CHAPTER 1

GILAH C. LEDER, ERKKI PEHKONEN, AND GÜNTER TÖRNER

SETTING THE SCENE

Abstract. Although the importance of engaging students both cognitively and affectively when they learn mathematics is now widely recognized, the place of beliefs in the teaching and learning of mathematics is not well researched. After a brief introduction in which some contextual issues are raised, the contents of the contributions that follow – each with a clear focus on beliefs in mathematics education - are described in this introductory chapter.

1. INTRODUCTION

Mathematics is widely recognized not only as a core component of the curriculum but also as a critical filter to many educational and career opportunities. Yet in recent years much concern has been expressed about students' reluctance to continue with the study of mathematics well beyond the compulsory years, a trend often described emotionally as the drift away from mathematics and from the physical sciences. The passage below, taken from a national Australian document, is representative of the sentiments voiced.

> Mathematics and science have a fundamental contribution to make both to understanding the world and to changing the world, particularly in the context of change and economic adjustment. The decline in interest in mathematics ... needs to be arrested. This is an urgent and complex matter related not only to education but to other issues. (Department of Employment, Education and Training, 1989, p. 14)

Lip service is certainly paid to the importance of engaging students of mathematics - of all ages - affectively as well as cognitively (see e.g., Jensen, Niss, & Wedege, 1998; the National Council of Teachers of Mathematics [NCTM], 2000). But (how) will that make a difference? What do we know about the interplay between beliefs and behaviors? Only a decade ago, McLeod (1992) noted: "(a)lthough affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field" (p. 575). Schoenfeld (1992) similarly argued that there was "a fairly extensive literature on" student beliefs, "a moderate but growing literature" about teacher beliefs, and as yet relatively little exploration of "general societal beliefs about doing mathematics" (p. 358). Research on mathematics and gender has been singled out as one area where "some aspects of beliefs about self have been researched quite thoroughly" (McLeod, 1992, p.580). A review of this work is beyond the scope of this

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introductory chapter, but can be found in, for example, Fennema and Hart (1994), Forgasz and Leder (2001a, 2001b), Leder (1992), and Leder, Forgasz, and Solar (1996). At the same time, there are areas in research on beliefs, for example teacher change, where extensive research has been carried out over some two decades, but no consistent pattern has yet been identified for facilitating teacher change. Research reports about intervention programs commonly conclude that some teachers have changed, but others have not (e.g., Senger, 1999; Borko, Davinroy, Bliem, & Cumbo, 2000; Wood, 2001; Hart, this volume; Wilson & Cooney, this volume).

1.1. Genesis of the Book.

Each of the contributions in this book has a clear focus on beliefs about mathematics. Aspects of beliefs about mathematics, its teaching and learning, are examined broadly and from a variety of perspectives. Collectively, the contributions reflect the diverse approaches used in the conceptualization and study of beliefs. The genesis of the book can be traced to a specialist international meeting about mathematics-related beliefs held in the unique setting of the *Mathematisches Forschungsinstitut Oberwolfach* in November 1999 (Pehkonen & Törner, 1999). The majority of the authors of the book were invited participants to the Institute, and their chapters are based on the presentations they gave at the meeting. Thus the core ideas contained in the chapters, and the diverse ways in which beliefs in mathematics education can be explored, were discussed beforehand, during an international high-level forum. The presentations and discussions were subsequently formalized, elaborated into full papers, and subjected to extensive peer-review.

2. ABOUT BELIEFS

A careful reading of the early psychological literature reveals that beliefs and belief systems began to be explored in the beginning of this century, particularly by social psychologists (Thompson, 1992). However, as the more behavioral perspectives of learning attracted increased attention, research endeavors turned to the more readily observed parts of human behavior, with a consequent loss of interest in beliefs. With new developments in cognitive science in the 1970s, attention to beliefs and belief systems re-emerged (Abelson, 1979). Although work on beliefs can be found in areas as diverse as political science, history, psychology, sociology, and anthropology, it is "especially social psychologists, who have devoted much effort into studying the acquisition and change of beliefs, their structure, their contents, and their effects mainly on individuals' affect and behavior" (Bar-Tal, 1990).

