5.1(b). This topology is sometimes called a RLGC model.

The distributed RLGC model may be viewed as a chain of an infinite number of π or T sections, with each section representing a very small segment of transmission line. In Figure 5.2, T sections are used with the series R, with L elements equally divided in each arm.

The circuit in Figure 5.2 is a lowpass ladder filter made up of *constant-k* sections, and it's appropriate to use filter theory to determine the circuit's impedance, delay, and attenuation characteristics. To do so, it's first necessary to group the series and shunt impedances, as shown in Figure 5.3.

In transmission line work it's customary to represent the series element (Z_1) as an impedance (Z, with ohms as units, not to be confused with Z_o, the *characteristic impedance* described next) and the shunt (Z_2) as an admittance (Y, the inverse of impedance, with units of siemens). Frequency is expressed in radians/sec (ω) rather than hertz. With this in mind:

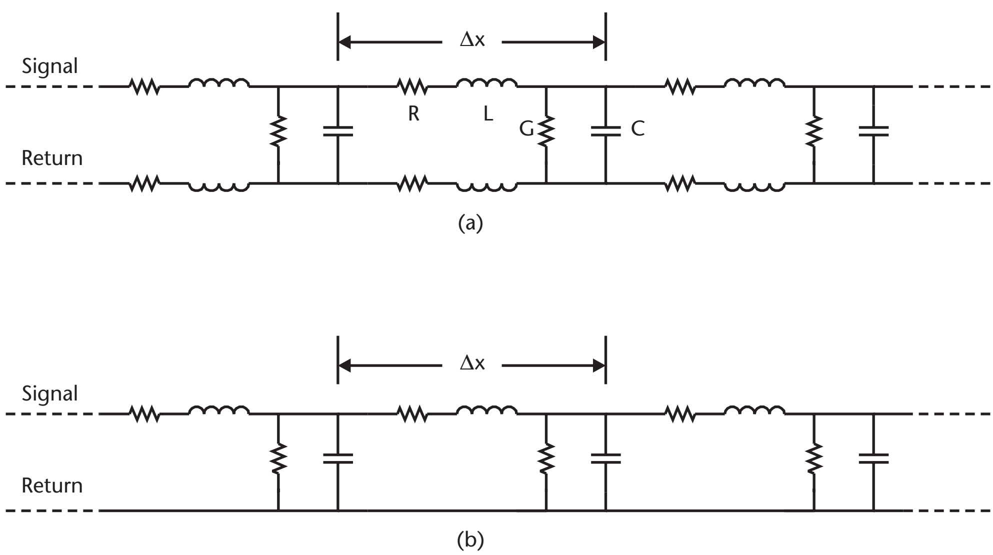

Figure 5.1 RLGC transmission line models: (a) general model, and (b) return R,L folded into signal.

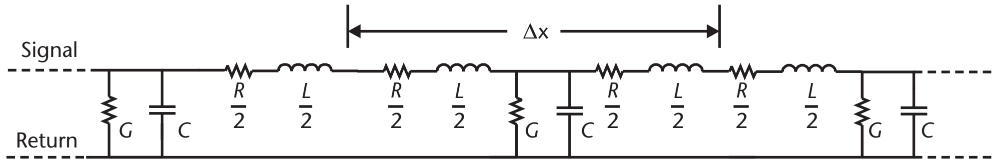

Figure 5.2 T network representation of a transmission line.

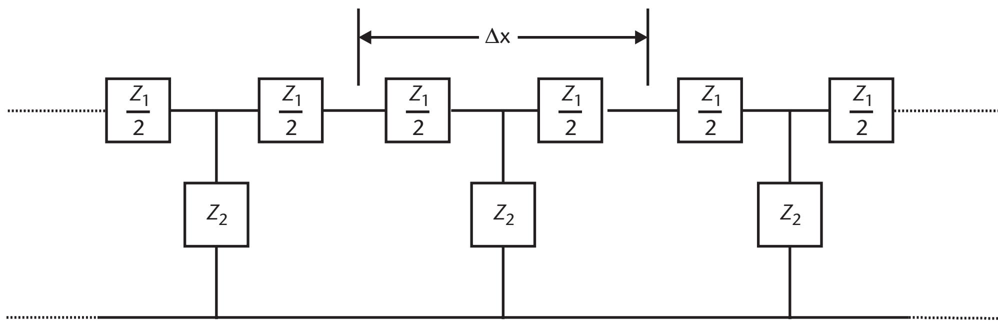

Figure 5.3 Transmission line generalized model.

$$\omega = 2\pi f \tag{5.1}$$

$$Z_1 \equiv R + j\omega L \tag{5.2}$$

$$Y \equiv \frac{1}{Z_2} = G + j\omega C \tag{5.3}$$

where j is the imaginary operator $\left(\text{equal to } \sqrt{-1}\right)$.

From network theory, Figure 5.3 has three properties of interest:

- Its *characteristic impedance* (Z_o) is the impedance necessary to properly match the line so that reflections are not produced when a wave reaches the line's end. When the line is properly terminated, the impedance also determines the relationship between the voltage and current waves traveling down the line.
- It is a lowpass filter: Higher frequencies will be attenuated more than lower frequency ones. The *attenuation constant* (α, units of nepers or decibels) describes the amount of attenuation at each frequency.
- It introduces a phase shift between the waveform launched at the input and the waveform recovered at the output. The phase shift is defined by the *phase constant* (β, with units of radians). As we'll see, this phase shift represents a time delay (*td*).

The attenuation and phase constants are specified on a per–unit length basis and are often jointly represented by a single value called the *propagation constant*

(γ). The delay is specified per unit length, while the impedance has a value that is independent of length.

5.2.1 Relationship Between ωL and R

The imaginary terms in (5.2) and (5.3) suggest the impedance and propagation constant will be complex numbers, made up of real and imaginary parts, and in general this is true.

However, as we'll see, for PWB trace at very high frequencies, the imaginary terms dominate the real terms, allowing them to be ignored. The $j\omega$ terms then cancel, so that with some approximations it's possible to develop equations for Zo, α, and β that do not use them. In order to do so, it's necessary to determine the frequencies where ωL is larger than R and ωC is larger than G. We begin with the relationship between ωL and resistance across frequency and then examine ωC and G.

