INTRODUCTION

A demanding, prudent, "experimental" attitude is necessary; at every moment, step by step, one must confront what one is thinking and saying with what one is doing, with what one is. (Foucault, 1984, p. 374)

As teachers, we rightly value the ways in which our students bring meaning to the mathematical situations they encounter. There is much scope for judgment, insight and creativity in the style of mathematical work being introduced in many schools and we may aspire to encourage these qualities in the students' learning. Yet there is still a need for an individual to reconcile her own personal mathematical understanding with the ideas and traditions which have grown out of centuries of mathematical exploration and invention (cf. Ball, 1993). Whilst students can be creative mathematicians there is still a need to be able to do everyday calculations and understand aspects of conventional mathematical thinking. We are often torn between attempting to focus on our students' own way of seeing their mathematical endeavours, and seeing these endeavours with our own eyes, inspired perhaps by a "correct" view of mathematics. There are inevitably difficulties for us in making sense of students' own developing understanding without using our own "expert" overview as a yardstick, especially when we pose the tasks that they are working on. Teacher descriptions of students' learning often presuppose an adult overlay framing the mathematical ideas supposedly being addressed. Meanwhile, the sort of constructions which students are likely to generate for themselves are a function of their own particular concerns in relation to the sorts of tasks with which they are presented. Whilst we may wish to encourage students to pursue their own mathematical concerns at times, we retain the option of blowing the whistle and denying that their work is indeed mathematics in the sense that we as teachers intend. Furthermore, as the apparent relevances of different aspects of mathematics, as perceived by society, grow or decline, the nature of tasks on offer, and the values associated with them, will alter.

In the climate of rapidly accelerating social change that we now face, conventions get replaced with alarming regularity and, as teachers, we face challenge in any supposed role as experts in the worlds our students will encounter. In meeting this we need to continually reassess how our intentions might be concealing yet promoting initiation into existing structures which in turn support the reproduction of those structures. Our ways of describing the world need to constantly readjust to meet new demands. At the heart of this is a readjustment in the relationship between the rate at which we grow and the pace at which our environment changes; the very relationships between adult-child, teacher-student, mother-daughter are brought into question, as are the ways these relationships underpin assumptions about approaches to teaching. When social change was slow it was reasonable to suppose that what was good for the father was good enough for the son. As it speeds up, however, conflicts arise; the weakening of the family being but one example. Brookes (1994, p. 45) suggests that for education systems to be compatible with the world as we experience it we need to

> accept the twin constraints of an environmental framework that is changing non-repetitively and accelerating and a generational framework which is cyclically repeatable and only gently changing.

This entails an on-going renegotiation of social roles and reevaluation of how we construct and utilise knowledge. The task of education becomes ever more concerned with enabling us to understand the changes of which we are part and to see how we might have some influence over them.

In academia at large it is no coincidence that the study of language has become so prominent in our examination of the social world; the world does not rest for long enough to allow descriptions of it to settle and become familiar. We become immersed in multiple "feedback" as our attempts to describe things grapple with a world entangled in its own self descriptions. As mathematicians we have often sought immunity to these shifts. The apparent sturdiness of mathematics has somehow resisted pressure to be more responsive to changes in demands made upon it and mathematical activity has retained an image of being anchored by various mathematical truths. But it is now becoming more apparent that mathematics is created and utilised through history according to time dependent needs which change ever faster. Mathematics education research has responded by promoting less "positivistic" understandings of its host discipline, but as yet has seemed reluctant to go the whole way to understanding itself as an integral part of a social web subject to fundamental and continuous reevaluation. As such one might argue that we have not fully addressed the "strong programme" presented by Bloor (1976), which sought an extension of the practices of sociologists of scientific knowledge to include an examination of the content and nature of human scientific knowledge, and not just the circumstances surrounding its production.

Broadly, this book concerns the way in which language and interpretation underpin the teaching and learning of mathematics. In tackling this theme I wish to position some issues arising from research in mathematics education in relation to some major writers in continental philosophy. In particular, issues of language, understanding, communication and social evolution, all of which are tackled by recent mathematics education research under the banner of constructivism and related areas, are central themes in post-war western thinking on philosophy and the social sciences, yet research in mathematics education seems to under-utilise the resource of work done in the broader context. Whilst there is a growing recognition that such work is of importance (for example, Walkerdine, 1988; Skovsmose, 1994), we are still in the early days of such developments. In developing my theoretical framework I will be calling on certain key writers such as: Gadamer and Ricoeur on hermeneutics, Habermas on critical social theory, Saussure on linguistics, Derrida, Foucault and Barthes in post-structuralism and Schütz on social phenomenology. I seek to show how language is instrumental in developing mathematical understanding and also how both chronological and spatial dimensions of classroom experience condition ideas being met. I examine how language functions in orienting action within the normative constraints of a given situation and suggest that the task of the learner could be seen as reconciling experience with both conventional and potential ways of describing it. Classroom examples offered show school students seeking to capture their understanding in symbolic form. Meanwhile, college examples show teacher education students capturing their understanding in reflective writing. Recognising that the perspective of participants is becoming more central within analyses of social situations, this book offers a theoretical approach to discussing the world as understood through the eyes of participants. Within mathematics education research this means attending to the way in which students and teachers experience the classroom situation which they are in. These perspectives, it is suggested, are imbued with culturally derived or institutionally imposed structures, present both in the words used by inhabitants and in the physical space they occupy.

