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BY

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PREFACE

WHEN I was asked to write this tract I was given the privilege of making any possible use of the tract by my old friend J. E. Wright, which was published under the same title about twenty years ago. In these twenty years, however, so much has happened to change our view of the subject that I am sure Wright would have written an entirely new tract if he had lived—and that is what I have done.

It is not merely that many new discoveries have been made, but since the advent of Relativity the subject has been so much studied and expounded with a view to its applications that it now seems possible to say that certain methods are definitely accepted as of primary interest and certain others left to one side as of less consequence to science as a whole. I have tried to set forth the parts of the subject which are important for the applications as fully as the space available would permit and therefore have been forced to leave out several of the questions which Wright included. I have also tried to make the tract elementary in the sense that fundamental definitions are carefully formulated. This has necessarily made the preliminary part of the book long as compared with the rest, and has crowded out material on the applications of the subject which I wrote with more pleasure than some of the pages actually included. However, there are so many books on Relativity, and doubtless will be so many others applying differential invariant theory to Electromagnetic theory, Dynamics, and Quantum theory that one may perhaps be forgiven for not trying to include the applications in these few pages.

Differential geometry has also been crowded out. It seemed important to illustrate the general ideas by the simple case from which they are generalized, namely, elementary geometry. This left no room for higher differential geometry, not even for a discussion of infinitesimal parallelism. But the geometrical point of view is accessible in several recent books* with which this one is not intended to compete. Its purpose is rather to assist the students of differential geometry and mathematical physics by setting forth the underlying differential invariant theory. So it is not entirely by accident that the book ends with a formula which can be of interest only to a reader who intends to go forward to the problems in which it is used.

My thanks are due to several of my colleagues and students at Princeton who have made helpful suggestions either when reading the manuscript or during my lectures on the subject. I am particularly indebted to Dr J. M. Thomas and Mr M. S. Knebelman who have read the whole of the manuscript, and the proof sheets as well.

* On differential geometry we may mention D. J. Struik, *Mehrdimensionale Differentialgeometrie*, Berlin, 1922; J. A. Schouten, *Der Ricci-Kalkül*, Berlin, 1924; E. Cartan, *La Géométrie des espaces de Riemann*, Paris, 1925; T. Levi-Civita, *Lezioni di Calcolo Differenziale assoluto*, Rome, 1925 (English translation, London, 1927); L. P. Eisenhart, *Riemannian Geometry*, Princeton, 1926: on differential invariants in general, R. Weitzenböck, *Invariantentheorie*, Groningen, 1923.

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