Chapters 2 and 4 showed that at high frequency, the series resistance increases as the square root of frequency because of the skin effect, but the inductance remains constant.

At low frequencies ω is small, making ωL small, so at some arbitrarily low frequency ωL is less than the conductor's dc resistance. However, as the frequency increases, ω increases linearly, while R is only increasing as the square root. This means ωL will gradually overtake R, until finally the frequency becomes high enough for ωL to exceed (or even greatly exceed) R.

At frequencies where the skin effect is well developed [F_{skin}, see (2.12) and (2.13)] appearing in Chapter 2), ωL is much greater than R, even for narrow, low-impedance microstrip. This is the worst case for comparing R to ωL across frequency because in general, narrow, low-impedance trace has higher loop ac resistance and lower inductance than wide, higher impedance trace. Such a low-inductance, high-resistance trace will require a higher frequency before ωL is larger than R.

The ratio of ωL to R across frequency is shown in Figure 5.4 for four stripline and microstrip traces. This is example data for half-ounce copper trace on FR4 with a copper return plane. The traces are either 4 mils or 10 mils wide. It's apparent that ωL exceeds R for frequencies in the 10s of MHZ region, even for narrow PWB trace, but it's not until the frequency is in the 100- to 200-MHz range before $\omega L >> R$ for all of the traces shown. The narrow, 50-Ω microstrip (ms2) is the most resistive and thus requires the highest frequency before R exceeds ωL.

A laminate having a lower ε_r than that used to create Figure 5.4 will shift the curves down, requiring a higher frequency before ωL exceeds R.

It's evident in Figure 5.4 that microstrip requires a frequency higher than stripline before ωL exceeds R. This is because for a given impedance, width, and thickness, microstrip trace has both a lower loop inductance and higher loop resistance than stripline (one return path versus two in parallel, hence higher return path resistance).

5.2.2 Relationship Between ωC and G

We now turn our attention to the relationship between ωC and G, with the goal of showing that at high frequencies, ωC is much greater than G. Notice that in this discussion ωC is just the product of the frequency and capacitance and is not the capacitive reactance X_c discussed in (3.10) in Chapter 3.

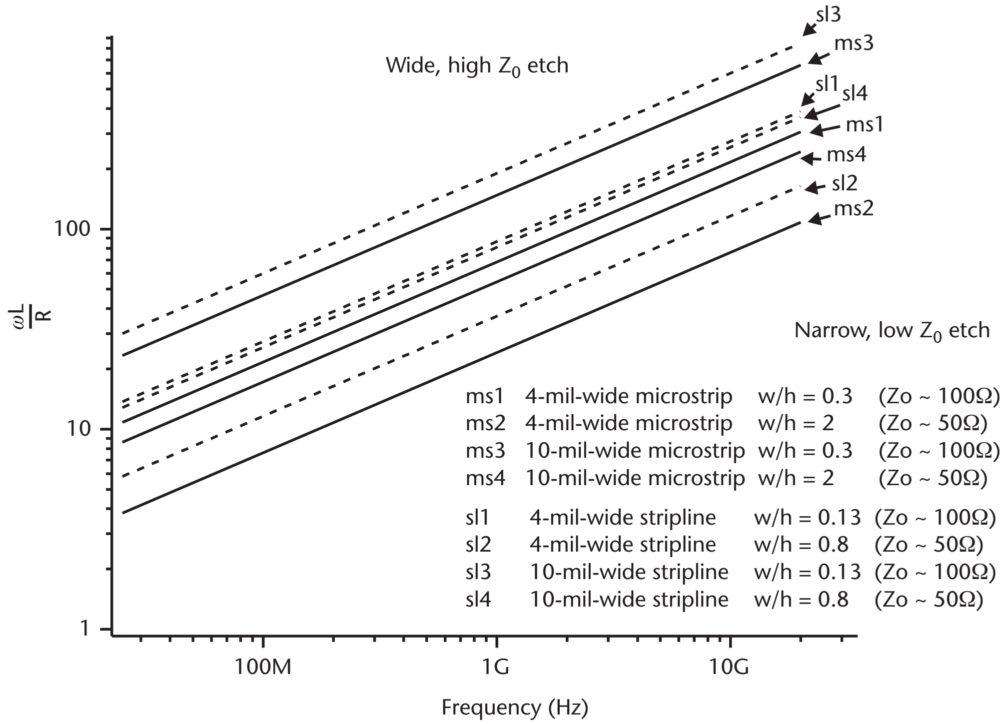

Figure 5.4 *R* and ωL for narrow and wide microstrip and stripline trace on FR4.

Recall from (3.15) in Chapter 3, that *tan* (δ) (the loss tangent) relates the dielectric losses *G* to capacitance as $G = \omega C \tan(\delta)$. Because the dielectric losses in PWB laminates are much less than one, and because they do not increase rapidly with frequency (from Table 3.2, FR4 has a *tan* [δ] value of 0.015 at 1 MHz and 0.025 at 1 GHz, for example), the quantity ωC will always be greater than *G* except at very low frequencies where ω is very small. In fact, for FR4 at 1 MHz and above, ωC will be at least 40 times larger than *G* at high frequencies and is well over 100 times greater for higher performance dielectrics.

Sections 5.2.1 and 5.2.2 have shown that when signaling over PWB trace at high frequencies, the inductive and capacitive reactances dominate the resistive and dielectric losses. This simplifies the mathematics and will lead to straightforward equations for impedance, loss, and phase shift. But, even though ωC and ωL dominate the losses represented by the *R* and *G* terms, the losses can't be ignored. In fact, they are the cause of signal distortion, as we'll see next.

5.3 Impedance

Applying a voltage to the transmission line shown in Figure 5.2 causes current to flow as capacitor C charges. A much smaller leakage current is also drawn by *G*. Assuming no reflections are present on the line, the voltage-to-current ratio is called the line's *characteristic impedance* (Z_0, with units of ohms, Ω). The characteristic impedance is independent of the line's length and has the same value everywhere along a uniform line, regardless of its length.