Most of the examples offered will describe students in the age range six to twelve learning mathematics in schools, although there will be some discussion of work taking place within teacher education courses including some detailed descriptions of approaches taken. I am writing as someone immersed in the British educational system and drawing on research published in English. Nevertheless, much of the empirical work was carried out in an English speaking Caribbean country (Dominica). The remaining school based material was collected in England from lessons observed in culturally diverse classrooms in London and Manchester, and some from my own classroom teaching in the Isle of Wight. The work reported with initial training students and practising teachers took place at Dominica Teachers College and the Manchester Metropolitan University. As such the majority of situations reported are connected to my own professional concerns as a teacher and teacher trainer. These situations are not typical of classrooms in general; rather my observations were motivated by an intention to locate and analyse examples of students expressing their understandings verbally or in writing. In a sense I am identifying what I see as examples good practice based around a struggle by students to combine mathematical activity with reflection on it. It is hoped that these examples be interpreted as possible strategies for developing the linguistic dimension of mathematical activity in line with a major theme of this book which sees this broadening of mathematical concerns as an approach to situating mathematics in a more developed notion of society.

Chapter one presents an exposition of hermeneutics together with an outline of its roots in phenomenology. It emphasises language as manifested in action rather than as a transcendentally existing system. This provides a platform for a social scientific analysis of mathematical learning seen as interaction, linguistically mediated and governed by social norms, that seeks to reconcile evolving understanding with static forms.

Chapter two further examines how language functions in orienting mathematical thinking and acting, and offers practical examples of how this is achieved. After outlining how Saussurian linguistics provides the roots of post-structuralism, it employs this theoretical perspective in developing the notion of linguistic framings of mathematical ideas being more stable than the thinking generating them. Further, the self-reflexive qualities of language that position the individual in her society are examined as an important dimension within that individual's self formation. This is used in addressing the difficulty of distinguishing between the individual learner creating and inheriting mathematical ideas.

Chapter three focuses on sharing mathematical perspectives and questions the potential which language displays in locating and holding on to mathematical ideas in doing this. For example, it examines the difficulties teachers face in alerting their pupils' attentions to mathematical ideas that the teachers seek to share. It also asks how students represent their understandings. A detailed description of a lesson is offered in which students seek to reconcile their mathematical experiences with various ways of capturing it for sharing with others. This commences a substantive analysis of children doing mathematics which spans the next four chapters.

Chapter four seeks to capture the classroom through a preliminary study of how students experience mathematics in their classroom environment. It is suggested that the physical and social dimensions of the classroom frame and so condition the mathematics being encountered. A number of examples are offered of students being guided by verbal instructions, peer interaction, physical apparatus, learnt rituals, and so on.

Chapter five introduces a more theoretical treatment of this classroom experience. Utilising a framework from social phenomenology, it examines how students orientate themselves within the space within which they see themselves working. Their learning is described as a hermeneutic reconciliation through time between their expectations and their actual experience; between this experience and their attempts to capture and share it.

Chapter six pursues the phenomenological approach in examining how the teacher maps out a picture of the individual student's understanding through the evidence available in the immediate classroom situation, with view to guiding and shaping the emerging mathematical thinking of the student.

Chapter seven, which comprises new material, examines a theoretical perspective on the ways in which children progress in learning mathematics. It suggests that there is a difficulty in associating teaching discourses with the mathematics they locate. This can result in an incommensurability between alternative perspectives being offered. The chapter resists attempts to privilege any particular account but rather demands an analysis of these discourses and their presuppositions. In developing these themes the chapter invokes Ricoeur's analysis of time and narrative as an analytical approach to treating notions such as transition, development and progression in mathematical learning.

Chapter eight argues for seeing all mathematical experience as being linguistic. Leaning primarily on the work of Derrida the

CHAPTER 2

THE PRODUCTION OF MATHEMATICAL MEANING: A POST-STRUCTURALIST PERSPECTIVE

Where therefore is truth? A mobile army of metaphors, metonymies, anthromorphisms ... truths are illusions of which one has forgotten that they are illusions (Nietzsche, quoted by Spivak in her translator's introduction to Derrida, 1976, p. xxii)

As we have seen, in recent work in the study of language there have been moves away from seeing words within a language as being a labelling device to denote preexisting phenomena. Rather, language and phenomena bring each other into existence through time, a process through which meaning is produced. I have shown, within an hermeneutic analysis, how language comes to the fore in mediating mathematical experience. Indeed, it becomes difficult to locate mathematics outside of a linguistic frame. Mathematics is accessed through accounts humans offer about it. Any attempt to locate the underlying truth of mathematics results in us encountering what Lacan calls the "lack"; the emptiness which emerges after the final layers of description ("stories") are peeled away (Brown, Hardy and Wilson, 1993). Whilst hermeneuticians might hold onto notions of truth anchoring our thoughts, post-structuralist accounts resist the notion of such centring and instead occupy the "play" of accounts offered through history. Rather, we enter what Foucault calls "regimes of truth", or discourses governed by their own internal structures, consequential to society's view of itself. Such a view results in educational discourses which emphasise the individual constructions of the student, less constrained by concerns for the teacher's intention, or by any notion of universal "truth" (Gallagher, 1992).

In this chapter, following an introduction to post-structuralism and its implications for education, I will set out the structuralist roots of post-structuralism through outlining Saussure's linguistic model. In particular, I introduce Saussure's notion of the "sign"; an association between a mental concept and a mental image of a word or symbol. I consider the implications of Lacan's assertion that the linguistic signifier is more stable than the concept with which it is associated. An example is given of a particular mathematical expression being used by students where the meaning of the expression evolves as the students' work progresses. I also outline the importance of Saussure's distinction between language as a system and the performance of language. After drawing an analogy with mathematics I conclude with a discussion of how learning mathematics is always associated with performance in mathematical activity, where individuals reveal how they see things and so introduce their own perspectives into the ideas they tackle. Finally, I suggest that this results in a self-reflexive dimension to learning mathematics which further compromises any presumed anchorage in universal truth. Most of the more abstract general theoretical discussion is confined to the first short section.

