Contents

Preface xvii
Acknowledgements xxi

Part 1 The surface vorticity method for inviscid ideal fluid flow

Chapter 1 The basis of surface singularity modelling
1.1 Introduction 3
1.2 The source panel or Douglas–Neumann method 5
1.3 The surface vorticity or Martensen method 8
1.4 Physical significance of the surface vorticity model 10
1.5 Vorticity convection and production in a shear layer 14
1.6 Surface vorticity model for plane two-dimensional flow 17
1.6.1 Self-induced velocity of a surface vorticity element due to curvature 22
1.6.2 Computational scheme for surface vorticity analysis 25
1.7 Comparison of surface vorticity analysis with Douglas–Neumann scheme 27
1.8 Calculation of streamlines and velocities within the flow field 32
1.9 Flows with symmetry about the x axis 35
1.10 Generalised equations for surface vorticity modelling in curvilinear coordinates 39

Chapter 2 Lifting bodies, two-dimensional aerofoils and cascades
2.1 Introduction 44
2.2 Circular cylinder with bound circulation – Flettner rotor 45
2.3 Flow past a thin ellipse 49
2.3.1 Reconsideration of non-lifting ellipse 50
## Contents

2.3.2 Use of sub-elements ........................................ 55
2.3.3 Back diagonal correction .................................... 56
2.4 Thin ellipse as a lifting aerofoil ............................... 59
  2.4.1 Kutta condition, method 1 – prescribed bound circulation $\Gamma$ ................................. 59
  2.4.2 Kutta condition, method 2 – trailing edge unloading ............. 60
  2.4.3 Method 3 – Wilkinson’s Kutta condition ...................... 63
2.5 Aerofoils ................................................................ 66
  2.5.1 Unit solutions ..................................................... 67
  2.5.2 Specification of aerofoil geometry ......................... 68
  2.5.3 Comparison with Joukowski aerofoils ....................... 69
2.6 Turbomachine linear cascades .................................... 75
  2.6.1 Cascade coupling coefficients .............................. 75
  2.6.2 Cascade dynamics and parameters ......................... 79
  2.6.3 Program Bladerow.pas and sample calculations ............ 81
2.7 Multiple bodies and aerofoils with slots and flaps ............ 92
  2.7.1 Internal circulation correction for bodies in close proximity .... 94
  2.7.2 Assemblies of lifting aerofoils ............................... 96

### Chapter 3: Mixed-flow and radial cascades

3.1 Introduction ................................................................ 99
3.2 Transformation of a mixed-flow cascade into a straight cascade ............... 102
  3.2.1 Axial and radial blade rows ................................. 105
3.3 Sample calculation for an outflow radial diffuser vane cascade ............... 108
  3.3.1 Surface pressure distribution ................................ 109
  3.3.2 Inlet and outlet angles ........................................ 109
3.4 Rotor/stator interference in centrifugal compressors ......................... 110
3.5 Mixed-flow and radial rotor blade rows ................................ 112
  3.5.1 Transformation of the ‘relative eddy’ to the straight cascade plane ...................... 114
  3.5.2 Correction for irrotationality of the inner blade profile region ......................... 117
  3.5.3 Influence of meridional streamline thickness (AVR) .................. 122
  3.5.4 Unit solutions for mixed-flow cascades and prediction of flow angles ........ 124
## Contents

3.5.5 More precise method for removal of profile internal vorticity 126  
3.6 Comparison with exact solutions for radial cascades by conformal transformation 134  
3.6.1 Flow analysis of the transformation 137  
3.6.2 Sample solutions 139  
3.6.3 Comparisons with experimental test 141  
3.7 Effects of AVR in compressor cascades 143  