5.3.1 Calculating Impedance

Using network analysis, it can be shown (see [1], for example) that if the lumps in Figure 5.3 are small, the characteristic impedance is related to Z_1 and Z_2 as shown in (5.4):

$$Z_0 = \sqrt{Z_1 Z_2} \tag{5.4}$$

Combining (5.2) and (5.3) into (5.4) yields Z_0 in terms of the transmission line's distributed *R, L, G,* and *C* components (5.5):

$$Z_0 = \sqrt{Z_1 Z_2} = \sqrt{\frac{Z_1}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{5.5}$$

In those cases where the frequency is high enough so that ωL is larger than ωR and C is larger than *G*, (5.5) reduces to the familiar equation for the impedance of a lossless transmission line (5.6):

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \tag{5.6}$$

Example 5.1 compares results from (5.5) and the approximation in (5.6).

Example 5.1

A 5-mil-wide stripline built on FR4 has the following parameters per inch at 100 MHz:

R = 422 mΩ
G = 38 μS
L = 10.8 nH
C = 3 pF

Compute the impedance for this transmission line using (5.5) and (5.6).

Solution

From (5.1) $\omega = 6.28 \times 10^8$ rad/sec at 100 MHz.
Using (5.5):

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{422\ m\Omega + j\omega 10.8\ \text{nH}}{38\ \mu S + j\omega 3\ \text{pF}}} = 60 - 1.26 j\Omega$$

At 100 MHz, this transmission line is seen to have both real and imaginary parts to its impedance. The 60-Ω real portion is the resistive part and the imaginary portion (–1.26 $j\Omega$) represents a small capacitive reactance (inductive reactance would be shown as positive) and so indicates that the voltage and current are propagating out of phase. In this case, converting from the (60–1.26j) rectangular form to polar form (by taking the arctangent of *imag/real*) yields an angle of –1.2°. That is, the voltage lags the current by 1.2°.

Such a small angle shows the nearly perfect alignment between the voltage and current waves, just as they would be if the line's impedance were purely resistive. Because of this, the lossy impedance calculation given by (5.5) should closely match the lossless impedance given by (5.6). This is in fact the case:

From (5.6) $$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{10.8\text{ nH}}{3\text{ pF}}} = 60\Omega$$

In this example, the complex portion of the impedance is small enough to be ignored, and the line has an impedance that is nearly purely resistive. This will generally be the case for PWB microstrip and stripline and is especially so as frequency increases. But this assumption does not always hold for thin, narrow traces, especially at low frequency where the resistance is high and ωL is small. This type of trace can be found in some micro packages (where the traces are essentially thin films and have high resistance) or some flexible tape type interconnects that may have high inductance. Depending on frequency, the impedance of these types of trace can have a significant imaginary component, making (5.5) more appropriate than (5.6).

5.4 Traveling Waves

Generally in digital systems, the signals start out essentially rectangular in shape but often arrive at the load with rounded corners and reduced in amplitude. Rectangular waveforms are made from the sum of many sine waves (*harmonics*), each having a specific amplitude and frequency relationship with the waveform's fundamental frequency. The frequency content of pulses is discussed in Chapter 7, but for now it's enough to note that for the signal to retain its original shape, the transmission line must attenuate and phase shift each harmonic by the appropriate amount. Otherwise, the original relationship between the harmonics will not be preserved, and the received signal will be a distorted version of the original.

In fact, lossy lines do not treat the harmonics equally, so pulses do arrive at the load distorted. The amount of distortion depends on the line length, as that determines the degree to which the harmonics are exposed to the incorrect phase and amplitude adjustments. This is evident in Figure 5.5, which shows a 6-ns pulse as it appears at various points along a 36-in-long transmission line. It's plain that distortion increases as the pulse makes its way down the line, with the pulse showing progressively more rounding as it travels. This is caused by the upper frequency harmonics being attenuated more severely than the lower frequency ones. Also evident is the loss of height (amplitude) as the pulse travels down the line, and the pulse shows spreading at its base (smearing) as it travels. As discussed in Chapter 9, successive pulses traveling along this line are more likely to interfere with one another as the boundaries between them (bit times) becomes blurred.

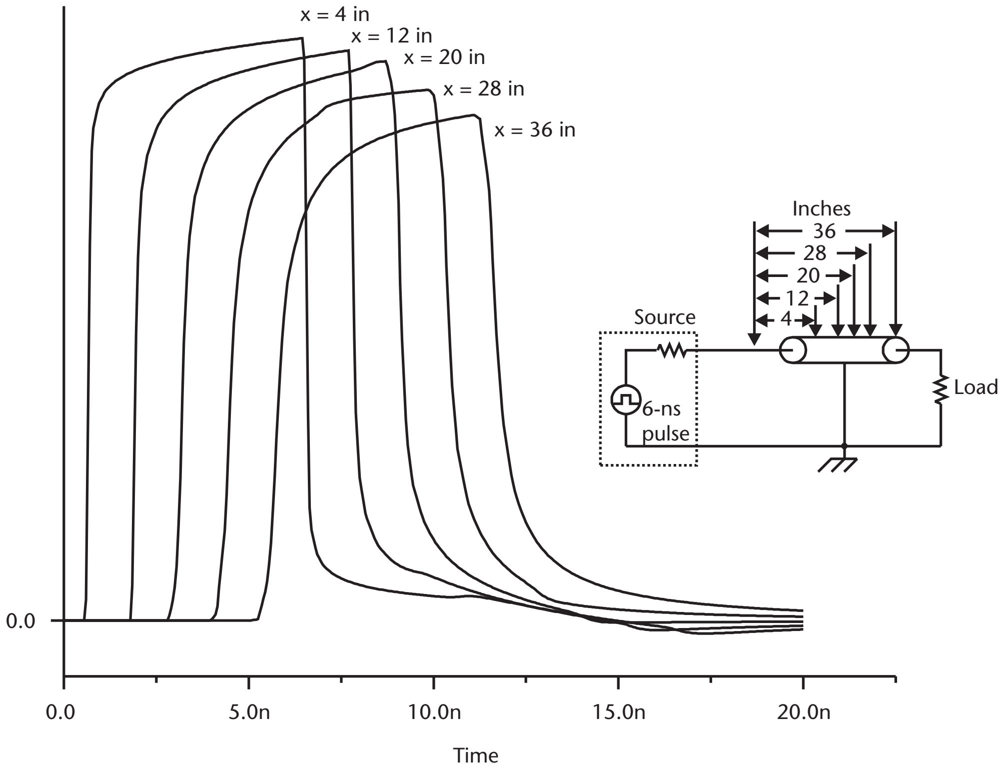

Figure 5.5 A 6-ns pulse propagating down a 36-in transmission line.