Given the variety of perspectives and disciplines within which beliefs have been studied, it is not surprising that the field abounds with subtly different definitions and classifications of beliefs. A detailed overview of this work is again beyond the scope of this chapter, but can be found in later chapters in this volume (Furinghetti & Pehkonen; Op't Eynde, de Corte, & Verschaffel). Here it is convenient to reproduce some key features highlighted by Bar-Tal (1990):

- The study of beliefs can be classified into four areas: "(a) acquisition and change of beliefs, (b) structure of beliefs, (c) effects of beliefs, and (d) content of beliefs[1]" (p. 12)
- "Beliefs have been viewed by social psychologists as units of cognition. They constitute the totality of an individual's knowledge, including what people consider as facts, opinions, hypotheses, as well as faith" (p. 12). Descriptions such as this highlight the difficulty often shown in distinguishing between beliefs and knowledge[2].
- "Beliefs [can] be differentiated on the basis in which they are formed: (a) Descriptive beliefs are formed on the basis of direct experience.... (b) Inferential beliefs ... are based on rules of logic that allow inferences.... (c) Informational beliefs are formed on the basis of information provided by outside sources..." (based on the work of Bem (1970) and Fishbein & Ajzen (1975), as summarized by Bar-Tal, p. 12).
- "Psychologists have suggested different features and dimensions to characterize beliefs.... Krech and Crutchfield (1948) proposed the following seven characteristics to describe beliefs: kind, content, precision, specificity, strength, importance, and verifiability" (p. 15).
- It is useful to focus on four characteristics of beliefs: "Confidence, centrality, interrelationship, and functionality. Confidence differentiates beliefs on the basis of truth attributed to them; centrality characterizes the extent of beliefs in individuals' repertoire...; interrelationship indicates the extent to which the belief is related to other beliefs; functionality differentiates beliefs on the basis of the needs they fulfill" (p. 21).

Almost any one of these characterizations could have been used as an organizational theme for the various contributions in this volume. The grouping we finally selected is one we consider to be particularly constructive for the examination of beliefs about mathematics, its teaching and learning from a variety of different perspectives. We settled on a three clusters: contributions with a major focus on the concept of beliefs in mathematics education; on teachers' beliefs; and on students' beliefs.

3. ABOUT THE BOOK

As already indicated, the book is divided into three main sections, with different yet overlapping themes. The broad international mix of the contributing authors ensures a diversity of perspectives, as well as reference to relevant research beyond that published in English. Such coverage is less likely to be achieved with a culturally more homogeneous group of contributors. Thus the book offers a variety of different perspectives into the concept of beliefs, and into methods of investigating the place of beliefs in the teaching and learning of mathematics. A synthesis/critique chapter which, inter alia, highlights common and diverse themes, concludes each section.

CHAPTER 2

PETER OP 'T EYNDE, ERIK DE CORTE, AND LIEVEN VERSCHAFFEL

FRAMING STUDENTS' MATHEMATICS-RELATED BELIEFS

A Quest For Conceptual Clarity And A Comprehensive Categorization

Abstract. Despite the general agreement among researchers today that students' beliefs have an important influence on mathematical problem solving there is still a lack of clarity from a conceptual viewpoint. In this chapter we present a literature review of available categorizations or models of students' beliefs related to mathematics learning and problem solving. These reveal that although they all cover a broad spectrum of relevant beliefs, there appears to be no consensus on the structure and the content of the relevant categories of students' beliefs. A philosophical and psychological analysis of the nature and the structure of beliefs enables us to come to a deeper understanding of the development and the functioning of students' beliefs and to clarify the relation between beliefs and knowledge. The insights developed through this analysis result in an elaborated and concrete definition of students' mathematics-related beliefs and allow us to develop a theoretical framework that coherently integrates the major components of prevalent models of students' beliefs. We differentiate between students' beliefs about mathematics education, students' beliefs about the self, and students' beliefs about the social context, i.e., the class context.

1. INTRODUCTION

Recent theories on cognition and learning (e.g., Greeno, Collins, & Resnick, 1996; Salomon & Perkins, 1998) point to the social-historical embeddedness and the constructive nature of thinking and problem solving. According to these theories, each form of knowing and thinking is constituted by the meanings and rules that function in the specific communities in which they are situated (e.g., the scientific community, the class, the group). Acquiring knowledge or learning, therefore, consists of getting acquainted with the concepts and rules that characterize the activities in the different contexts. As such, learning becomes fundamentally a social activity.

