

### **Atmospheric Dynamics**

In his book, John Green presents a unique personal insight into the fundamentals of fluid mechanics and atmospheric dynamics. Generations of students have benefited from his lectures, and this book, many years in the making, is the result of his wide teaching and research experience.

The theory of fluid flow has developed to such an extent that very complex mathematics and models are currently used to describe it, but many of the fundamental results follow from relatively simple considerations: these classic principles are derived here in a novel, distinctive, and at times even idiosyncratic, way. In order to resist blindly following the ever more complex computer simulations of atmospheric dynamics, the author advocates that students need to keep in touch with these fundamental derivations and thought processes. These self-same lower level derivations can then often be useful in developing strategies for creation of more complex systems.

John Green's book is an introduction to fluid mechanics in the atmosphere for students and researchers that are already familiar with the subject, but who wish to extend their knowledge and philosophy beyond the currently popular development of conventional undergraduate instruction.

JOHN GREEN graduated in mathematics from Imperial College, London, where he also received a PhD from the Meteorology Department, and went on to become Reader in Meteorology. He is now a lecturer in fluid dynamics and mathematical modelling of natural processes at the School of Environmental Sciences, University of East Anglia. He has published extensively, mainly in the *Royal Meteorological Society Quarterly Journal* and the popular journal *Weather*, and has been awarded the Buchan prize of the Royal Meteorological Society for work on baroclinic instability and thunderstorms.



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John Green





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