POST-STRUCTURALISM: A RADICAL FORM OF HERMENEUTICS

Although Derrida and Foucault might themselves question the description, they are generally described as post-structuralist writers. Certain writers (for example, Caputo, 1987, pp. 117-150; Gallagher, 1992 b, pp. 278-284) may well be adding insult to injury by arguing that post-structuralism can be seen as a radical form of hermeneutics. Many others would argue that hermeneutics and post-structuralism are incommensurable traditions. Eagleton (1983) provides an introduction, Spivak (1996, pp. 74-104) a more sophisticated analysis of this question. Nevertheless, whilst I would agree that we are examining here another case of "irreducible perspectives", post-structuralism does, I would argue, display many important similarities with the hermeneutics of Gadamer and Ricoeur as described in the last chapter, which makes it difficult to draw strict boundaries. The universality of language and linguisticality of interpretation are central in both camps. The principal distinction of interest here is Derrida's and Foucault's fairly extreme stand in not seeing interpretation as being governed by the motivation of locating the ultimate "truth". To them, writers like Ricoeur and Gadamer, with their theological leanings, are conservative because of their reluctance to abandon their supposed points of anchorage. Whilst Ricoeur, for example, acknowledges that it is possible to make many interpretations, he asserts that some will be better than others - some are closer to the "truth". Ricoeur and Gadamer attempt to hold on to some notion of underlying reality, albeit obscured by the media through which we attempt to access it. In post-structuralism, however, this quest is abandoned, the meaning is to be found in the text itself. Nevertheless, Foucault's early work (for example, "*Madness and*

Civilisation") pursued a largely hermeneutic quest, but he later rejected the "promise" of such enquiry. He came to feel that no matter how deeply one penetrates below the surface of the text one could not encounter reality outside the discourse itself. For example, in his book "*The Birth of the Clinic*" he no longer sought "madness itself behind the discourse about madness" (Habermas, 1985, p. 241). Foucault's work went through a number of other radical transformations before his early death.

Other writers, such as Barthes (e.g. 1976, 1977, pp. 79-124), Lacan and Levi-Strauss commenced with a more overtly structuralist enterprise, where they attempted to develop the linguistic model offered by Saussure into a "science of signs which goes beneath the surface events of language (parole) to investigate a variety of concealed signifying systems (langue)" (Urmson and Ree, 1989, p. 311). In Levi-Strauss's work on structural anthropology, for example, there was some belief in a structure (say of a particular society) that could be observed from the outside with some fixed relation between its outward manifestations and its inner workings. The task here was to translate the disorder of empirical experience into the order of systematic structures. The post-structuralists, meanwhile:

> rejected the binary oppositions between surface and depth, event and structure, inner and outer, conscious and unconscious ..(and) renounced the structuralist quest for a science of signs, celebrating instead the irreducible excesses of language as a multiple play of meaning (ibid.).

Given the focus of this book I should perhaps add "internalist" and "externalist", "individual" and "social" to this list of rejected binary oppositions. Derrida (1978, p. 287) pinpoints a significant shift in the work of Levi-Strauss which moved structuralist concerns in to more complex territory:

> The study of myths raises a methodological problem, in that it cannot be carried out according to the Cartesian principle of breaking down the difficulty into as many parts as may be necessary for finding the solution. There is no real end to the methodological analysis, no hidden unity to be grasped once the breaking down process has been completed. Themes can be split up *ad finitum*. Just when you think you have disentangled and separated them, you realise that they were knitting together again in response to the operation of unexpected affinities. Consequently the unity of the myth is a never more than tendential and projective and cannot reflect a state or a particular moment of the myth. It is a phenomenon of the imagination, resulting from the

attempt at interpretation; and its function is to endow the myth with synthetic form and to prevent its disintegration into a confusion of opposites. ... it follows that this book on myths is itself a kind of myth. (Levi-Strauss, 1970, pp. 5-6)

Derrida sees this observation by Levi-Strauss as a challenge to any notion of a "centre" orienting structural analysis. Such moves, which characterised the shifts, in the late sixties, in to what became known as post-structuralism, resulted in the whole idea of a systematic structure being undermined since there could be no agreed relationship, as any such view presupposed a particular individual perspective. Indeed, any supposed meaning itself becomes forever elusive. More importantly however, each individual can only describe the world of which they are part and so there is necessarily, a reflexive dimension to any such description. The observer is not so much describing a structure but rather their view of it, and by implication they are describing a bit of themselves. Foucault's (1972) book "*The Archaeology of Knowledge*" emphasises that forms of knowing occur in a multiply interpretable historical context. The very language we use is rife with conventional ways of classifying things, concealing existing power relations. This view echoes Habermas, although the two writers often found themselves in debate (e.g. Habermas, 1985).

POST-STRUCTURALISM, EDUCATION AND MATHEMATICS

Post-structuralism resists the supposed anchorages of tradition in describing social situations. Rather, meaning is to be found in the way in which different accounts interact with each other. For example, Barthes' notion of teaching is akin to the Gadamerian concept of a conversation; a dialogue where the "correcting and improving movement of speech is the wavering of a flow of words" (Barthes, 1977, p. 191). Barthes however does not see this process heading towards foreseeable outcomes. There are definite constraints on this apparent free play. Post-structuralist accounts of education tend to be rooted in the Marxist notion of society forming consciousness, where individuals are absorbed in social norms. For Althusser, (human) individuals are understood as "bearers" of a system of social relations which exist prior to and independently of their consciousness and activity (Urmson et al., p. 7). For an individual seen in this way, knowing, and its evolution, is closely associated with action, since the social practices which host specific actions are imbued with the society's preferred ways of doing things. Whereas critical hermeneutics sees education as risking the reproduction of the institutional power relations through its content, post-structuralism identifies a more all-embracing reproduction, namely the metaphysical framework which conditions all understanding.

For what can be oppressive in our teaching is not, finally, the knowledge or the culture it conveys, but the discursive forms through which we propose them (Barthes, quoted by Gallagher 1992 b, p. 300).

