Editorial Introduction

Daniel Alpay and Victor Vinnikov

> *Tout problème profane un mystère; à son tour,*
> *le problème est profané par sa solution.*
>
> (Cioran, Syllogismes de l'amertume, [5, p. 40])

This present volume contains six papers written on the occasion of the workshop *Operator theory, system theory and scattering theory: multidimensional generalizations and related topics, 2005*, held at the Department of Mathematics of the Ben-Gurion University of the Negev during the period June 26–July 1, 2005. As for the previous conferences held in 2001 and 2003, we wish to thank the participants for a great scientific atmosphere and acknowledge the support of the Center of Advanced Studies in Mathematics of Ben-Gurion University of the Negev, which made the workshop possible. A volume of papers written on the occasion of the 2003 conference appeared in the series *Operator Theory: Advances and Applications*; see [2].

The papers can be divided into the following topics, which had an important place in the conference, namely:

1. Schur analysis
2. Quaternionic analysis
3. Multidimensional operator theory.
4. Moment problems

Schur analysis: Schur analysis originates with the papers [7], [8] where I. Schur associates to a function analytic and bounded by one in modulus in the open unit disk an infinite sequence of numbers in the open unit disk, or if the function is a finite Blaschke product, a finite sequence of numbers in the open unit disk and a number on the unit circle. It is an active field of research, see, e.g., [1] for recent developments. The first paper, **The transformation of Issai Schur and related topics in an indefinite setting**, by A. Dijksma, H. Langer and the first named editor, reviews recent developments in the Schur algorithm in the indefinite case. The authors focus on the scalar case. This hypothesis allows to obtain uniqueness results for the factorization of certain 2×2 matrix functions which are unitary with respect to an indefinite metric on the unit circle or on the real line. The

theory of reproducing kernel Pontryagin spaces of the kind introduced by L. de Branges and J. Rovnyak in the Hilbert space case, see [3], [4], play an important role in the theory. The next paper, **A truncated matricial moment problem on a finite interval. The case of an odd number of prescribed moments**, by A. Choque Rivero, Y. Dyukarev, B. Fritzsche and B. Kirstein, deals with the matrix-valued case. The authors use V. Potapov's Fundamental Matrix Inequality method combined with L. Sakhnovich's method of operator identities. The Schur algorithm has been conducive to important applications in signal processing, in particular in relationships with structured matrices. In the paper **Algorithms for hierarchically semi-separable representation**, Z. Sheng, P. Dewilde and S. Chandrasekaran consider hierarchical semi separable matrices, which are structured matrices for which various efficient algorithms can be developed, for instance to compute the Moore-Penrose inverse. The study is in the setting of time-varying systems.

Quaternionic analysis: Linear algebra when the complex field is replaced by the skew field of quaternions has recently been the object of various papers; see for instance [6], [9]. Such results are important *per se* and also in the wider setting of hypercomplex analysis, when the Cauchy operator is replaced by the Cauchy–Fueter (resp. the Dirac) operator and analytic functions are replaced by hypercomplex functions (resp. by regular functions). In **Canonical forms for symmetric and skewsymmetric quaternionic**, L. Rodman studies canonical forms for pairs of quaternionic matrices, and gives applications to various problems where symmetric matrices are involved and to joint numerical cones of pairs of skewsymmetric quaternionic matrices.

Multidimensional operator theory: We discussed at length the various extensions of multidimensional operator theory in the editorial introduction of [2]. In the present volume two directions are represented: the first is the theory of homogeneous operators while the second is related to abstract moment problems. More precisely, in the paper **On the irreducibility of a class of homogeneous operators**, G. Misra and S. Shyam Roy use Hilbert module techniques to study homogeneous d-uples of operators. In **Unbounded normal algebras and spaces of fractions**, F. Vasilescu considers algebras of fractions of continuous functions to study representations of normal algebras and operator moment problems.

Moment problems: Moment problems permeate much of mathematical analysis. They have both motivated many of the more abstract and technical developments during the last hundred years, and benefited in their turn from such developments. In the present volume they are discussed in the first, second and last paper.

References

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Daniel Alpay and Victor Vinnikov
Department of Mathematics
Ben-Gurion University of the Negev
Beer-Sheva, Israel

e-mail: `dany@math.bgu.ac.il`
e-mail: `vinnikov@math.bgu.ac.il`