THE DEVELOPMENT OF NEWTONIAN CALCULUS IN BRITAIN 1700–1800

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CAMBRIDGE UNIVERSITY PRESS

Cambridge

New York Port Chester Melbourne Sydney

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1989
First paperback edition 2002

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data
Guicciardini, Niccolò.
The development of Newtonian calculus in Britain 1700–1800 /
Niccolò Guicciardini.
p. cm.
Originally presented as the author's thesis (Ph. D.–Council for
National Academic Awards, 1987)
Bibliography: p.
Includes index.
ISBN 0 521 36466 3
1. Calculus – Great Britain – History – 18th century. I. Title.
QA303.G94 1989
515′.0941′09033–dc20 89-7085 CIP

ISBN 0 521 36466 3 hardback
ISBN 0 521 52484 9 paperback

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INTRODUCTION

EIGHTEENTH-CENTURY British mathematics does not enjoy a good reputation. The eighteenth century, a 'period of indecision'[1] as many historians would say, is said to have witnessed 'the crisis' or the 'decline' of mathematics in the country of Newton, Wallis and Barrow. However, even a glance at the following list of names should be sufficient to refute the prevailing image of eighteenth-century British mathematics. To the imported Abraham de Moivre one can add the native Brook Taylor, James Stirling, Edmond Halley, Roger Cotes, Thomas Bayes, Colin Maclaurin, Thomas Simpson, Matthew Stewart, John Landen and Edward Waring. Through their work they contributed to several branches of mathematics: algebra, pure geometry, physical astronomy, pure and applied calculus and probability.

I devote this work to a theory that all these natural philosophers knew very well: the calculus of fluxions. This was the British equivalent of the more famous continental differential and integral calculus. It is usually agreed that the calculus of fluxions was clumsy in notation and awkward in methodology: the preference given to Newton's dots and to geometrical methods engendered a period which was eventually labelled as the 'Dot-Age'.[2] Furthermore, the calculus of fluxions is usually indicated as the principal cause of the decadence of British mathematics: the 'Dot-Age' was the price paid for a chauvinistic attachment to Newton's theory.

The origin of this depressing image of the Newtonian calculus can be easily traced back to the irreverent writings of the Cambridge Analytical Society's fellows who, at the beginning of the nineteenth century, tried to introduce into Great Britain the algebraical methods of Lagrange and Arbogast.[3] Like all the reformers, they offered a pessimistic view of the past. Since then, many historians have behaved as loyal members of the Analytical Society, and a standard account of the eighteenth-century fluxional calculus has been given in the histories of mathematics. For instance, in Koppelman (1971) we find stated that the 'quiescence' of English mathematics in the eighteenth century depended upon the isolation of English mathematicians from the continent. The reason for this isolation is attributed to the 'bitterness engendered by the Newton–Leibniz priority controversy' and to the 'insularity of the English'. The result of this situation was, according to Koppelman, that 'the Newtonian school clung to a clumsy notation and, perhaps even more important, to a reliance on geometric methods out of a misguided belief that these represented the spirit of Newton' (Koppelman (1971), pp. 155–6).

The difference between Newton's and Leibniz's notation has been given too much importance. Even though there are some reasons for preferring the differential notation, it is certain that the progress of the calculus of fluxions was not dependent upon the choice between the dots and the d's. Indeed the fluxional notation is still successfully used in mechanics to express the derivatives as a function of time.

Another commonplace misinterpretation is that British mathematicians used geometrical methods. It is not clear to me how the researches of Stirling on interpolation, or of Taylor on finite differences, the second book of Maclaurin's *Treatise of Fluxions* (1742), the work of Simpson on physical astronomy and geodesy, the results of Landen on infinite series and elliptic integrals, and those of Waring on fluxional equations could be defined as geometrical. Many British mathematicians consciously departed from the geometrical methods of the *Principia*, and they did so with different motivations and different results.

