CAMBRIDGE TRACTS IN MATHEMATICS

71. *Finite Free Resolutions*

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CAMBRIDGE UNIVERSITY PRESS
CAMBRIDGE
LONDON·NEW YORK·MELBOURNE

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1976
First paperback edition 2004

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication Data
Northcott, Douglas Geoffrey.
Finite free resolutions.
(Cambridge tracts in mathematics; no. 71)
Bibliography: p.
Includes Index.
1. Algebra, Homological. 2. Modules (Algebra)
I. Title. II. Series
QA169.N66 512′.55 75–31397

ISBN 0 521 21155 7 hardback
ISBN 0 521 60487 7 paperback

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Preface

This Cambridge Tract originated in a seminar given by J. A. Eagon while he was visiting Sheffield University during the session 1972/3. The aim of the seminar was to report on some recent discoveries of D. A. Buchsbaum and D. Eisenbud concerning finite free resolutions. I found myself fascinated by the subject and Eagon and I had many discussions on different aspects of it. In the end we were able to construct what we considered to be a simplified treatment of certain parts of the theory, and our ideas appeared subsequently in a joint paper.

I continued to think about these matters after Eagon had left Sheffield, and during 1973/4 and 1974/5 gave seminars covering an enlarged range of topics, but still using what I regard as elementary methods. The elementary approach was based on the belief that, for those sections of the theory I was considering, Noetherian conditions were never really necessary; consequently I was committed to showing that, where such considerations had previously been used, a way of getting rid of them could be found. Now the parts of the theory in which Noetherian properties had originally played an apparently vital rôle were, to a considerable extent, concerned with applications of the concept of grade; and I had, for some time, known of M. Hochster's approach to a theory of grade in which it was not necessary to restrict oneself to Noetherian modules. Thus the programme I had set myself hinged on adapting Hochster's ideas in order to produce a simplified theory of grade applicable to general modules over an arbitrary commutative ring. At the same time the theory had to be rich enough for the applications that I had in mind.

The results of the attempt to construct an appropriate theory of grade will be found in Chapter 5, and with the aid of what is contained there the original programme was carried through. One advantage that has emerged is that the account in the following pages demands remarkably little in the way of prerequisites. All that is required is a knowledge of the basic properties of modules and linear mappings, and, in a few places, a facility in working with tensor products is presupposed. Otherwise, with the exception of the Appendices, the account is quite self-contained. In particular I have resisted any temptation to use the theory of exterior algebras in the main text because, at this level, it is not particularly helpful and there are some key places where its use seems to have definite disadvantages. Of course it is to be expected that, as the theory grows, general structural considerations will come to play a more dominant rôle and that *ad hoc* computational arguments will be increasingly hard to find. I have tried, at appropriate places, to indicate how the subject of finite free resolutions has grown and to mention the names of those who have contributed to its development. Here I would like to thank those who came to my lectures and who encouraged me by their continued interest. I must add a special word of thanks to P. Vámos whose wide knowledge proved most helpful and whose comments on particular points led to many improvements; and to D. W. Sharpe for discussions on the rôle of grade in the theory of linear equations, for bringing various errors to my attention, and for help with reading the proofs. Also I am much indebted to C. J. Knight who invariably assists me with queries that have to do with Topology. But above all I wish to place on record my appreciation of the work done by my secretary, Mrs E. Benson, who, by producing the whole typescript with her characteristic skill and good judgement, has again enabled me to turn the notes for a seminar into a book.

Sheffield D. G. NORTHCOTT
July 1975