From such a perspective, learning is primarily defined as a form of engagement that implies the active use of certain cognitive and metacognitive knowledge and strategies, but cannot be reduced to it. Indeed, more and more researchers (e.g., Bereiter & Scardamalia, 1993) are convinced that referring only to cognitive and metacognitive factors does not capture the heart of learning. Several studies (e.g.,

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Connell & Wellborn, 1990; Schiefele & Csikszentmihalyi, 1995) point to the key role conative and affective factors play as constituting elements of the learning process, as well as and in close interaction with (meta)cognitive factors. Motivation and volition (i.e., the conative factors) are no longer seen as just the fuel or the engine of the learning process, but are perceived as fundamentally determining the quality of learning. In a similar way, self-confidence and positive emotions (affective factors) are no longer considered as just positive side effects of learning, but become important constitutent elements of learning and problem solving.

Recent developments in the field of research on mathematical problem solving tend to illustrate this change in perspective. Studies on students' beliefs about mathematics (e.g., Garofalo, 1989; Kouba & McDonald, 1986; Schoenfeld, 1985a) and on their motivational beliefs (e.g., Pintrich & Schrauben, 1992; Seegers & Boekaerts, 1993), as well as research on the influence of emotions (Cobb, Yackel, & Wood, 1989; DeBellis, 1996) and on other affective factors such as "students' perceived confidence" (Vermeer, 1997) aim at unraveling the role of conative and affective factors in mathematical problem solving. On a conceptual level researchers try to capture the interrelated influence of (meta)cognitive, conative and affective factors on mathematical learning and problem solving in line with the notion of a "mathematical disposition". Such a disposition refers to the integrated mastery of five categories of aptitude (De Corte, Verschaffel, & Op 't Eynde, 2000):

1. A well-organized and flexibly accessible knowledge base involving the facts, symbols, algorithms, concepts, and rules that constitute the contents of mathematics as a subject-matter field.
2. Heuristics methods, i.e., search strategies for problem solving which do not guarantee, but significantly increase, the probability of finding the correct solution because they induce a systematic approach to the task.
3. Metaknowledge, which involves knowledge about one's cognitive functioning (metacognitive knowledge), on the one hand, and knowledge about one's motivation and emotions that can be used to deliberately improve volitional efficiency (metavolitional knowledge), on the other hand.
4. Mathematics-related beliefs, which include the implicitly and explicitly held subjective conceptions about mathematics education, the self as a mathematician, and the social context, i.e., the class-context.
5. Self-regulatory skills, which embrace skills relating to the self-regulation of one's cognitive processes (metacognitive skills or cognitive self-regulation), on the one hand, and of one's volitional processes (metavolitional skills or volitional self-regulation), on the other hand.

Acquiring such a mathematical disposition is necessary for students to become competent problem solvers, equipped to recognize and tackle mathematical problems in different contexts, and, as such, able to overcome the well-known phenomenon of inert knowledge (see the National Council of Teachers of Mathematics [NCTM], 2000). After all, according to Perkins (1995), the integrated mastery of these different kinds of knowledge (i.e., domain-specific, metacognitive, metavolitional), skills and beliefs results in a sensitivity to the occasions when it is appropriate to use them and an inclination to do so. This sensitivity to situations and contexts, and the inclination to follow through, are both determined by the concepts and beliefs a person holds. A person's beliefs about what counts as a mathematical context and what (s)he finds interesting or important will, as such, have a strong influence on the situations (s)he will be sensitive to, and whether or not (s)he will engage in them. The relevance of beliefs as a component of a mathematical disposition and their impact on mathematics learning is echoed in the *Curriculum and evaluation standards for school mathematics* (NCTM, 1989) in the U.S.A.: "These beliefs exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition" (p. 233).

It is nowadays generally assumed that the impact of students' mathematics-related beliefs on their learning and problem-solving behavior is mediated through cognitive as well as conative and affective processes. First, several researchers have shown how students' "beliefs about mathematics" determine how they choose to approach a problem and which techniques and cognitive strategies will be used (e.g., Garofalo, 1989; Schoenfeld, 1985a). Secondly, others have pointed to the implications of students' "mathematically related beliefs" for their motivational decisions in mathematics learning and problem solving (e.g., Kloosterman, 1996). Finally, it is argued that "students' beliefs related to mathematics education" provide an important part of the context within which emotional responses to mathematics develop (e.g., Isoda & Nakagoshi, 2000; McLeod, 1992). Moreover, this relationship between beliefs and emotions seems to be reciprocal. It is explained that local emotional experiences over time provide the context for the development and strengthening of more stable "global" affect as attitudes and beliefs (Goldin, this volume).

