This monograph introduces modern developments on the bound state problem in Schrödinger potential theory and its applications in particle physics.

The Schrödinger equation provides a framework for dealing with energy levels of N-body systems. It was a cornerstone of the quantum revolution in physics of the 1920s but re-emerged in the 1980s as a powerful tool in the study of spectra and decay properties of mesons and baryons. This book begins with a detailed study of two-body problems, including discussion of general properties, level ordering problems, energy-level spacing and decay properties. Following chapters treat relativistic generalizations, and the inverse problem. Finally, three-body problems and N-body problems are dealt with. Applications in particle and atomic physics are considered, including quarkonium spectroscopy. The emphasis throughout is on showing how the theory can be tested by experiment. Many references are provided.

The book will be of interest to theoretical as well as experimental particle and atomic physicists.

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PARTICLE PHYSICS AND THE SCHRÖDINGER EQUATION

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Particle Physics and the Schrödinger Equation

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To Heidi and Schu

Contents

Preface

Until 1975 the Schrödinger equation had rather little to do with modern particle physics, with a few exceptions. After November 1974, when it was understood that the J/ψ was made of heavy quark–antiquark pairs, there was a renewed interest in potential models of hadrons, which continued with the discovery of the b quark in 1977. The parallel with positronium was obvious; this is the origin of the neologism "quarkonium". However, in contrast to positronium, which is dominated by the Coulomb potential, the potential between quarks was not known and outside explicit numerical calculations with specific models, there was a definite need for new theoretical tools to study the energy levels, partial widths, radiative transitions, etc. for large classes of potentials. This led to the discovery of a large number of completely new rigorous results on the Schrödinger equation which are interesting not only for the qualitative understanding of quarkonium and more generally hadrons but also in themselves and which can be in turn applied to other fields such as atomic physics. All this material is scattered in various physics journals, except for the *Physics Reports* by Quigg and Rosner on the one hand and by the present authors on the other hand, which are partly obsolete, and the review by one of us (A.M.) in the proceedings of the 1986 Schladming "Internationale Universitätswochen für Kernphysik", to which we will refer later. There was a clear need to collect the most important exact results and present them in an orderly way. This is what we are trying to do in the present book, or least up to a certain cut-off date, since new theorems and new applications continue to appear. This date may look rather far away since it is the beginning of 1995; for instance, the results of J.M. Richard and one of us (A.M.) on the Ω_c particle are not included.

There are two focuses of the book. On the one hand we have rigorous theorems. On the other hand, we have applications to atomic and particle physics which were spectacularly successful, but there is absolutely no attempt to justify at a fundamental level the use of potential models in hadron physics because we feel that its main justification is its success. In addition we felt that we could not avoid presenting a short review of more classical problems like the counting of bound states in potentials, where progress has been made in the last 20 years.

This book does not contain all the material collected in the reviews we mentioned. For instance, the behaviour of the energy levels for large quantum numbers is not reproduced (see the review of Quigg and Rosner and the work of Fulton, Feldman and Devoto both quoted later). The reader will certainly notice, from chapter to chapter, differences in style. However, this book has the merit of being the only one making it possible for a newcomer to become acquainted with the whole subject. Another of its merit is that it does not need any preliminary sophisticated mathematical knowledge. All that is required in most of the book is to know what a second-order differential equation is.

We must warn the reader of the fact that, contrary to common usage, theorems are not numbered separately but like equations, on the right-hand side of the page.

We have to thank many people and primarily Peter Landshoff, who asked us to write this book, and kept insisting, as years passed, until we started working seriously. Our wives, Schu and Heidi, also insisted and we are grateful for that.

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Vienna and Geneva H. Grosse and A. Martin