For example, Walkerdine (1988) describes categories such as "child", "teacher", "learning", etc. as constructions situated within historically and culturally specific discourses - the very fabric of the language we use presupposes manifold assumptions about our classification of the world (see also, Evans and Tsatsaroni, 1994). The reflexivity inherent in the language we use positions us in relation to the world we see our selves in. Foucault (1979, pp. 170-194) describes the regulation implicit in the classifications in every aspect of schools from the architectural design to the administrative structures, for example, examinations categorising students according to ability, dress codes and separate facilities according to sex, as strict codes regulating behaviour. Similarly, Althusser (1971) sees schools as an essential part of the "ideological state apparatus", asserting particular forms of "hegemony", that is, power relations held in place by common assent.

Conventional mathematical ideas are all culturally derived but have become so embedded within the fabric of our culture that it is hard for us to see them as anything other than givens. These historical choices can never be eradicated and will forever condition and mediate our experiences of mathematics. Derrida (1978, p. 281) has spoken of the difficulty of analysing linguistic structures since there is always a need to use the elements of the structures themselves in dismantling these very structures. Similarly, we can never analyse mathematics without using the culturally derived components of this very mathematics. Any "new" mathematical construction is always made within an inherited language which means that it is always already partially constructed. The culture provides the building blocks and the final building is a function of these. This applies to both the "objective" components of mathematics for example, symbols like "cos", "+", "$\sum$" or "$\geq$"

CHAPTER 3

SHARING MATHEMATICAL PERSPECTIVES

As I pointed out in the last chapter Saussure saw language as having the dual function of articulating meaning (process) and communication (product). However, Saussure and also his modern day advocates in post-structuralism see both process and product as contingent on each other. As educators we are often concerned not so much with the learning itself as with giving an account of what learning has taken place (cf. Garfinkel, 1967, pp. 24-31, who discusses the difficulties of moving between what is said and what is being spoken about). We need to decide what has been learnt by a student and how this has been demonstrated through tangible product, whether this be the application of a method, a reproduction of a famous result, or some verbal explanation of work completed, etc. Learning by the student, however, continuously evolves, oscillating between understanding and explanation; that is, between an on-going learning process and statements generated within this process, which become as if frozen in time. Further, this learning encompasses concerns beyond the frame which maybe anticipated by the teacher.

Questions were raised in earlier chapters concerned with the problematic relationship between mathematical ideas and the symbols which represent them. Such ideas, it has been suggested, cannot be transferred "ready-made" but, rather, are susceptible to interpretive modification as they move between people and through time. Hermeneutical views of mathematical learning emphasise the individuality and time dependency in our understanding of specific mathematical ideas. In assessing mathematical work we are thus faced with a task of finding an adequate way of locating mathematical knowledge - with seeing how it is held between the perspectives of teacher and student. Further, if we see mathematical work as linguistic activity, our view of developing understanding is dependent on the way in which we see language functioning in relation to reality and also on the way in which we see mathematics relating to its symbolic and physical embodiments. This chapter commences by briefly reviewing how the range of views of language offered by the various hermeneutical schools provide a framework for understanding mathematical learning and how they condition what mathematics is. This is followed by an examination of the task teachers and students face in sharing mathematical perspectives. The chapter concludes with an example of a lesson featuring students capturing their mathematical understanding in words, diagrams and pictures.

DISCOURSE OR REALITY?

In his book, "*Post-Modernism, or the Cultural Logic of Late Capitalism*", Jameson (1971, pp. 6-10) makes an interesting distinction between Modernism and Post-Modernism. He associates hermeneutical depth interpretation with the former and the textuality of post-structuralism with the latter. He illustrates this by contrasting two paintings; "A Pair of Boots" by van Gogh and "Diamond Dust Shoes" by Warhol. He suggests that van Gogh's painting of a pair of peasants boots gives rise to the possibility of various interpretations. He offers the magnificence of the bucolic landscapes we might normally associate with the paintings of van Gogh or alternatively, the stark peasant lifestyle suggested by such clothing. Either view can be developed as a fairly full account of what the painting might be seen as evoking. Warhol's effort, however, a dark, sparse, shadowy affair that may have been produced with the help of an X-Ray machine seems to defy any such generation of stories. It seems to be *all in the surface* - it begins and ends with the painting.

With this as our analogy in mind how might we distinguish between Modernist and Post-Modernist presentation of mathematics? Interpretation of mathematics might be seen as a relatively new idea, unless you are talking about statistics or mechanics. As suggested in previous chapters there needs to be an emphasis on the activity of mathematics before interpretation seems tenable. It is only when we consider people choosing to use some piece of mathematics or other that alternatives present themselves. This choosing however, has only recently been reintroduced into the vocabulary of school mathematics. The emphasis has generally been on mathematics where the choices have already been made. The students have typically been delivered to the already formed ideas and told to work with them.

In another reflection on post-modernism, Zizek (1989, p. 96) explores the Coca Cola advert which declares "This is it". What is "it"? he asks. He suggests that "it" is nothing other than America itself and the associated glossy lifestyle of which Coke is supposedly part. I suggest that "mathematics" has pulled off a similar *coup d'etat*. Mathematics is ordered, it is logical, obeys strict methods, is fully decidable... or so they say! After carrying out mathematics for years we have looked back on it and claimed certain features as being "it". These features however, seem completely devoid of the humans and their struggle that brought them into existence. Mathematics which has been derived from the activity that has given rise to it, is, it might be claimed, *all in the surface*.

Another feature which Jameson sees as distinguishing Modernism from Post-Modernism is that styles of the former have become icons of the latter - for example, a process of iteration becomes a button on a calculator. Things shrink as the field they are in expands into ever greater complexity. Interpretation comes firmly into play as we are forced to choose between an ever increasing number of things. The activity of choosing at all levels forces a permanent oscillation between interpretable mathematical activity and making statements *as if* free from the situation which gave rise to them. It may be that this manifestation of the hermeneutic circle asserts the dynamics which prevent mathematics seen as a discipline standing still long enough to be defined in either way.

