Modelling with Differential and Difference Equations

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Contents

Preface

This book provides an introduction to modelling with both differential and difference equations. Our approach to mathematical modelling is to emphasize what is involved by looking at specific examples from a variety of disciplines. From each discipline enough background is provided to enable students to understand both the assumptions and the predictions of the models. Exercises have been included at the end of each section. They are intended to provide a balanced development of some of the main skills used in mathematical modelling, and hence they are an essential part of the book.

The main mathematical tools used in the book are differential and difference equations. *Differential equations* have their origins in mechanics: Newton's laws of motion lead to differential equations whose solutions can be used to predict the position of a body at some later time. Differential equations have been closely associated with the rise of physical science in previous centuries and they are now being used as models for real world problems in a variety of other disciplines. *Difference equations* are the discrete analogues of differential equations. They have risen to prominence in the last decade, during which it has become generally known that solutions of even very simple difference equations can exhibit complex chaotic behaviour.

To allow time for the development of other modelling skills besides solving the equations arising from the models, we have selected only models involving differential equations which are relatively easy to solve. Although our treatment of differential equations is intended to be self-contained, it is only fair to point out that our students were taking concurrently a first course in mathematical methods (beginning with the elements of differentiation and ending with some practice at solving separable and linear constant-coefficient differential equations, towards the end of the year). Some chapters of the book assume a knowledge of linear equations, complex numbers, vector algebra, or the elements of probability theory. The mathematical prerequisites are listed at the start of each chapter.

This book grew out of notes prepared for a first-year course given at La Trobe University for each of the last six years. The total time allotted to the course was 65 hours, including lectures and practice sessions. Not all chapters were covered in the same year and some choice is possible. A longer course could be organized by covering all the material in the book and including some computer work where relevant.

Each year we refined the material and its presentation, based on our experience in teaching the course during the previous years. Some curious incidental difficulties were faced each year by some students. These included (a) correct use of minus signs in setting up equations (b) sketching diagrams to illustrate the choice of a particular coordinate in a mechanics problem (c) distinguishing between the parameters of a problem and its unknowns. We have attempted to address these and other difficulties in both the text and the exercises. We have also analysed the steps involved in solving various types of problem, at least in the early part of the book, and we find this helps students to present their solutions to exercises clearly.

We wish to thank Sid Morris and Ed Smith for assistance with the overall planning of the original course and Alan Andrew, Jeff Brooks, Peter Stacey and John Strantzen for improvements in certain sections. One of the authors (G.F.) also wishes to thank Colin Pask for his encouragement. We also thank Dorothy Berridge and Annabelle Lippiatt for assisting with the typing.

G.F., P.F., A.J.
1996