To understand the cause of these effects, it's necessary to determine how the transmission line treats each harmonic. For this reason, the remainder of this chapter will focus on the transmission line's response to single frequencies (sinusoids). In Chapter 7, this insight is applied to signaling with pulses.

5.4.1 Propagation Constant

Solving the differential equations relating the voltage across a small segment of transmission line to the current flowing through it yields the equation of a wave traveling along the line's length [2]. For a line with no reflections, the voltage at a distance x is attenuated exponentially from the sending voltage V_s as shown by (5.7):

$$V_x = V_s e^{-\sqrt{YZ}x} \tag{5.7}$$

where Z is the impedance given in (5.2) and Y the admittance given in (5.3).

The quantity $\sqrt{YZ}$ is called the *propagation constant* (this was briefly mentioned at the start of Section 5.2) because it governs the way voltage and current waves propagate down the line. The propagation constant (a misnomer because it varies with frequency) is represented by γ (5.8):

$$\gamma = ZY = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{5.8}$$

The propagation constant is a complex number having two parts: the real portion is the *attenuation constant* (α, with units of neper per unit length), while the imaginary portion is called the *phase constant* (β, units of radians per unit length).

The attenuation constant α determines the way a signal is reduced in amplitude as it propagates down the line, while the phase constant β shows the difference in phase between the voltage at the sending end of the line and at a distance x.

Because it includes ω, (5.8) determines γ at one specific frequency. A pulse contains harmonics of many frequencies. To determine the effect a lossy transmission line has on a pulse, (5.8) must be applied individually to each harmonic. As discussed in Chapter 7, the harmonics are then recombined at the load with unique values of α and β for each frequency to yield a composite waveform at the load [3].

Equation (5.8) produces a total value for α that is the sum of the series resistance and dielectric losses, and it is valid for any TEM transmission line, regardless of the values of R and G. The resistive and dielectric loss contributions are broken out in Section 5.4.4.

Example 5.2

Find γ, α, and β for the transmission line described in Example 5.1

Solution

As the frequency in Example 5.1 is given as 100 MHZ, from (5.1) $\omega = 6.28 \times 10^8$ rad/sec.

A scientific calculator or a scientific software calculation package such as Mathcad [4] or MatLab [5] make it straightforward to compute γ with (5.8):

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(11m + j\omega 10.8n)(38\mu + j\omega 3p)} = 0.0047 + 0.113j$$

The real part of γ is α, and the imaginary portion is β. As the RLCG values were all given per inch length of transmission line, the computed values for γ (and so α and β) are the values for 1-in worth of line.

So:

α = 0.0047 nep/in
β = 0.113 radians/in

5.4.2 Phase Shift, Delay, and Wavelength

The phase constant $\beta\, x$ shows the phase shift of the voltage (or current) at a point located at a distance x along a transmission line with respect to the sending voltage (or current). A phase shift of 360° (or 2π radians) equals one wavelength and, as shown in Figure 5.6, marks the distance between successive points on the waveform (such as zero crossings).

The wavelength is the distance x required to make the phase angle βx increase by 2π radians. A wavelength is therefore:

$$\lambda = \frac{2\pi}{\beta} \tag{5.9}$$

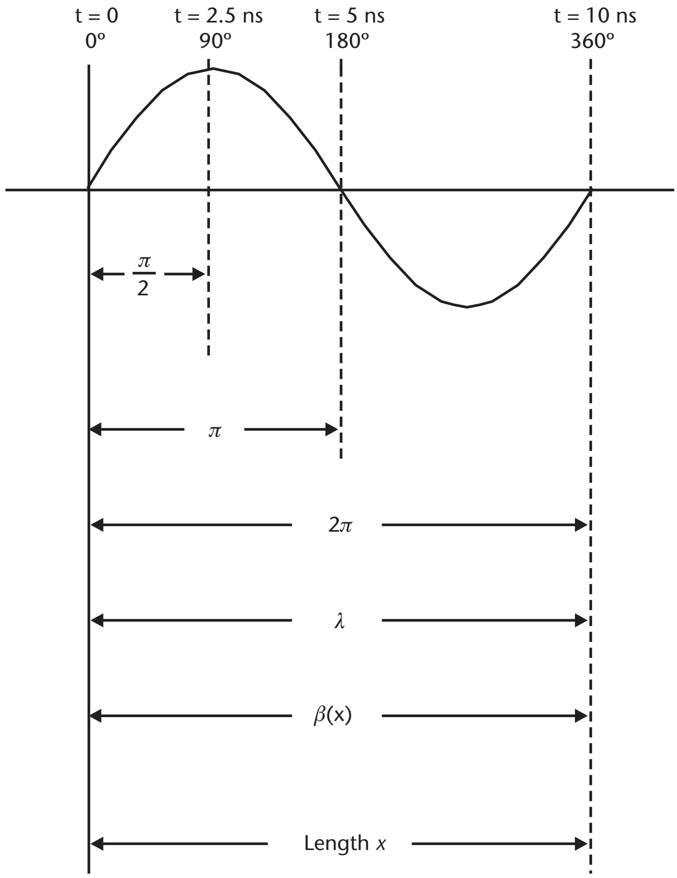

Figure 5.6 Relationship between degrees, radians, phase shift (β), and wavelength (λ).