The current account of the decline of the calculus of fluxions also includes sociological discussions. It is maintained that the practical bent of a country dominated by the industrial revolution together with the chauvinistic isolation of British scientists caused the stagnation of mathematics. However, many British scientists cultivated a deep interest in pure research, for instance in pure geometry or cosmology, and in Great Britain there was a considerable interest in mathematics as the many 'philomaths' mathematical serials show. The existence of a chauvinistic myth for the *Philosophia Britannica*[4] is undeniable, but this does not imply that there was a total separation between continental and British scientists. For instance, continental and British astronomers were in close contact. Furthermore, the theory of the 'golden isolation' of the fluxionists does not explain why there should be so many letters from continental mathematicians in the correspondence of Stirling and Maclaurin and why there existed several translations from continental mathematical works into English using Newton's notation.[5]

It is disappointing that the only work devoted to the eighteenth-century British calculus, Florian Cajori's *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse* (1919), restates the usual account. For instance, on p. 254 Cajori simply says that 'Newton's notation was poor and Leibniz's philosophy of the calculus was poor', a statement which historians of Leibniz's mathematics would not easily accept; while on p. 279 we find that 'the doctrine of fluxions was so closely associated with geometry, to the neglect of analysis, that, apparently, certain British writers held the view that fluxions were a branch of geometry'.

Furthermore, Cajori is interested only in the definitions of the term 'fluxion'. Since these definitions did not change very much during a whole century and were generally unsatisfactory from a modern point of view, he takes it as an argument in favour of the thesis of the decline of the British calculus. Cajori's quotations are invariably taken from introductions and prefaces of treatises on fluxions. The reader is left without any information about the authors, the length and contents of their works, and the purposes for which they were written.

Thanks to the recent works of Schneider (1968), Gowing (1983) and Feigenbaum (1985) we have acquired a very good knowledge of de Moivre, Cotes and Taylor. However, it seemed to me necessary to study the whole period from Newton's work to the reform of the calculus in the early nineteenth century. I will offer a general survey of the development of the calculus in Great Britain; I will not consider therefore the impact of the Newtonian calculus on continental mathematics. I will try to concentrate especially on aspects which are not covered in other works. Whenever it is possible, I will refer the reader to studies which cover specific subjects or authors. First of all, I will take for granted a knowledge of Newton's mathematical work, which has been extensively and masterfully studied by Whiteside in his well-known edition of Newton's mathematical papers. Other works which have been useful are: Tweedie (1922) and Krieger (1968) on Stirling; Eagles (1977a) and (1977b) on David Gregory; Clarke (1929) on Simpson; Tweedie (1915), Turnbull (1951) and Scott (1971) on Maclaurin; Grattan-Guinness (1969) and Giorello (1985) on Berkeley; Trail (1812) on Simson; Chasles (1875) on Simson, Stewart and Maclaurin; Smith (1980) on Bayes; and Bos (1974) on the differential calculus. The *Dictionary of National Biography*, the *Dictionary of Scientific Biography* and E.G.R. Taylor's *Mathematical Practitioners* (1954) and (1966), have been indispensable tools in this work. However, the most important source of information on the lives and works of British mathematicians is the monumental P.J. and R.V. Wallis's *Biobibliography of British Mathematics and its Applications* (1986), which I have been able to use at the final stage of my research.

The Overture is devoted to Newton's published work on the calculus of fluxions. Its aim is to present the fundamental elements of Newton's calculus.

The first chapter is concerned with the early diffusion of the calculus of fluxions from 1700 to 1730. The first attempts to popularize the Newtonian calculus were carried out by quite obscure mathematics teachers and itinerant lecturers, such as Charles Hayes, John Harris, Humphry Ditton and Edmund Stone. An analysis of their textbooks shows that they were influenced by the Leibnizian as well as by the Newtonian tradition. The second chapter deals with the research in pure mathematics done by the early Newtonians. Of particular importance are the researches on integration by Roger Cotes, on finite differences by James Stirling and Brook Taylor, and on higher ordered curves by Colin Maclaurin. It seems that early Newtonians, rather than researching the calculus of fluxions, developed related theories, especially the theory of series. In the third chapter space is given to the controversy on the foundations of the calculus originated by Berkeley's *Analyst* (1734). The most authoritative answer to Berkeley was in Maclaurin's *Treatise of Fluxions* (1742): the true manifesto of the fluxionists.

