Preface

These lecture notes are based on several courses and lectures given at different places (University Pierre et Marie Curie, University of Bordeaux, CNRS research groups GRIP and CHANT, University of Roma I) for an audience of mathematicians. The main motivation is indeed the mathematical study of Partial Differential Equations that arise from biological studies. Among them, parabolic equations are the most popular and also the most numerous (one of the reasons is that the small size, at the cell level, is favorable to large viscosities). Many papers and books treat this subject, from modeling or analysis points of view. This oriented the choice of subjects for these notes towards less classical models based on integral equations (where PDEs arise in the asymptotic analysis), transport PDEs (therefore of hyperbolic type), kinetic equations and their parabolic limits.

The first goal of these notes is to mention (and describe very roughly) various fields of biology where PDEs are used; the book therefore contains many examples without mathematical analysis. In some other cases complete mathematical proofs are detailed, but the choice has been a compromise between technicality and ease of interpretation of the mathematical result. It is usual in the field to see mathematics as a black box where to enter specific models, often at the expense of simplifications. Here, the idea is different; the mathematical proof should be close to the 'natural' structure of the model and reflect somehow its meaning in terms of applications.

Dealing with first order PDEs, one could think that these notes are relying on the burden of using the method of characteristics and of defining weak solutions. We rather consider that, after the numerous advances during the 1980s, it is now clear that 'solutions in the sense of distributions' (because they are unique in a class exceeding the framework of the Cauchy-Lipschitz theory) is the correct concept. They allow for abstract manipulations, which we justify in the first section of the chapter 'General mathematical tools', and we use them freely throughout the text. Then one can concentrate on the intimate mathematical structure of the models.

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