It's apparent from Figure 5.7 that a phase shift may also be seen as a delay. In fact, expressing β as a sinusoid and taking the derivative with respect to time yields the velocity at which the wave travels down the line [6]:

$$v_P = \frac{\omega}{\beta} \tag{5.10}$$

From Maxwell's equations, waves propagate along TEM transmission lines with a velocity equal to the speed of light in the dielectric. This leads to particularly useful equations for the velocity of propagation (5.11) and the *guide wavelength* (λ_g) (5.12) [7] for PWB trace where it's assumed the metals are nonmagnetic and so $\mu_r = 1$:

$$v_p = \frac{c}{\sqrt{\varepsilon_r}} \tag{5.11}$$

$$\lambda_g = \frac{c}{f\sqrt{\varepsilon_{r\_eff}}} \tag{5.12}$$

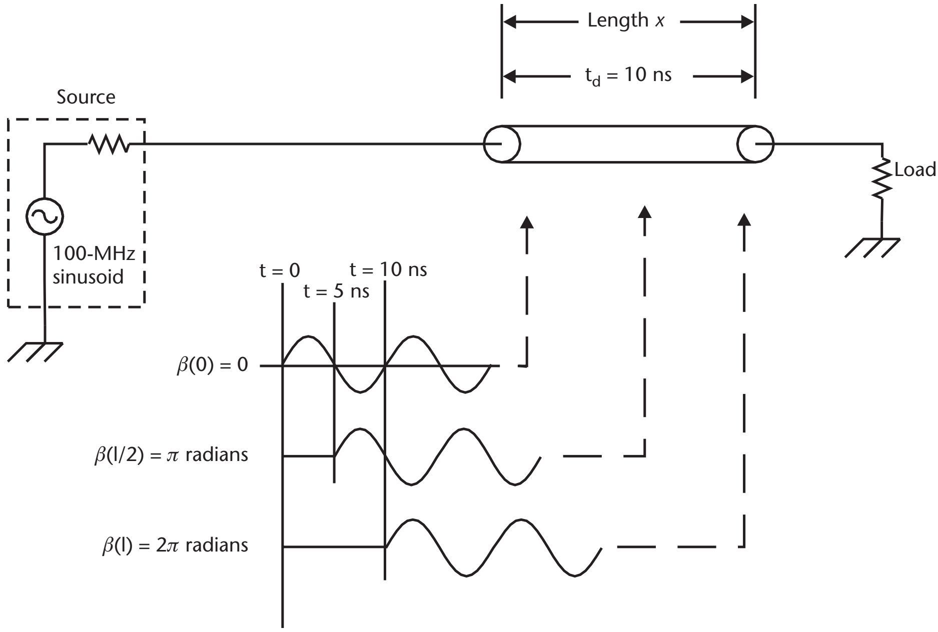

Figure 5.7 Phase shift as a delay.

In (5.11) and (5.12), f is the frequency in hertz, c is the speed of light (3×10^8 m/s or 11.8×10^9 in/sec), and ε_{r_eff} the *effective permittivity*. For stripline, ε_{r_eff} is just the dielectric constant ε_r of the laminate as described in Chapter 3, but this is not the case with microstrip. As discussed in Chapter 9, some of the microstrip electric and magnetic field lines propagate in air as well as the laminate, making ε_{r_eff} lower than ε_r of the laminate itself. In fact, the geometry of a given microstrip determines the value of ε_{r_eff}.

As delay is proportional to the inverse of velocity, the amount of delay a transmission line introduces per distance x is:

$$t_d = \frac{x}{v_p} = \frac{\beta x}{\omega} \tag{5.13}$$

It follows from (5.11) and (5.13) that for stripline, the velocity of propagation (and thus the delay per inch) is the same for all traces, but for microstrip the velocity (and thus the delay) depends on the trace's width and height above a return plane because that's what determines ε_{r_eff}. This is a fundamental difference between stripline and microstrip and is discussed in Chapter 9.

Example 5.3

Using (5.13), what is the time delay of the transmission line in Example 5.2, and what is the delay assuming the trace is a stripline on FR4 with $\varepsilon_r = 4.5$?

Solution

(a) The phase shift (β) and the frequency at which that phase shift is measured (ω) are required to calculate the delay from (5.13). From Example 5.2, the specification is for a 1-in-long line ($x = 1$), making $\beta = 0.113$ radians/inch, and $\omega = 6.28 \times 10^8$ radians/sec.

From (5.13) the delay is then: $t_d = \dfrac{\beta x}{\omega} = \dfrac{0.113 \text{ rad/in}}{6.28 \times 10^8 \text{ rad/sec}} = 180$ ps/in

(b) From (5.11) and (5.13) $t_d = \dfrac{\sqrt{\varepsilon_r}}{c} = \dfrac{\sqrt{4.5}}{11.8 \times 10^9 \text{ in/sec}} = 180$ ps/in

For the signal to appear with the same shape at the end of a transmission line, each harmonic must be delayed by the same amount. From (5.13), β must therefore increase linearly with frequency. Otherwise, t_d would be different for each harmonic, and each would arrive at the load at a different time, improperly altering their phase relationship and yielding a distorted waveform.

Example 5.4

Vias are to be placed every tenth of a wavelength along a 50-Ω stripline fabricated on a laminate having $\varepsilon_r = 4.0$. The highest harmonic has a frequency of 6 GHz. What is the required spacing? What time delay does that spacing represent?

Solution

The wavelength is found directly from (5.12) to be nearly an inch as follows:

$$\lambda_g = \frac{c}{f\sqrt{\varepsilon_{r\_eff}}} = \frac{11.8 \times 10^9 \text{ in/sec}}{6 \times 10^9 \text{Hz}\sqrt{4.0}} = 0.983 \text{ in}$$

So a via must be placed approximately every 100 mils (one tenth of an inch) to satisfy the 10th-wavelength requirement.

From Example 5.3, $t_d = \dfrac{\sqrt{\varepsilon_r}}{c} = \dfrac{\sqrt{4.0}}{11.8 \times 10^9 \text{ in/sec}} = 169.5 \text{ ps/in}$, making the delay for 100-mil separation $t_d = 169.5$ ps/in $\times$ 0.1 in = 17 ps.