This echoes a distinction drawn earlier between the hermeneutics of Gadamer and Habermas. In defining the scope of language we oscillate between seeing it as something of which we are part to seeing it as something we can operate on. I suggested Gadamer uses the former as his home base while Habermas uses the latter. Different breeds of hermeneuticians position themselves at varying points around this spectrum.

Post-structuralists analyse texts as individual performances of language (parole) but without any detailed investigation of any external reality to which they may refer. Their focus is exclusively on the the usage of language rather than on any externally defined meaning of language. Meanwhile, Gadamer prefers to emphasise how text resonates with the reader's experience. Language is seen as being in a dialectical relationship with reality, albeit a reality conditioned by language. Habermas prefers to assume a critical distance from language, seeking to understand how it functions in relation to its creators' intentions. A conservative view would neutralise language to the more limited function of labelling reality as in the work of Russell (1914, pp. 63-97) or the early Wittgenstein (1961). Nevertheless in all of these various positions we are concerned with how experience gets mapped into language and vice versa. If we take mathematics as a language we similarly move between seeing it as a dimension of human activity and as something *as if* free of human intervention - between seeing mathematics as discourse and seeing it as transcending human experience.

LOCATING MATHEMATICAL KNOWLEDGE

In assessing mathematics we seem at first to be caught between on the one hand, working with a style of mathematics where we assert a field of symbols *as if* devoid of humans and, on the other, speaking of mathematics as a depth interpretation of a certain style of human activity. This however, is not a satisfactory dichotomy, if only because we never have a choice of one over-arching symbolic framework. Mathematics can assume a multitude of linguistic styles, which can sometimes meet and intersect, but which very often conflict, or get confused. The choice is perhaps more accurately between on the one hand, depth interpretation of activity and on the other, composing fragments from alternative discourses. Both of these have an implicit interpretive dimension since choice is central in each.

If we emphasise mathematics as generative discourse we downplay the supposed permanence of its attributes. In the absence of hard mathematical knowledge which can be transported around intact we are faced with a difficulty in assessing the results of mathematical activity. If mathematics is seen as being interpretable it becomes more malleable and susceptible to variation according to who is presenting it. This causes problems for assessment of mathematical achievement. It might be suggested that the supposed transferability of mathematical topics influences the prominence they are given in the school curriculum. That is, those areas of mathematics more easily describable in clearly defined linguistic categories are more robust since they are more easily accounted for. Gattegno (1988, pp. 118-119), for example, argues that school education in general is mainly verbal and that many areas of mathematics sit uneasily in such a curriculum. As an example, he sees this as having led to a widespread deficiency in geometrical intuition - an area which needs to be taught yet does not lend itself to easy description. In Gattegno's view, school geometry is generally algebraic in nature and is mainly about categorising geometrical phenomena into discrete notions, a partitioning which frustrates intuition. Geometrical understanding is not fully classifiable in language and as soon as it becomes framed in language it is reduced into an algebraic style of thinking. Such an emphasis allows it to be mechanised, made repeatable, so that it is more manageable in a school setting. Geometrical intuition is harder to account for since any attempt to share it with others requires translation into algebra with the cost to "geometrical" experience that entails. Intuition thus becomes a "spin-off" of teaching rather than something easily targeted in didactic presentations, or described in curricula.

We are then faced with a question of how much of any mathematical experience can be held in the language which describes it. We are also concerned with how we might witness others attempting to capture their experience in language. Clearly, whatever view you take, mathematical expressions themselves do not mean the same to all people; individuals see expressions in the context of their own experience, cultural perspective and current intentions. Their intended meaning depends both on the way in which the individual perceives their task and on their familiarity with such expressions. Nevertheless, there remains an issue of how far we see such activity gravitating around "correct" or culturally specific meanings. In the teaching of mathematics it is the norm to assume that the teacher's task has something to do with drawing the student to a particular point of view. The teacher needs to find ways of enabling his students to share a perspective. Insofar as developing understanding is seen hermeneutically, however, learning is not necessarily about the reproduction of the teacher's knowledge in the mind of the student, but rather can be seen as a transforming of both positions. Knowledge is not a fully constituted object being confronted by a fully constituted student, rather, both change through a time-dependent process. Learning is not just about adding to knowledge, rather knowledge, or at least our state of knowing, can be transformed in many ways; one subtracts from it as well as adds to it, forgetting as well as remembering, one reorganises so that known "things" get new meanings - and knowing is not just about things.

In his early days I sometimes tried pointing out things to my baby son Elliot, but, it seemed, he just looked at my pointing hand. For pupils in classrooms there is a frequent conflict between attending to the teacher's understanding and attending to the object of that understanding. Should the child pay attention to the pointing hand or to the thing being pointed at? For the traditionalist anchored by some sort of positivistic reality, the object itself arbitrates. However, the further we move away from this towards views of language that see it constructing reality we are faced with more complex decisions as to the supposed location of knowledge.

CHAPTER 4

SOME LESSONS

Let me begin with an anecdote about two ten year old boys demonstrating various skills in a game of "Multiplication Snap" (Brown, 1994 c). Each had a share of playing cards and took turns to place a card on a central pile. My understanding of the teacher's intention was that if the product of two successive cards was in the range twenty to forty the first person to say "snap" collected the central pile. The game finished when one player ran out of cards.

It soon became clear that if I placed too much concern about what the teacher had in mind I would be distracted from what was really going on. To suggest that adherence to the teacher's rules would optimise the use of skill would be to ignore a considerable range of talents on offer. A variety of strategies were being employed by the boys towards winning the game. For it was the appearance of winning which governed much of what followed.