### Chapter 4 Bodies of revolution, ducts and annuli

4.1 Introduction 146  
4.2 The axisymmetric surface vorticity model 147  
4.2.1 Evaluation of complete elliptic integrals. Use of look-up tables 150  
4.2.2 Numerical representation of the integral equation for axisymmetric flow 153  
4.2.3 Self-induced velocity of a ring vorticity element 154  
4.3 Flow past a body of revolution 157  
4.3.1 Flow past a sphere 158  
4.3.2 Flow past a body of revolution 159  
4.4 Annular aerofoils or engine cowls 160  
4.5 The semi-infinite vortex cylinder 167  
4.6 Flow through a contraction 170  
4.7 Flow through an annulus 174  
4.8 Source panel solutions for plane two-dimensional and axisymmetric flows 176  
4.8.1 Source panel modelling of lifting aerofoils 177  
4.9 Source panel method for axisymmetric flows 183  
4.9.1 Source panel method for a body of revolution 185  
4.9.2 Source panel method for an annular aerofoil or engine cowl 187  

### Chapter 5 Ducted propellers and fans

5.1 Introduction 191  
5.2 The sucked-duct or pipe-flow engine intake facility 192  
5.3 Free vortex ducted propeller 198  
5.4 Non-free vortex ducted propeller – lifting surface theory 204
Table of Contents

Contents

5.4.1 Matching the helix angle
5.4.2 Propeller loading and vortex shedding
5.5 Vorticity production in axisymmetric meridional flows
5.5.1 Streamwise and smoke-ring vorticity
5.6. Non-free vortex actuator disc model for axial turbomachines and ducted propellers
5.7 Models to deal with the induced effects of distributed ring vorticity in axisymmetric meridional flows
5.7.1 Numerical representation of rectangular and circular ring vortex elements
5.7.2 Check on self-propagation of a smoke-ring vortex
5.7.3 Self-propagation of a sheet ring vortex element
5.7.4 Induced velocities close to a rectangular ring vortex
5.7.5 Flow of a shear layer past a body of revolution

Chapter 6 Three-dimensional and meridional flows in turbomachines

6.1 Introduction
6.2 Three-dimensional flow past lifting bodies
6.3 Three-dimensional flow past annular aerofoils and engine cowl
6.3.1 Numerical scheme using circumferential series expansions
6.4 Sweep and dihedral in turbomachine blade rows
6.4.1 Swept aerofoils and cascades of infinite aspect ratio
6.4.2 Swept cascade of finite aspect ratio
6.4.3 Analysis with constant spanwise loading
6.4.4 Analysis with variable spanwise loading in three-dimensional flow
6.5 Local blade rake and lean and blade forces
6.5.1 Local blade forces
6.6 Equations of meridional flow for bladed regions
6.7 Axisymmetric meridional flows in mixed-flow turbomachines
6.7.1 Flow through an actuator disc in a cylindrical annulus
6.7.2 Meridional flow through a mixed-flow turbomachine

x
# Table of Contents

## Part 2 Free shear layers, vortex dynamics and vortex cloud analysis

### Chapter 7 Free vorticity shear layers and inverse methods

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 Introduction</td>
<td>281</td>
</tr>
<tr>
<td>7.2 The free-streamline model</td>
<td>282</td>
</tr>
<tr>
<td>7.3 Free jets</td>
<td>289</td>
</tr>
<tr>
<td>7.4 Inverse aerofoil design</td>
<td>291</td>
</tr>
<tr>
<td>7.4.1 Basis of inverse surface vorticity design method for aerofoils and cascades</td>
<td>292</td>
</tr>
<tr>
<td>7.4.2 Further refinements</td>
<td>295</td>
</tr>
<tr>
<td>7.4.3 Angular constraints on leading and trailing edge elements</td>
<td>297</td>
</tr>
<tr>
<td>7.4.4 Aerofoil inverse design</td>
<td>301</td>
</tr>
<tr>
<td>7.5 Inverse design of cascades and slotted cascades</td>
<td>303</td>
</tr>
<tr>
<td>7.5.1 True inverse design method for cascades</td>
<td>304</td>
</tr>
<tr>
<td>7.5.2 Inverse cascade design by iterative use of the direct method</td>
<td>306</td>
</tr>
<tr>
<td>7.6 Inverse design of axisymmetric bodies</td>
<td>309</td>
</tr>
</tbody>
</table>