5.4.3 Phase Constant at High Frequencies When *R* and *G* Are Small

It was shown in Section 5.2 that for PWB trace at high frequencies, R and G were small compared to ωL and ωC. This allowed for the simplification of the impedance equation (5.5) to the well known lossless impedance equation given in (5.6). Simplifying the phase constant (5.8) to eliminate the imaginary terms is not as straightforward if R and G are small relative to ωL and ωC but not small enough to ignore. This is the usual case when signaling on PWB trace, especially at high and very high frequencies.

After some involved reductions, [8] eliminates the use of imaginary terms in (5.8) and produces (5.14) for the phase constant at high frequencies when R and G are small but nonzero, and $\omega L > R$ and $\omega C > G$:

$$\beta \approx \omega\sqrt{LC}\left(1 - \frac{RG}{4\omega^2 LC} + \frac{G^2}{8\omega^2 C} + \frac{R^2}{8\omega^2 L^2}\right) \tag{5.14}$$

The time delay for such a line is found by combining (5.14) and (5.13) to produce (5.15):

$$t_d \approx \frac{\beta}{\omega}\sqrt{LC}\left(1 - \frac{RG}{4\omega^2 LC} + \frac{G^2}{8\omega^2 C} + \frac{R^2}{8\omega^2 L^2}\right) \tag{5.15}$$

Equation (5.15) shows that if R and G are large, each harmonic in a signal will be delayed by a different amount and thus will arrive at the load at different times. Recombining these variously phase shifted harmonics would yield a distorted signal that is not merely a smaller version of the signal launched from the generator.

However, the frequency terms are squared in (5.14) and (5.15), so even at moderate frequencies they dwarf the R and G terms of practical PWB trace. In this case, the R and G terms in (5.14) drop out and the phase constant becomes (5.16):

$$\beta \approx \omega\sqrt{LC} \tag{5.16}$$

The delay for this line can be found by combining (5.13) and (5.16), this time to form the well-known delay of a lossless line (5.17):

$$t_d = \frac{\beta}{\omega} = \frac{\omega\sqrt{LC}}{\omega} = \sqrt{LC} \tag{5.17}$$

Because from (5.17) the time delay of a lossless line is not frequency dependent, all of the signal's harmonics will be delayed by the same amount and so will recombine in proper phase at the load. Taken by itself, this suggests that signal distortion, especially at high frequencies, should be negligible. Of course, the opposite is true: signals are significantly distorted by PWB trace, especially so by long trace carrying high-frequency signals. As discussed next, this distortion is chiefly caused by unequal attenuation of each harmonic rather than the improper phase shift at high frequency. But at low frequencies or when signaling over very resistive interconnect, the ω^2 term does not swamp out R, and (5.15) shows that each harmonic will be delayed by a different amount. Such a signal is said to experience *phase distortion*.

5.4.4 Attenuation

Intuitively, signals propagating down lossy transmission lines experience attenuation by an amount that is strongly dependent on the line's length. A line twice as long as another attenuates a signal not a factor of two, but rather by a factor of greater than seven, assuming both lines are properly matched. Matching is important because reflections can change the load voltage, making it appear as if a lossy line has lower (or sometimes greater) attenuation than calculated.

In fact, a sine wave is attenuated exponentially as it travels down a lossy line, as shown in (5.18):

$$V_{fe} = V_{ne}\, e^{\alpha x} \tag{5.18}$$

where α is the loss factor found from (5.8) and is expressed in *nepers* (Np, in honor of John Napier, the first developer of logarithms [9]) per unit length. The signal travels a distance x from the *near end* to the *far end* of a line. The value given to α is negative for losses and positive for a gain.

Equation (5.18) is easily solved to find the voltage loss in nepers (5.19):

$$\text{Voltage loss in nepers} = \ln\left(\frac{V_{fe}}{V_{ne}}\right) \quad (5.19)$$

This is illustrated in the following example.

Example 5.5

Equation (5.8) is used to find γ for a certain transmission line at a specific frequency. From that calculation, α is found to be 0.0115 Np/in at that frequency. What is the voltage at the far end at that frequency if the line is 10 in long?

Solution

Because from (5.18) the signal swing reduces as $e^{\alpha x}$, the signal will be reduced to $e^{-0.0115 \times 10} = 0.891$ times its original value. A 1-V input swing would therefore appear on the output with an 891-mV swing. Loss is taken as negative in (5.18) to show the signal attenuates.

It would be convenient to have a way to calculate α directly without first having to calculate γ. In fact, an approximation for α appears as part of the simplification process used previously to obtain β. At high frequencies (where $\omega L >> R$ and $\omega C >> G$), [2] shows:

$$\alpha \approx \frac{R}{2Z_0} + \frac{GZ_0}{2} \quad \text{Np/length} \quad (5.20)$$

Example 5.6

Use (5.19) to compute for the transmission line in Example 5.1.

Solution

In Example 5.1, R and G are given at 100 MHz, so α can only be computed at that one specific frequency. Because R and G are specified per inch, the value computed for α will have units of nepers per inch.

$$\alpha \approx \frac{R}{2Z_0} + \frac{GZ_0}{2} = \frac{422m}{2(60)} + \frac{38\mu(60)}{2} = 0.0047 \text{ Np/in}$$

This matches the result from Example 5.2 and shows the good agreement at high frequency between (5.8) and (5.20).

5.4.5 Neper and Decibel Conversion

It's common for voltage loss to be specified in decibels (dB, one tenth of a Bel). Just as nepers express the ratio of the far-end voltage to the near-end voltage on a scale based on natural logarithms, decibels express that ratio on a scale based on common logarithms. Accordingly, (5.21) shows how voltage loss is expressed in decibels:

$$\text{Voltage loss in dB} = 20\log\left(\frac{V_{fe}}{V_{ne}}\right) \tag{5.21}$$

As with losses expressed in nepers, a negative decibel value represents a loss, while a positive one indicates gain.

The conversion between nepers and decibels is 20log(e) = 20(0.4343) = 8.686. That is, multiply the value in nepers by 8.686 to convert it to decibels.

It is often necessary to find the voltage ratio if the loss in decibels (or, sometimes, in nepers) is known. Equation (5.22) shows how to perform this conversion:

$$\text{Voltage ratio} = \frac{V_{fe}}{V_{ne}} = 10^{\frac{db}{20}} = e^{np} \tag{5.22}$$

The following two-part example illustrates the use of these equations.