One of the key strategies was to slap a hand on the table each time a new card was placed on the central pile. Invariably, the boy placing the new card had most success with this since it arose prior to any detailed concern about whether or not the product of the two numbers was in the required range. There was no shame in asserting an incorrect pair, rather it displayed confidence and engagement. This ritual persisted throughout the game although as time progressed more obvious pairings, such as two picture cards which scored 10x10, escaped the slap. Whilst the initial slap often appeared as a demonstration of absolute certainty both players saw through this. On each occasion the successful slapper reluctantly withdrew his hand if confirmation was not forthcoming in the following few seconds, to make way for "reflection".

The ensuing period of relative calm permitted the search for some sort of justification - calculators appeared, jottings took place, friends were asked and searching looks were directed towards me (as the absolute authority!). This process of validation was full of tension and any half baked notion was worthy of an airing if only as a holding device. After all, it would appear from the play that any concern for your partner's ideas distracted you from coming up with your own. Each new declaration was preceded by a renewed slap of the hand on the central pile with varying degrees of decisiveness. Degrees of certainty seemed to displace any sort of right/wrong dichotomy. Uncertainties were often resolved by the loudest granting themselves the benefit of the doubt.

On closer inspection quite a few other strategies were being employed. These included; slapping a hand down as soon as it appeared the opponent was about to slap, offering new interpretations of the original rules, proceeding rapidly through controversial decisions, bluffing, claiming ownership of arguments offered by the opponent, blatant cheating such as changing the order of cards on the central pile. The quest was to convince others rather than to be correct and it was important to present a good case regardless of whether or not you had grounds for actually thinking you were right. The pressure was on to offer convincing arguments and there was no absolute authority available to offer any final confirmation.

The mathematics was inseparable from the social activity which generated it. In social situations generally, negotiation skills and the ability to appear correct are as important as actually being correct. One might suggest that modern day economics has less to do with statistical facts than with assertions of particular interpretations. A recent finance minister in the United Kingdom was sacked for lacking the required political aptitude to supplement his economic skill. (For a classroom example of an economics activity with young children discussed from this perspective, see Brown and Mears, 1994.) However, in classroom mathematics there are many ways of concealing ineptitude (cf. Holt, 1969). But, of course, these strategies should not detain us here since we are concerned with the teaching of mathematics!

In my analysis so far I have introduced some perspectives in which mathematical objects and the perception of them is softened, as the objects and perception of them, evolve together through time. It is this approach which will be taken further in this chapter. Our scope of interest however, will be a little broader. Rather than restricting ourselves to the individual's experiencing of mathematics per se, we step back a little further towards examining more closely how ideas emerge for the individual in the broader context of the mathematics classroom. In this, the classroom will be understood as an environment of signs, comprising things and people, which impinge on the reality of the individual student and influence the way in which ideas are identified and experienced. Conventional views of mathematical phenomena will not be presupposed, nor will physical embodiments of mathematical ideas be seen as transparent. Also, in line with radical constructivist philosophy I will not be relying on an expert overview of mathematics motivating this task, since such an overview is not available to the learner. I will seek to avoid assumptions about an independent, preexisting world outside the mind of the knower. Consequently, mathematical ideas will be seen as being held in the minds of teacher and students, without the anchoring of "actual" ideas. In addressing these issues I shall lean on the seminal work on social phenomenology by Alfred Schütz (1962, 1967).

A particular focus will be on how physical apparatus and language intervene in the process of developing mathematical thinking. I follow Kaput (1991) in seeing physical instructional apparatus as contributing to the "architectural" environment within which students build their own constructions, where physical apparatus guides thinking in much the same way as furniture guides movement around a room. Similarly, following on from earlier discussion, I see language as guiding rather than holding thinking. I will suggest that the characteristics and relative importances of phenomena perceived by the student in the classroom, evolve through time, and, in due course, some of these phenomena may be treated as "mathematical" as they are seen to be displaying particular qualities. However, even in work presented as "mathematical" to students by teachers, the mathematical qualities may not necessarily be immediately apparent for the student. For this reason, I focus on the initiation for the individual which takes place prior to becoming part of a (mathematical) *consensual domain* (cf. Kaput, 1991). That is, before fully formed ideas have been derived from the complexity of classroom engagement and been understood as being mathematical. I will consider how such ideas develop in the mind of a student, through time, in relation to that seen in immediate perception. This activates a concern that will span the next four chapters where I question both how mathematical thinking develops in time and space to produce language, but also how language is produced to create notions of progression through time and movement in space.

In the next chapter I shall introduce a framework through which mathematical thinking is seen as taking place in the imagined world through the filter of the world in immediate perception, with reference to the work of Schütz and Goffman. I will suggest that mathematical ideas are contained and shaped by the student's personal phenomenology, which evolves through time. In particular, I will question how students become aware of mathematical ideas in the complex environment of the classroom. Further, I will argue these ideas are never encountered directly, but rather, are met through a circular hermeneutic process of reconciling expectation with experience. A theoretical framework will be offered which accommodates the time-dependency implicit in this.

In this chapter, I offer some classroom examples of students doing mathematics as a prelude to examining how the student reads the situation they are in and how the significance of the teacher's input shows itself in their activity. I also consider how a teacher's intention is framed in the instructions she gives to students and in the physical instructional apparatus employed. We shall see that whilst the teacher's instructions may appear to be associated by them with very specific actions to be carried out by students, the students' reading and related actions may not be so precise before achieving any ultimate sharing of the teacher's way of seeing things.