### Chapter 8 Vortex dynamics in inviscid flows

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1 Introduction</td>
<td>316</td>
</tr>
<tr>
<td>8.2 Vortex convection</td>
<td>319</td>
</tr>
<tr>
<td>8.2.1 Convection of a vortex pair</td>
<td>320</td>
</tr>
<tr>
<td>8.3 Convection and stability of vortex sheets</td>
<td>326</td>
</tr>
<tr>
<td>8.3.1 Roll-up of a free-ended vortex sheet</td>
<td>327</td>
</tr>
<tr>
<td>8.3.2 Kelvin–Helmholtz instability of a vortex sheet</td>
<td>329</td>
</tr>
<tr>
<td>8.4 Convective interaction of free vortices with solid bodies</td>
<td>337</td>
</tr>
<tr>
<td>8.4.1 Potential flow past a cylinder due to a nearby vortex</td>
<td>339</td>
</tr>
<tr>
<td>8.4.2 Convection of a free vortex near a circle or an ellipse</td>
<td>347</td>
</tr>
<tr>
<td>8.4.3 Convection of vortices in very close proximity to a body</td>
<td>351</td>
</tr>
<tr>
<td>8.5 Simple vortex cloud modelling for two-dimensional bodies with prescribed separation points</td>
<td>354</td>
</tr>
<tr>
<td>8.5.1 Vorticity shedding from a sharp edge separation point</td>
<td>355</td>
</tr>
</tbody>
</table>
Contents

8.5.2 Simple vortex dynamics scheme for simulation of wake development 356
8.5.3 Vorticity shedding from a smooth surfaced bluff body 358

Chapter 9 Simulation of viscous diffusion in discrete vortex modelling

9.1 Introduction 364
9.2 Diffusion of a point vortex in two-dimensional flow 366
  9.2.1 Random number generation 370
  9.2.2 Radial distribution of vorticity $\omega(r)$ 371
  9.2.3 Diffusion over a series of time steps 372
9.3 Diffusion of a vortex sheet 374
9.4 Boundary layers by discrete vortex modelling 377
  9.4.1 Vorticity creation and shedding (Step 2) 378
  9.4.2 Viscous diffusion (Step 3) 381
  9.4.3 Vortex convection (Step 5) 381
  9.4.4 Vortices in close proximity (Step 6) 382
  9.4.5 Calculation of velocity profile (Step 9) 384
  9.4.6 Selection of element size and time step 387
  9.4.7 Some considerations for high Reynolds number flows 388

Chapter 10 Vortex cloud modelling by the boundary integral method

10.1 Introduction 393
10.2 Vortex cloud modelling with prescribed separation points 395
  10.2.1 Introduction of reduced circulation 398
  10.2.2 Time growth of the vortex core 400
10.3 Application of fixed separation point analysis to a lifting aerofoil 400
10.4 Full vortex cloud modelling by the surface vorticity boundary integral method 404
  10.4.1 Potential flow analysis in the presence of a vortex cloud 407
  10.4.2 Vortex shedding from body surface 409
  10.4.3 Convection schemes. Method 1, strict Eulerian convection. Method 2, simplified Eulerian convection 410

xii
# Contents

10.5 Calculation of surface pressure distribution and body forces 413
  10.5.1 Pressure distribution – full vortex cloud model 414
  10.5.2 Pressure distribution – vortex cloud modelling with fixed separation points 415
  10.5.3 Pressure and force fluctuations due to numerical noise 417
  10.5.4 Data reduction of unsteady pressures and forces for bluff body flows 420

10.6 Application of vortex cloud analysis to flow past a circular cylinder 422

## Chapter 11 Further development and applications of vortex cloud modelling to lifting bodies and cascades

11.1 Introduction 428

11.2 Flow past a lifting aerofoil by vortex cloud analysis 428

11.3 Alternative vortex cloud modelling techniques by Spalart & Leonard 434
  11.3.1 NACA 0012 aerofoil in dynamic stall 437

