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This book provides a self-contained introduction to cellular automata and lattice Boltzmann techniques.

Beginning with a chapter introducing the basic concepts of this developing field, a second chapter describes methods used in cellular automata modeling. Following chapters discuss the statistical mechanics of lattice gases, diffusion phenomena, reaction-diffusion processes and nonequilibrium phase transitions. A final chapter looks at other models and applications, such as wave propagation and multiparticle fluids. With a pedagogic approach, the volume focuses on the use of cellular automata in the framework of equilibrium and nonequilibrium statistical physics. It also emphasizes application-oriented problems such as fluid dynamics and pattern formation. The book contains many examples and problems. A glossary and a detailed list of references are also included.

This will be a valuable book for graduate students and researchers working in statistical physics, solid state physics, chemical physics and computer science.

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# Cellular Automata Modeling of Physical Systems

Bastien Chopard and Michel Droz  
*University of Geneva*



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To Viviane  
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*Everything should be made as simple as possible but not simpler*  
*A. Einstein*

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## Preface

The cellular automata approach and the related modeling techniques are powerful methods to describe, understand and simulate the behavior of complex systems. The aim of this book is to provide a pedagogical and self-contained introduction to this field and also to introduce recent developments. Our main goal is to present the fundamental theoretical concepts necessary for a researcher to address advanced applications in physics and other scientific areas.

In particular, this book discusses the use of cellular automata in the framework of equilibrium and nonequilibrium statistical physics and in application-oriented problems. The basic ideas and concepts are illustrated on simple examples so as to highlight the method. A selected bibliography is provided in order to guide the reader through this expanding field.

Several relevant domains of application have been mentioned only through references to the bibliography, or are treated superficially. This is not because we feel these topics are less important but, rather, because a somewhat subjective selection was necessary according to the scope of the book. Nevertheless, we think that the topics we have covered are significant enough to give a fair idea of how the cellular automata technique may be applied to other systems.

This book is written for researchers and students working in statistical physics, solid state physics, chemical physics and computer science, and anyone interested in modeling complex systems. A glossary is included to give a definition of several technical terms that are frequently used throughout the text. At the end of the first six chapters, a selection of problems is given. These problems will help the reader to become familiar with the concepts introduced in the corresponding chapter, or will introduce him to new topics that have not been covered in the text. Some problems are rather easy, although they usually require some programming

effort, but other problems are more involved and will demand significant time to complete.

Most of the cellular automata simulations and results presented in this book have been produced on the 8k Connection Machine CM-200 of the University of Geneva. Others have been computed on an IBM SP2 parallel computer, also installed at the University of Geneva. Although a parallel supercomputer is quite useful when considering large scale simulations, common workstations and even modern personal computers are well adapted to perform cellular automata computations, except for on-line display which is always very desirable. Dedicated hardware is also available but, usually, less flexible than a general purpose machine.

Despite our effort, several errors and misprints are still likely to be present. Please report them to us\* (as well as any comment or suggestion).

We would like to thank all the people who have made this book possible and, in particular Claude Godrèche who gave us the opportunity to write it. Special thanks go to Pascal Luthi and Alexandre Masselot who made several original and important simulations which are presented in this book. Other people have played a direct or indirect role in the preparation of the manuscript. Among them, we thank Rodolphe Chatagny, Stephen Cornell, Laurent Frachebourg, Alan McKane, Zoltan Racz and Pierre-Antoine Rey.

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