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**FREE CHOICE PETRI NETS**

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# Preface

Free-choice Petri nets have been around for more than twenty years, and are a successful branch of net theory. Nearly all the introductory texts on Petri nets devote some pages to them. This book is intended for those who wish to go further. It brings together the classical theorems of free-choice theory obtained by Commoner and Hack in the seventies, and a selection of new results, like the Rank Theorem, which were so far scattered among papers, reports and theses, some of them difficult to access.

Much of the recent research which found its way into the book was funded by the ESPRIT II BRA Action DEMON, and the ESPRIT III Working Group CALIBAN.

The book is self-contained, in the sense that no previous knowledge of Petri nets is required. We assume that the reader is familiar with naïve set theory and with some elementary notions of graph theory (e.g. path, circuit, strong connectedness) and linear algebra (e.g. linear independence, rank of a matrix). One result of Chapter 4 requires some knowledge of the theory of NP-completeness.

The book can be the subject of an undergraduate course of one semester if the proofs of the most difficult theorems are omitted. If they are included, we suggest the course be restricted to Chapters 1 through 5, which contain most of the classical results on S- and T-systems and free-choice Petri nets. A postgraduate course could cover the whole book.

All chapters are accompanied by a list of exercises. Difficult exercises are marked with asterisks.

We would like to express our warmest thanks to the many people who have helped us to write the book. Eike Best encouraged us, offered advice and criticism, and was a good friend. Raymond Devillers flooded us with helpful comments, and corrected many mistakes. Glenn Bruns, Ekkart Kindler, Maciej Koutny, Agathe Merceron, Alan Paxton, Anette Renner, P.S. Thiagarajan and Walter Vogler made useful suggestions.

This book was written while the first-mentioned author was at the Technical University of Munich and the Humboldt University of Berlin, and the second author was at the University of Hildesheim and the University of Edinburgh. We thank our colleagues Eike Best, Julian Bradfield, Glenn Bruns, Hans-Günther Linde-Göers, Wolfgang Reisig, Colin Stirling and Rolf Walter for creating a very enjoyable working atmosphere.

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Finally, many thanks to Jaco de Bakker for suggesting that we publish the book as a Cambridge Tract in Theoretical Computer Science, and to David Tranah and Roger Astley, from Cambridge University Press, for their help in the production.