11.4 Mixed vortex cloud and potential flow modelling 441
  11.4.1 Lifting aerofoil by the hybrid potential flow/vortex cloud method 443
  11.4.2 Aerofoil with airbrake spoiler by the hybrid potential flow/vortex cloud method 445
  11.4.3 Aerofoils with moving spoilers 450

11.5 Application of vortex cloud modelling to turbomachinery blade rows 451
  11.5.1 Vortex cloud analysis for periodic flow through linear cascades 451
  11.5.2 Rotating stall in compressors 459

11.6 Flow induced acoustic resonance for a bluff body in a duct 463

11.7 Potential for future development of vortex cloud analysis 467

## Chapter 12 Use of grid systems in vortex dynamics and meridional flows

12.1 Introduction 469

12.2 Cell-to-cell interaction method for speeding convective calculations 470
## Contents

12.3 Cloud-in-cell (CIC) method ............... 476
  12.3.1 Vortex re-distribution to cell corners .... 477
  12.3.2 Convection with grid distribution of vorticity 478
12.4 Cellular modelling of viscous boundary layers .... 483
  12.4.1 Numerical solution for a diffusing vortex sheet 483
  12.4.2 Diffusion coupling coefficient matrices .... 487
  12.4.3 Boundary layer simulation by the cell method 489
  12.4.4 The Blasius boundary layer .......... 492
  12.4.5 Similarity boundary layers ........ 493
  12.4.6 Selection of grid data for cellular boundary layer computational schemes 497

### Appendix Computer Programs

<table>
<thead>
<tr>
<th>Program</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Flow past a circular cylinder including surface velocity, comparison with exact solution.</td>
<td>499</td>
</tr>
<tr>
<td>1.2</td>
<td>Flow past a circular cylinder by the Douglas-Neumann source panel method.</td>
<td>502</td>
</tr>
<tr>
<td>1.3</td>
<td>Flow past an ellipse, including surface velocity comparison with exact solution and streamline pattern.</td>
<td>505</td>
</tr>
<tr>
<td>2.1</td>
<td>Calculation of flow past a cylinder with bound circulation.</td>
<td>508</td>
</tr>
<tr>
<td>2.2</td>
<td>Flow past an ellipse with prescribed bound circulation.</td>
<td>510</td>
</tr>
<tr>
<td>2.3</td>
<td>Potential flow past an aerofoil.</td>
<td>512</td>
</tr>
<tr>
<td>2.4</td>
<td>Potential flow through a turbomachine cascade.</td>
<td>515</td>
</tr>
<tr>
<td>4.1</td>
<td>Calculation of complete elliptic integrals of the first and second kinds.</td>
<td>518</td>
</tr>
<tr>
<td>4.2</td>
<td>Flow past a body of revolution.</td>
<td>521</td>
</tr>
<tr>
<td>4.3</td>
<td>Flow past an axisymmetric cowl or duct.</td>
<td>524</td>
</tr>
<tr>
<td>4.4</td>
<td>Flow through a contraction or diffuser.</td>
<td>526</td>
</tr>
<tr>
<td>4.5</td>
<td>Flow past a body of revolution in a uniform stream.</td>
<td>530</td>
</tr>
<tr>
<td>5.1</td>
<td>Potential flow through an engine intake sucked from downstream by a cylindrical duct and located in a uniform stream (Pipe flow test rig).</td>
<td>533</td>
</tr>
<tr>
<td>5.2</td>
<td>Potential flow through a free-vortex ducted propeller in a uniform stream.</td>
<td>537</td>
</tr>
</tbody>
</table>
## Contents

Program 8.1 Program for experimentation with convection of vortex clouds. 540
Program 9.1 Program to generate a set of random numbers and sort them. 543
Program 9.2 Diffusion of a point vortex. 544

**Bibliography** 547

**Index** 560