Example 5.7

A transmission line connects a source to a load as shown in Figure 5.8. The voltage at the input to the transmission line (V_{ne}) is measured as 800 mV peak/peak, while the voltage at the output (V_{fe}) is measured as 650 mV peak/peak.
(a) What is the transmission line's loss in nepers and decibels?

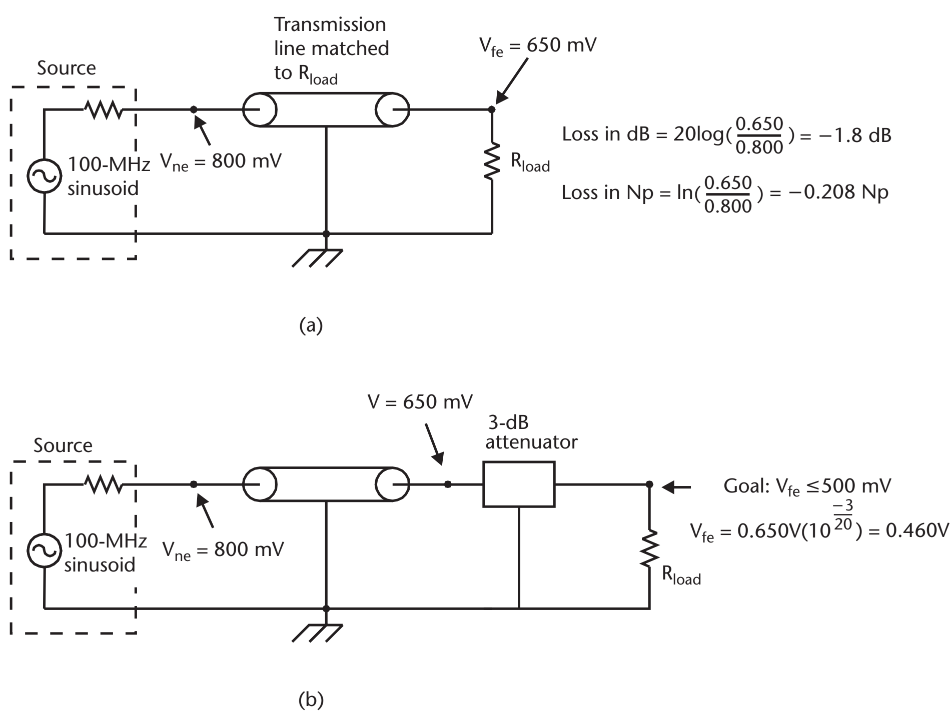

Figure 5.8 Circuit setup for attenuation calculations in Example 5.7.

Solution

From (5.21), the loss in decibels is $dB = 20\log\left(\frac{0.650}{0.800}\right) = -1.8$ dB, and from (5.19) in nepers it's $nep = \ln\left(\frac{0.650}{0.800}\right) = -0.208$ Np.

As a check, –0.208 nep × 8.686 dB/nep = –1.8 dB.

(b) It is desired to reduce the 650-mV signal to below 500 mV by adding a radio frequency (RF) attenuator at the load. Attenuators on 1-dB increments are available from stock. Which one should be selected?

Solution

The required loss is calculated by using (5.21) with V_{ne} = 650 mV and V_{fe} = 500 mV as –2.28 dB, so a 3-dB attenuator will be selected from stock. From (5.21), a 3-dB loss will reduce the 650-mV signal to $0.65 \times \left(10^{\frac{-3}{20}}\right) = 0.460V$, some 40 mV lower than the minimum requirement.

5.5 Summary and Worked Examples

The following four examples summarize the material in this chapter. Example 5.8 is a simple computation for the loss exhibited by a 12-in-long line when the RLCG parameters are known on a per-inch basis. Example 5.9 shows how to compute RLCG values (and so frequency-dependent loss) at new frequencies when the value is only known at one frequency. In doing so, it draws on material presented in Chapters 2 through 4. Example 5.10 computes propagation delay across frequency for a very resistive line and compares the results to the lossless case. In the final example of the chapter, Example 5.11 breaks out the total loss into its resistive and dielectric loss portions across frequency.

Example 5.8

A 12-in-long transmission line having the characteristics per inch calculated in Example 5.1 is used to connect a source to a 60-Ω load resistor, as indicated in Figure 5.9. When connected as shown, the source output voltage at V_{ne} is 800 mV peak/peak. Determine the voltage at the load (V_{fe}).

Solution

To calculate the far end voltage, it's first necessary to see if the transmission line has the same impedance as its load, because the attenuation and phase equations assume no reflections are present on the line.

In Example 5.1, the line impedance Z_o was found to be very close to 60Ω, so a simple 60-Ω load resistor can be assumed to provide a perfect match. Because there will be no reflections, the attenuation constant can be used to accurately determine the voltage at the load.

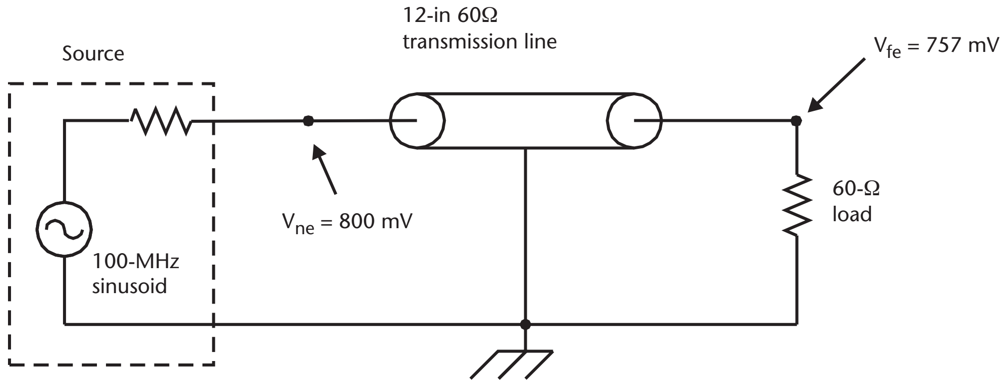

Figure 5.9 Transmission line connecting source to load for Example 5.8.