Whilst working with trainee teachers in Dominica, I became involved in supervising a project investigating the role teacher's speech has in the management of a primary mathematics lessons. It was addressing the specific issue of finding alternatives to the "chalk and talk" strategies prevalent in a country where school based apprenticeship models of teacher training resulted in many teachers adopting styles similar to how they were themselves taught at school. At the beginning of the project it was common among these teachers to have a heavy reliance on their speech, both in their teaching and as a management technique, yet it seemed that much of this speech was ignored by the students, perhaps exacerbated by many children speaking a French Patois as their mother tongue (Brown, 1984, 1987 d). There often seemed to be a considerable gulf between the teachers' intentions as represented in their speech and the actual work being carried out by the students. The teachers on the project worked on the task of exploring the consequences of reducing their own speech in the classroom towards expressing themselves more economically and relying more on other strategies. By observing each other teach and through taking time out to look at their own teaching they became more aware of the way in which speech was used, but also, of the other factors governing the management of the class. This project, and in particular its concern with how classroom participants understand the space they are in, later became the basis of my doctoral dissertation (Brown, 1987 b). Lessons given by some of the teachers involved in this project will provide examples for the discussion which follows. My intention is to trace out some of the facets of the filter which translates teacher intention into responses by students.

Initially, I explore some situations where the teacher's verbal instructions combine with physical materials to influence the activity of the students. My focus here is on how the environment is read as functioning in shaping ideas in the activity of the students. I offer some anecdotes, produced at the time of the research, in an attempt to capture some of the issues that concerned us during the enquiry. They offer brief accounts of interludes within lessons, followed by some discussion of how the activity of students was being guided. These may be viewed as preliminary notes for the more rigorous treatment in the subsequent chapters. The first example features students representing and adding numbers using base ten strips (a home-made 2-D version of Dienes materials). The teacher's requests for students to make specific arrangements are greeted by substantial delays and much deliberation. Precise requests failed to receive precise responses. In the second example students are producing the sequence of square numbers. A precise course of action was prescribed by the teacher, yet the pupils interpretation of this or their inability to arrange the pieces as requested, stood in the way of smooth responses. The activity is shaped by the materials and the teacher's verbal requests but considerable scope for manoeuvre results from both intellectual and technical difficulties. The third example focuses on a counting exercise where students become immersed in the space created through their own actions, as the teacher's intentions become ever more distant. In particular, I focus on how the individual's perceived space is shaped by the actions of others. In the fourth example, a group of students, confronted with a task of ordering sticks by length, are governed as much by learnt rituals as by physical constraints. The final two examples describe college sessions led by me and attended by the teachers involved in the project. The focus in these two sessions was on how the teacher's intention is captured, transmitted and received in language.

Example 1: Addition using base ten strips

A group of six year old students were working on some problems set by their teacher. These involve using "base ten strips" in tackling double digit addition. The teacher's speech was very brief and sparse, consisting, almost entirely, of requests such as "Make 34", "Now make 21", "Now put them together". The students,

CHAPTER 5

THE PHENOMENOLOGY OF THE MATHEMATICS CLASSROOM

This chapter's primary purpose is to offer some preliminary work in theoretising the individual learner's perspective in mathematics lessons within a model derived from Schütz's seminal work in social phenomenology (for example, 1962, 1967). Here, the mathematics classroom is seen as an environment of signs, comprising things and people, which impinge on the reality of the individual student (cf. Brown 1996 c). The chapter introduces a framework through which mathematical work is seen as taking place in the imagined world through the filter of the world in immediate perception. This provides an approach to structuring evolving mathematical understanding. It is suggested that mathematical ideas are contained and shaped by the student's personal phenomenology, which evolves through time. Further, I argue that these ideas are never encountered directly but rather are always met through a circular hermeneutic process involving the reconciliation of expectation with experience.

In particular, I examine Schütz's framework used in describing how an individual experiences their world, as an approach to understanding how the student experiences the mathematics classroom. The focus in this paper is on the socio-cognitive aspects of learning mathematics seen from the individual learner's perspective as he builds an understanding of mathematics. Seeing a student as an insider of a particular way of life, I employ this perspective as a basis for offering a description of the process through which he develops mathematical ideas. It is this perspective that will be used as a home base in this enquiry rather than any sort of mathematical framework. That is, we shall concern ourselves with the task of the novice as he sees it, moving from a state of relative naivete, without the benefit of the expert mathematician pinpointing for us the mathematical objective governing the teacher's intention. From the outset this chapter should be understood as a one-sided enterprise, focusing on the insider point of view. In line with radical constructivist philosophy I will not be relying on such an expert overview of mathematics overseeing the students' work, since this is not available to the learner. In this particular chapter I will be proceeding *as if* there is no "independent, preexisting world outside the mind of the knower" (Lerman, 1989, p. 211), where mathematical phenomena can only ever be perceived from particular positions and perspectives by observers with individual interests, from specific historically and culturally determined backgrounds. Whilst certain perspectives presuppose a social plane capable of producing a social view, social phenomenology focuses on the individual's experience of this social plane.

In this chapter I examine an approach to describing how individual students create mathematics in the physical and social situation they inhabit. Mathematical activity is seen as mediating access by the individual to any supposed externally defined objective mathematics. Extending an earlier metaphor, I suggest that their task is to identify (or even build) the furniture as well as find their way around it. Conventional views of mathematical phenomena are not presupposed, nor are physical embodiments of mathematical ideas seen as transparent (cf. Voigt, 1994, pp. 172-176). Rather, I build a framework for describing how these phenomena develop in the mind of a student, through time, in relation to that seen in immediate perception. I suggest that the student faces a whole variety of things and people which hold his attention in different ways. The characteristics and relative importances of these things, as perceived by the student, evolve through time and, in due course, some of these may be treated as "mathematical" as they are seen to be displaying particular qualities. However, even in work presented as "mathematical" to students by teachers, the mathematical qualities are not necessarily immediately apparent for the student. This chapter focuses on a theoretical framework for describing mathematics which accommodates the shifts in form and meaning that mathematical notions undergo in the mind of the individual. The influence of the work of Goffman (1975) and Schön (1983, 1987) and their notions of *frame* and *re-framing* will be evident in my discussion.

