A COMPOSITIONAL APPROACH TO PERFORMANCE MODELLING

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JANE HILLSTON
University of Edinburgh

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Table of Notation

$\mathcal{C}$	set of possible components
$\mathcal{A}$	set of possible action types
$\mathcal{A}ct$	set of possible activities
$\mathcal{A}(C)$	set of current action types of component C
$\mathcal{A}ct(C)$	multiset of current activities of C
$\mathcal{A}ct(C_i \mid C_j)$	multiset of current activities of C_i with derivative C_j
$\vec{\mathcal{A}}(C)$	complete action type set of C
τ	unknown action type
$\top$	unspecified activity rate
w_i	weight of a passive activity
$r_\alpha(C)$	apparent rate of action type α in component C
$ds(C)$	derivative set
$\mathcal{D}(\mathcal{C})$	derivation graph
Sys_P	the system component represented by P
$\mathcal{C}/\cong$	set of equivalence classes induced by $\cong$ on $\mathcal{C}$
$\mathcal{C}/\mathcal{R}$	set of equivalence classes induced by $\mathcal{R}$ on $\mathcal{C}$
$(\alpha, r).P$	activity prefix
$P + Q$	component choice
$P \underset{L}{\bowtie} Q$	cooperation between P and Q on the set of action types L
$P \parallel Q$	parallel composition of P and Q, cooperation on $\emptyset$
P/L	activities of P with types in L appear as unknown type
$E\{P/X\}$	every occurrence of X in E is replaced by P
$\tilde{X}, \tilde{P}$	indexed sets of variables and components respectively
$A \stackrel{\text{def}}{=} P$	defining equation for the constant A
$Id_{\mathcal{C}}$	identity function on components
$P \equiv Q$	syntactic equivalence
$P = Q$	P is isomorphic to Q
$C \leq P$	C is a compact form of P
$P \approx Q$	P is weakly isomorphic to Q
$P \sim Q$	P is strongly bisimilar to Q
$P \cong Q$	P is strongly equivalent to Q
$\overline{P}$	the compact form of component P
$\widehat{P}$	the lumped component of P
$V_{(\tau,r)}(C)$	visible (τ, r)-derivative of C
$\mathcal{A}ct_{\cong}(T)$	lumped activity set
$ds(\mathcal{S})/\cong$	lumped derivative set
$\mathcal{D}_{\cong}(\mathcal{S})$	lumped derivation graph
$\vec{\mathcal{A}ct}_{\cong}(\mathcal{S})$	complete lumped activity set

$\mathbb{R}^+$	set of activity rates, $\{x \mid x > 0; x \in \mathbb{R}\} \cup \{\top\}$
$\mathbb{N}$	natural numbers, $\{1, 2, 3, \ldots\}$
$F_a(t)$	probability distribution function associated with a
$f_a(t)$	probability density function associated with a
X_i	state in a Markov process
$\mathbf{Q}$	infinitesimal generator matrix
q_{ij}	transition rate between state X_i and X_j
$\Pi(\cdot)$	steady state probability distribution
$\Pi_j(\cdot)$	conditional steady state probability distribution
$X_{[j]}$	aggregated state in a Markov process
x_n	state in a generalised semi-Markov process (GSMP)
s	active element in a GSMP
$p(x_i, s, x_j)$	transition probability in a GSMP
$q(C)$	exit rate from component C
$q(C_i, C_j)$	transition rate from C_i to C_j
$q(C_i, C_j, \alpha)$	conditional transition rate via activities of type α
$q(C, \alpha)$	conditional exit rate via activities of type α
$q[C, S]$	total transition rate from C to the set of derivatives S
$q[C, S, \alpha]$	total conditional transition rate via activities of type α
$p(C, a), p(C, \alpha)$	conditional probabilities that C completes a, or an activity of type α
$p(C_i, C_j)$	transition probability from C_i to C_j
$p[C, S]$	total transition probability from C to the set of derivatives S
$p[C, S, \alpha]$	total conditional transition probability via activities of type α
ρ_i	reward associated with derivative C_i
R	total reward
$\uplus$	multiset union
$\{\mid \ldots \mid\}$	multiset delimiters

Preface

This book is, in essence, the dissertation I submitted to the University of Edinburgh in early January 1994. My examiners, Peter Harrison of the Imperial College, and Stuart Anderson of the University of Edinburgh, suggested some corrections and revisions. Apart from those changes, most chapters remain unaltered except for minor corrections and reformatting. The exceptions are the first and final chapter.

Since the final chapter discusses several possible directions for future work, it is now supplemented with a section which reviews the progress which has been made in each of these directions since January 1994. There are now many more people interested in stochastic process algebras and their application to performance modelling. Moreover, since these researchers have backgrounds and motivations different from my own some of the most interesting new developments are outside the areas identified in the original conclusions of the thesis. Therefore the book concludes with a brief overview of the current status of the field which includes many recent references. This change to the structure of the book is reflected in the summary given in Chapter 1. No other chapters of the thesis have been updated to reflect more recent developments. A modified version of Chapter 8 appeared in the proceedings of the 2nd International Workshop on Numerical Solution of Markov Chains, January 1995.

I would like to thank my supervisor, Rob Pooley, for introducing me to performance modelling and giving me the job which brought me to Edinburgh initially. Many colleagues on the IMSE project provided stimulating discussions which influenced this work. My second supervisor, Julian Bradfield, provided support and advice in large quantities for which I am very grateful. Many other people also influenced this work through helpful comments, discussions and encouragement; they include Graham Birtwistle, Stephen Gilmore, Peter King, James McKinna, Faron Moller, Michael Rettelbach, Ben Strulo and Nico van Dijk. Stephen also provided the tools which made constructing and solving the large models in Chapter 4 possible.

I would never have finished this thesis without the support, encouragement and distractions provided in appropriate proportions by my parents and many friends, during the four and a half years it took to complete.

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Jane Hillston
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