OCEAN WAVES AND OSCILLATING SYSTEMS

LINEAR INTERACTIONS INCLUDING WAVE-ENERGY EXTRACTION

JOHANNES FALNES
Department of Physics
Norwegian University of Science and Technology NTNU
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Preface

This book is intended to provide a thorough consideration of the interaction between waves and oscillating systems (immersed bodies and “oscillating water columns”) under conditions where amplitudes are sufficiently small that linear theory is applicable. In practice, this small-wave assumption is reasonably valid for most of the time, during which, for example, a wave-energy converter is generating most of its income. During the rather rare extreme-wave situations, however, non-linear effects may be significant, and such situations influence design loads, and hence the costs, for ships and other installations deployed at sea. This matter is treated in several other books.

The present book is mainly based on lecture notes from a postgraduate university course on water waves and extraction of energy from ocean waves, which I have taught many times since 1979. For the purposes of this book, I have selected those parts of the subject which have more general interest, rather than those parts of my course which pertain to wave-power conversion in particular. I hope that the book is thus of interest to a much wider readership than just the wave-energy community.

Except in 1983, my course has been taught every second year, mainly for doctorate students at the university in Trondheim, but other interested students have also attended. Moreover, a similar two-week course was given in 1986 with participants from Norwegian industry. Another two-week course, with international participation, was held at the Chalmers University of Technology in Gothenburg, Sweden, in 1998.

In February 1980, the lecture notes were issued in a bound volume entitled *Hydrodynamisk teori for bølgjekraftverk* (“Hydrodynamic theory for wave power plants”) by L. C. Iversen and me. One hundred copies were published by the University of Trondheim, Division of Experimental Physics. Later I revised the lecture notes, while translating them into English. In 1993 this process resulted in a two-volume work entitled *Theory for extraction of ocean-wave energy*.

I wish to thank the course participant Knut Bønke for his inspiring encouragement to have the lecture notes typed in 1979 and issued in a bound volume, and
for his continued encouragement over many years to write a textbook based on the notes. Moreover, I would like to thank Jørgen Hals, a course participant from 1997, for working out the subject index of the present book. I am also in debt to my other students for their comments and proof-reading. In this connection I wish to mention, in particular, the following graduate students (the years they completed their doctorate degrees are given): L. C. Iversen (1980), Å. Kyllingstad (1982), O. Malmo (1984), G. Oltedal (1985), A. Brendmo (1995) and H. Eidsmoen (1996). Also my collaborator over many years, P. M. Lillebekken, who attended the course in 1981, has made many valuable comments.

Most of all, I am in debt to my late colleague Kjell Budal (1933–1989), whose initiative inspired my interest in wave-power utilisation at an early stage. During the oil crisis at the end of 1973, we started a new research project aimed at utilising ocean-wave energy. At that time we did not have a research background in hydrodynamics, but Budal had carried out research in acoustics, developing a particular microphone, whereas I had studied waves in electromagnetics and plasma physics. During 1972–4 we jointly authored a (Norwegian) textbook Bølgjelære (“Wave Science”) for the second-year undergraduate students in physics. This is an interdisciplinary text on waves, with particular emphasis on acoustics and optics.

With this background our approach and attitude towards hydrodynamic waves have perhaps been more interdisciplinary than traditional. In my view, this has influenced our way of thinking and stimulated our contributions to the science of hydrodynamics. This background is also reflected in the present book, notably in Chapter 3, where interaction between oscillations and waves is considered in general; water waves, in particular, are treated in subsequent chapters (Chapters 4–7).

I am also grateful to Elsevier Science for permission to reuse, in Sections 4.9, 5.5 and 7.2, parts of my own contributions to papers in Applied Ocean Research. Moreover, I wish to thank Professor J. N. Newman and Dr. Alain Clément for suggesting the use of computer codes WAMIT and AQUADYN, which have been used to compute the numerical results presented in Subsections 5.2.4 and 5.7.3, respectively. I am also grateful to Dr. Stephen Barstow for permission to use Problem 4.4, which he formulated.

Finally, I wish to thank my wife, Dagny Elisabeth, for continuous support during the many years I have worked on this book. Our oldest son, Magne, took the photographs used in composition of the front cover of the book.

4 September 2001   Johannes Falnes