In the first part I introduce the notion of "personal space"; the space in which an individual sees himself acting. This is derived from Husserl's Cartesian phenomenology and developed in relation to Schütz's extension of this work. I show how it can provide a model for describing how students proceed through the classroom environment of phenomena towards establishing mathematical sense. Mathematical ideas are seen as developing for the individual within activity, where activity is seen as being "held in" by various kinds of constraints; imagined or real, seen or unseen, some imposed by the teacher, some by other students, some by the physical environment and some by the student herself. As such, his world is captured in an evolving phenomenological frame, where there is a mutual dependency between the overarching frame and the components within it. The effect of these various constraints on an individual student depends on how he interprets and responds to them. The negotiation of these very constraints and the identification of the components of this space result in mathematical ideas being shaped in the mind of the individual. I seek to illustrate this process with an example of some students working on a mathematics task where the notion of "the line of symmetry" is seen as being embodied in some physical apparatus distributed to the children.

In the second part I introduce Schütz's theoretical structure. This model provides a framework for differentiating between the world as seen in immediate perception and the world as interpreted as a space for action (physical or mental). I also demonstrate how this provides a useful mechanism for structuring time and change. Following this model I suggest that the individual acts in the world he imagines to exist. I further suggest that mathematics resides in this imagined world and is in an interactive relation with the world of surface appearance. I develop this discussion in relation to the lesson on symmetry and show how the physical apparatus employed in this lesson can be seen as anchoring, although not determining, the students' mathematical constructions.

In the third part, I develop the discussion by proposing a mismatch between the individual's expectations and experience. Whilst the individual might (voluntarily) act in the world as they imagine it to exist, the world may resist these actions in an unexpected way (involuntary response) and so cause a shift in the way in which an individual perceives the world. This extends to the individual's use of a mathematical idea. In this process I suggest that the individual never reaches a final definitive version of any mathematical idea, but rather, is destined to be always working with his most recent version.

PERSONAL SPACE

In a classroom situation each person is acting according to how the world appears to him. In this section I focus on the student's insider view of his classroom situation. I wish to introduce a notion of *personal space*, an extension of that which Schütz (1962, p. 224) calls the *world within reach;*

the stratum of the world of working which the individual experiences as the kernel of his reality.... This world of his includes not only Mead's *manipulatory area* (which includes those objects which are both seen and handled) but also things within his view and the range of his hearing, moreover not only the realm of the world open to his actual but also the adjacent ones of his potential working. Of course, these realms have no rigid frontiers, they have their halos and open horizons and these are subject to modifications of interests and attentional attitudes. It is clear that this whole system of "world within my reach" undergoes changes by any of my locomotions; by displacing my body I shift the centre O of my system of coordinates, and this alone changes all the numbers (coordinates) pertaining to this system.

So viewed, the notion "personal space" lean's firmly on Cartesian notions as developed by Husserl. A unified subject is implied; a thinking subject who therefore is (Descartes' "cogito ergo sum"). This is an idea treated with a certain disdain by post-structuralist writers insofar as it supposes any "completed and finished identity, knowing always where it is going" (Coward and Ellis, 1977, pp. 108-109). As I have indicated Derrida would reject the binary opposition between individual and social perspectives. Lacan (1977, pp. 1-7), meanwhile, stresses the importance of Descartes' notion, but places much more emphasis on the formation of the thinking subject in the reflexivity of the thinking done. Nevertheless, the thinking subject may not be aware of this theoretical perspective on his actions and so assumes he has more control over his own destiny than may be supposed in more post-structuralist formulations. It is this personal perspective I wish to examine now (cf. Brown and Jones, 2001).

In my formulation I incorporate the accents and emphases the individual places on the elements he perceives to be forming in this space according to his particular phenomenological frame; that is, the way in which the individual carves up his own particular perceptual field. Such a frame is consequential to the "biographically determined position" and current motives of the individual, which taken together form what Schütz calls the individual's *interest*. (Schütz, 1962, pp. 76-77; Goffman, 1975, pp. 8-9). This notion is akin to someone having an "interest" in a business - an interest which governs that person's actions in respect of the business. It is through this interest that various associations give rise to phenomena not in immediate perception. This interest also motivates the individual's will to act. Habermas (1972) sees knowledge in general as being flavoured by the interests it serves -

a notion pertinent to what follows here. Such an interest may be, for example, a student's desire to solve a particular mathematical problem as quickly as possible so as to satisfy his teacher. This could be qualitatively different to the interest of someone wishing to solve a problem for its own sake and seeking to understand the experience of being "inside" a mathematical problem (cf. Mason, 1992).

The personal space of any individual also incorporates some concern about other people sharing the social situation and how these people contribute to the perceived constraints. This concern may be about the way in which they impinge on the physical space, or be more directly about social interactions. This is discussed fully by Schütz (1967, pp. 97-207, 1962, pp. 312-329). Schütz's analysis is based on an individual society member "guided by the system of typical relevances prevailing within our social environment", who assumes he uses language in much the same way as everyone else (1962, pp. 327-328). He is cautious about the objective character of the reality of which he speaks. Goffman (1975, pp. 4-5) pinpoints this:

> We speak of provinces of meaning and not of sub-universes because it is the meaning of our experience and not the ontological structure of the objects which constitute reality (Schütz, 1962, p. 230),

attributing its priority to ourselves, not the world:

> For we will find that the world of everyday life, the common sense world, has a paramount position among the various provinces of reality, since, only within it does communication with our fellow men become possible. But the common sense world is from the outset a socio-cultural world, and the many questions connected with the intersubjectivity of the symbolic relations originate within it, and find their solution within it (Schütz, 1962, p. 294)

Similarly, here I work from the premise that it is the individual's experience of the world, of mathematics and of social interaction which govern his actions rather than externally defined notion of mathematics itself.

I wish to offer some notes from my classroom based research to assist me in demonstrating the character of this notion of personal space as it might be for a student in a mathematics classroom. In the lesson described below some students are working together on a mathematical activity. I will discuss an extract from a transcript as a