Notwithstanding the general agreement among researchers that students' beliefs have an important influence on mathematical learning and problem solving, there is still a lack of clarity from a conceptual viewpoint (see also Furinghetti & Pehkonen, this volume). The diversity in the terms used to describe relevant beliefs, sometimes referring to the same, at other times to different beliefs, is symptomatic of the actual state of the research domain. Despite, or maybe precisely because of the attention paid to the multiple ways in which different student beliefs influence mathematical learning and problem solving, research on this topic has not yet resulted in a comprehensive model of, or theory on, students' mathematics-related beliefs. As a matter of fact, most of the studies are situated in, respectively, cognitive, motivational or affective research traditions and in many cases they operate in relative isolation from each other. This results in a conceptual and theoretical confusion that is not at all advantageous for the development of a comprehensive theory or model of mathematics-related beliefs and the impact of these beliefs on students' learning and problem solving. We are well aware that the state of the art of the research field does not allow the development of a comprehensive theory at the moment. Nevertheless an attempt will be made in this chapter to clarify the conceptual discussion and to introduce a framework for students' mathematics-related beliefs that might be a further step in that direction.

CHAPTER 3

FULVIA FURINGHETTI AND ERKKI PEHKONEN

RETHINKING CHARACTERIZATIONS OF BELIEFS

Abstract. In this chapter we consider beliefs and the related concepts of conceptions and knowledge. From a review of the literature in different fields we observe that there is a diversity of views and approaches in research on these subjects. We report on a small research project of our own attempting to clarify the understanding of beliefs among specialists in mathematics education. A panel of 18 mathematics educators participated in a panel that we termed "virtual", since the participants communicated with us only by e-mail. We sent nine characterizations related to beliefs, selected from the literature, to the panelists, asked them to express their agreement or disagreement with the statements, and also asked each to give their own characterization of the term. The answers were analyzed, searching for the elements around which the concept of beliefs has developed along the years. We discuss issues on which there was agreement and disagreement and conjecture what lies behind the differences. As a final step we make some suggestions relating to characterization of the term belief and ways of dealing with it in future research.

1. INTRODUCTION

The purpose of this chapter is to draw attention to theoretical deficiencies in belief research. First, the concept of belief (and other related concepts) is often left undefined (e.g., Cooney, Shealy, & Arvold, 1998) or researchers give their own, possibly contradictory, definitions (e.g., Bassarear, 1989; Underhill, 1988). A second important problem is the inability to clarify the relations between belief and knowledge. At the end of the chapter, we describe the results of an empirical research study in which we tried to characterize the concept of belief based on views that emerged from written reports of mathematics education specialists in the field of beliefs.

2. BELIEFS AND CONCEPTIONS

Beliefs and belief systems began to be examined, to some extent, at the beginning of this century, mainly in social psychology (Thompson, 1992). But before long, behaviorism began to dominate research in the psychological domains. The focus turned to the observable aspects of human behavior, and beliefs were nearly forgotten. New interest in beliefs and belief systems emerged mainly in the 1970s, through developments in cognitive science (Abelson, 1979).

Individuals continuously receive signals from the world around them. According to their perceptions and experiences based on these messages, they draw conclusions

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about different phenomena and their nature. Individuals' subjective knowledge, i.e., their beliefs (including affective factors), is a compound of these conclusions. Furthermore, they compare these beliefs with new experiences and with the beliefs of other individuals, and thus their beliefs are under continuous evaluation and may change. When a new belief is adopted, this will automatically form a part of the larger structure of their subjective knowledge, i.e., of their belief system, since beliefs never appear fully independently. Thus, an individual's belief system is a compound of her conscious or unconscious beliefs, hypotheses or expectations and their combinations (Green, 1971).