The fourth, fifth and sixth chapters are devoted to the middle period of the fluxional school, roughly from 1736 to 1785. The production of new treatises and the improvements in the applications of the calculus of fluxions occupy, respectively, the fourth and fifth chapters. Particular importance is given to Maclaurin's and Simpson's study on the attraction of ellipsoids. The sixth chapter is concerned with the attempts made by some British mathematicians to develop new techniques in the calculus. A comparison with the progress on the continent shows that the Leibnizian calculus developed into a new form: it became an analytical tool dealing with multivariate functions. Interest in the work of continentals stimulated Thomas Simpson, John Landen and Edward Waring. However, they largely failed to understand the novelty of the analytical techniques of the continentals.

Chapters 7, 8 and 9 are devoted to the reform of the calculus which took place in the period 1775–1820. Four schools of reformers were involved, geographically situated in Edinburgh, the military schools of Woolwich and Sandhurst, Cambridge and Dublin. This part of the book is based on completely unknown material, the contribution of two generations of

British mathematicians having been ignored by historians. It is argued that the work of these mathematicians, including Charles Hutton, John Playfair, James Ivory, William Wallace, John Brinkley and Robert Woodhouse, laid the foundations for the resurrection of British mathematics in the first half of the nineteenth century.

In Appendix A I have grouped the tables which give information on the content, and in appendix B I have given the prices of some textbooks on fluxions. Appendix C lists the Chairs of Mathematics in Cambridge, Oxford, Edinburgh, Glasgow, St Andrews, Aberdeen and Dublin, and Appendix D gives information on the teaching of mathematics in the military schools at Woolwich, Sandhurst and Portsmouth. A subject index of the primary literature is given in Appendix E, and a list of the manuscript sources used is in Appendix F.

After these appendices the reader will find the endnotes, the general bibliography and the index.

This book therefore covers more than a century. From necessity I have been extremely selective in the analysis and discussion of the works connected with the development of the fluxional calculus. I have chosen those which appeared to me more exemplary of the level of research and style of a determinate mathematician or group of mathematicians. In compiling the bibliography, on the other hand, I have tried to be as complete as possible. I hope that my work will be useful as a first survey and historical assessment of the contributions (and failures) of British mathematicians in the eighteenth century.

This book is an improved version of my Ph.D. thesis submitted in June 1987 to the Council for National Academic Awards. A three year scholarship of the Italian Ministero della Pubblica Istruzione (D.M. 27.1.83) and a two year appointment as part-time research assistant at Middlesex Polytechnic (UK) provided the financial support which allowed me to complete my doctorate. My interest in Newton's calculus originates from the thesis I wrote in 1981–2 under the supervision of Prof. Corrado Mangione at Milan University. I was then encouraged by several friends, among whom the most encouraging was Giulio Giorello, to pursue and extend my research. Next I must mention Allan Findlay and Ivor Grattan-Guinness, the supervisors of my Ph.D. thesis during the years 1984 to 1987. Ivor followed my every step and gave to me all his encyclopaedic assistance: I owe very much to his competence, but especially to his friendship. I would like also to thank Eric Aiton, the external examiner of my Ph.D., who directed my attention on many points which needed amendments and additions. During several stays in Cambridge I received advice from Michael Hoskin and Simon Schaffer. In Cambridge I had the privilege of meeting Tom Whiteside, the great authority on Newton's mathematics, who, with great kindness and generosity, criticized several drafts of my book. I also owe a great deal to Roger Bray for giving me important information on the military schools, to Jennifer Carter for her kind letters on Aberdeen University, to Marco Panza for sending me early drafts of his (1989), and to Eric Sageng for his advice on James Gregory and Colin Maclaurin. Luca Bianchi, Umberto Bottazzini, Michele Di Francesco, Massimo Galuzzi and Angelo Guerraggio have been important in the progress of my research in several ways. I would like also to thank the librarians of Cambridge University Library (most notably Stephen Lees), the British Library (London), the Senate House Library (London), and the Royal Society Library (London). Thanking all these people is the part of my research which gives me the most satisfaction.