From Examples 5.2 and 5.6, α = 0.0047 np/in × 8.686 = 0.041 dB/in, so the total loss of the 12-in line will be 12 times that (0.49 dB). From (5.22), the voltage ratio corresponding to a loss of 0.49 dB is 0.945 [noting that the loss is used as a negative value in (5.22)]. If the near-end voltage is 800 mV, the far-end voltage will therefore be reduced to 800 mV × 0.945 = 756 mV.

Alternatively, the same result is obtained by using (5.22) with α in nepers. In this case, α = –0.0047 Np/in, and x is 12.

The second example shows how losses increase with frequency and demonstrates how they can be calculated across frequency even if their characteristics are known at only one frequency.

Example 5.9

Determine how the transmission line in Figure 5.9 attenuates frequencies of 500 MHz, 1 GHz, and 1.5 GHz. Assume the line is 1 in long and is properly matched at all of these frequencies, so no reflections will occur. Further assume the generator output remains “flat” across these frequencies (i.e., stays constant at 800 mV at the frequencies of interest).

Solution

To solve this problem, α must be determined for the three frequencies, but the RLGC values are only known at one frequency. From Example 5.1, the characteristics per inch at 100 MHz are:

R = 422 mΩ

G = 38 μS

L = 10.8 nH

C = 3 pF

We’ll first assume the inductance and capacitance values given at 100 MHZ are still valid at the higher frequencies. This is a good physics-based assumption for the inductance and is a good first-order assumption for the capacitance. In Table 3.2, the dielectric constant is seen to decrease as frequency increases, and this will cause *C* to be lower at the higher frequencies. However, the change is small for a good laminate and typically only results in an impedance change of a few percent when going from 1 MHz to 10 GHz. The change can be greater in lower performance laminates. To simplify things in this example, we'll assume a high-performance laminate and that capacitance remains fixed at 3 pF/in.

Although *L* and *C* can be considered constant, the resistance and conductance will vary significantly with frequency. From (3.15), *G* is seen to increase linearly with frequency, but from Chapter 2 we know that because the frequency is well above F_{skin}, the resistance increases as the square root of frequency.

This means the value of *G* at 500 MHz will be five times its value at 100 MHz, and *R* will be $\sqrt{\frac{500\text{ MHz}}{100\text{ MHz}}} = 2.24$ times as large. Table 5.1 shows the multipliers and corresponding computed values for *G* and *R* from 100 MHz to 1.5 GHz. Once *G* and *R* are known, (5.1) is used to compute α. That value (and the value when multiplied by 8.686 to convert it into decibels) is also presented in the table. Applying (5.22) yields the value shown for V_{fe} (assuming V_{ne} = 800 mV).

The higher frequencies are seen to be attenuated more than the lower ones. If these frequencies represent a signal's harmonics, the attenuation data in Table 5.1 shows an obvious distortion taking place, as the upper harmonics are attenuated far more than the lower frequency ones and thus will disproportionably recombine at the load.

Example 5.10

The transmission line described in Example 5.9 has a constant delay of 180 ps/in for all of the frequencies listed in Table 5.1. Recalculate the delay if *R* increases tenfold.

Solution

Table 5.2 shows the new results. The time delay (*td*) is found from (5.8) and (5.13) as was done in Example 5.3.

In this example, the delay is seen to decrease as frequency increases, and the table shows how higher frequencies travel more quickly than lower speed ones on lines having large resistive losses. This is significant to a waveform with a sub nanosecond rise time and will create a jittery, distorted signal. But it's

Table 5.1 Results for Example 5.9

Frequency	F_{mult} *(For G)*	$\sqrt{F_{mult}}$ *(For R)*	*G (μS) per inch*	*R(mΩ) per inch*	*α (Np) per inch*	*loss (dB) per inch*	V_{out} *(mV)*
100 MHz	1	1	38	422	0.0047	0.040	757
500 MHz	5	2.24	190	945	0.014	0.118	680
1,000 MHz	10	3.16	380	1,333	0.023	0.196	611
1,500 MHz	15	3.87	570	1,633	0.031	0.267	553

Table 5.2 Results for Example 5.10

Frequency	*G(μS)*	*R(Ω)*	*td (ps/in)*
100 MHz	38	4.22	187.29
500 MHz	190	9.45	181.46
1,000 MHz	380	13.33	180.69
1,500 MHz	570	16.33	180.44

worth noting that this came about because R was quite large compared to ωL. Even a narrow, thin PWB trace is unlikely to have such a large resistance by itself. However, as described in Chapter 2, the switching activity of neighbors sharing a common return path can have the effect of apparently increasing a trace's resistance. Especially for long lines, this effect can be large enough to cause a phase shift in important harmonics, which results in the kind of phase distortion evident in Table 5.2.

Example 5.11

Compare the conductor and dielectric losses for the transmission line of Example 5.9.

Solution

Equation (5.20) breaks out the conductor and dielectric losses. Setting G to zero yields the resistive loss portion, while setting R to zero yields the loss due just to the dielectric. The second column in Table 5.3 shows the total loss results in decibels using the R and G data originally appearing in Table 5.1. The third and fourth columns show the contributions due to resistive loss ($G = 0$) and dielectric loss ($R = 0$).

The total loss is seen to be the sum of the resistive and dielectric losses, and at the lower frequencies resistive loss is higher than the dielectric loss. In this example, that holds until the frequency reaches 1 GHz, at which point the total loss is evenly divided between the two. Dielectric losses are seen to dominate from that frequency on up. For this transmission line, a laminate having a lower loss tangent would improve losses for signal harmonics above 1 GHz.

Table 5.3 Example 5.11 Results

Frequency	*Loss (dB) (Total)*	*Loss (dB) (Resistive Losses, G = 0)*	*Loss (dB) (Dielectric Losses, R = 0)*
100 MHz	0.040	0.030	0.010
500 MHz	0.118	0.068	0.050
1,000 MHz	0.196	0.097	0.099
1,500 MHz	0.267	0.118	0.149

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