2.1. Different Understandings of Beliefs

As discussed in other chapters of this book (Leder & Forgasz; Op't Eynde, De Corte, & Verschaffel; Törner), there are many variations of the concepts belief and belief system used in studies in the field of mathematics education. As a consequence of the vague characterization of the concept, researchers have often formulated their own definition of belief which might even be in contradiction with others. For example, Schoenfeld (1985, p. 44) states that in order to give a first rough impression "belief systems are one's mathematical world view". He later adds explanations of his position, interpreting beliefs "as an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (Schoenfeld, 1992, p. 358). Hart (1989, p. 44) – under the influence of Schoenfeld's (1985) and Silver's (1985) ideas – uses the word belief "to reflect certain types of judgments about a set of objects". Lester, Garofalo, and Kroll (1989, p. 77) explain that "beliefs constitute the individual's subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements". Törner and Grigutsch (1994) label their research object as the "mathematical world view", as is done in Schoenfeld (1985). In a recent paper by Grigutsch, Raatz, and Törner (1998) this concept is elaborated further, and anchored into the theory of attitudes, as explained, for example, in Olson and Zanna (1993). Other researchers, Underhill (1988) for one, think that beliefs are some kind of attitudes. Yet another different explanation is given by Bassarear (1989) who sees attitudes and beliefs on the opposite extremes of a bipolar dimension.

When looking at these different, and in some case even contradictory (Underhill, 1988; Bassarear, 1989) characterizations of beliefs, one observes that most of them (Underhill, 1988; Lester et al., 1989; Thompson, 1992; Furinghetti, 1996; Lloyd & Wilson, 1998) refer to the static part of beliefs saying: beliefs are, constitute, are contained etc. The definition given by Schoenfeld (1992) stresses the dynamic part of beliefs, i.e., how beliefs function. The definition proposed by Hart (1989) puts forward the aspect of judgments. The place of beliefs on the dimension affective – cognitive may be seen in different ways. If we were to stress the connections between beliefs and knowledge, we would see beliefs mainly as representatives of the cognitive structure of individuals. However, to see beliefs as a form of reactions toward a certain situation means that we consider beliefs to be linked to the affective part of individuals.

In research, there are representatives of both viewpoints. Some researchers consider beliefs as a real part of cognitive processing. Most researchers acknowledge that beliefs contain some affective elements, since the birth of beliefs happens in the social environment in which we live (McLeod, 1989, 1992). Among the six definitions of belief given above, those of Underhill (1988), and Lester et al. (1989) stress the affective component, whereas the definitions of Bassarear (1989) and Thompson (1992) are more on the cognitive side. In this book the different orientations are present in Goldin's chapter (affective orientation), in the chapters by Op't Eynde et al. and by Törner (cognitive orientation), while the chapter of Leder and Forgasz has a more marked mixed orientation (affective/cognitive).

In his study, Saari (1983) tried to structure the central concepts of the affective domain. He grouped them using three categories: feelings, belief systems, and optional behavior. Belief systems are seen as being developed from simple perceptual beliefs or authority beliefs – via new beliefs, expectations, conceptions, opinions and convictions – to a general conception of life. Such a viewpoint, that attitude has a component structure, seems to be commonly accepted in psychology today. One may find the following definition in the dictionary of psychology (e.g., Statt, 1990, p. 11): Attitude is "a stable, long-lasting, learned predisposition to respond to certain things in a certain way. The concept has a cognitive (belief) aspect, an affective (feeling) aspect, and a conative (action) aspect". The same threefold structure is found in many definitions on attitudes within research on mathematics education (e.g., Hart, 1989; Olson & Zanna, 1993; Ruffell, Mason, & Allen, 1998).

2.2. *Different Characterizations of Conceptions*

Conceptions belong to the same group of concepts as beliefs, which are also used in different ways in mathematics education (and the wider) literature. For example, Thompson (1992) understands beliefs as a sub-class of conceptions. But she claims that "the distinction [between beliefs and conceptions] may not be a terribly important one" (p. 130). Thompson's idea is taken up by Furinghetti (1996) who explains an individual's conception of mathematics as a set of certain beliefs. A different understanding is given by Pehkonen (1994) who, in accordance with Saari (1983), characterizes conceptions as conscious beliefs.

Some trials for a definition of conceptions are given, among others, by Freire and Sanches (1992), Ponte (1994), and Lloyd and Wilson (1998). For example, Lloyd and Wilson (1998, p. 249) connect beliefs with conceptions saying: "We use the word conceptions to refer to a person's general mental structures that encompass knowledge, beliefs, understandings, preferences, and views". But there are other researchers who clearly distinguish the meaning of these two terms. For example, this is the position emerging from the following passage of Ponte (1994) who has used Pajares (1992) as an authority:

> They [beliefs] state that something is either true or false, thus having a prepositional nature. Conceptions are cognitive constructs that may be viewed as the underlying organizing frames of concepts. They are essentially metaphorical. (p. 169)