Preface

This Edited Volume is based on a workshop on "Mathematical and Physical Aspects of Quantum Gravity" held at the *Heinrich-Fabri* Institute in Blaubeuren (Germany) from July 28th to August 1st, 2005. This workshop was the successor of a similar workshop held at the same place in September 2003 on the issue of "Mathematical and Physical Aspects of Quantum Field Theories". Both workshops were intended to bring together mathematicians and physicists to discuss profound questions within the non-empty intersection of mathematics and physics. The basic idea of this series of workshops is to cover a broad range of different approaches (both mathematical and physical) to a specific subject in mathematical physics. The series of workshops is intended, in particular, to discuss the basic conceptual ideas behind different mathematical and physical approaches to the subject matter concerned.

The workshop on which this volume is based was devoted to what is commonly regarded as the biggest challenge in mathematical physics: the "quantization of gravity". The gravitational interaction is known to be very different from the known interactions like, for instance, the electroweak or strong interaction of elementary particles. First of all, to our knowledge, any kind of energy has a gravitational coupling. Second, since Einstein it is widely accepted that gravity is intimately related to the structure of space-time. Both facts have far reaching consequences for any attempt to develop a quantum theory of gravity. For instance, the first fact questions our understanding of "quantization" as it has been developed in elementary particle physics. In fact, this understanding is very much related to the "quantum of energy" encountered in the concept of photons in the quantum theory of electromagnetism. However, in Einstein's theory of gravity the gravitational field does not carry (local) energy. While general relativity is a local theory, the notion of gravitational energy is still a "global" issue which, however, is not yet well-defined even within the context of classical gravity. The second fact seems to clearly indicate that a quantum theory of gravity will radically chance our ideas about the structure of space-time. It is thus supposed that a quantum version of gravity is deeply related to our two basic concepts: "quantum" and "space-time" which have been developed so successfully over the last one-hundred years.

The idea of the second workshop was to provide a forum to discuss different approaches to a possible theory of quantum gravity. Besides the two major accepted roads provided by String Theory and Loop Quantum Gravity, also other ideas were discussed, like those, for instance, based on A. Connes' non-commutative geometry. Also, possible experimental evidence of a quantum structure of gravity was discussed. However, it was not intended to cover the latest technical results but instead to summarize some of the basic features of the existing ansätze to formulate a quantum theory of gravity. The present volume provides an appropriate cross-section of the discussion. The refereed articles are written with the intention to bring together experts working in different fields in mathematics and physics and who are interested in the subject of quantum gravity. The volume provides the reader with some overview about most of the accepted approaches to develop a quantum gravity theory. The articles are purposely written in a less technical style than usual and are mainly intended to discuss the major questions related to the subject of the workshop.

Since this volume covers rather different perspectives, the editors thought it might be helpful to start the volume by providing a brief summary of each of the various articles. Obviously, such a summary will necessarily reflect the editors' understanding of the subject matter.

The volume starts with an overview, presented by **Claus Kiefer**, on the main roads towards a quantum theory of relativity. The chapter is nontechnical and comprehensibly written. The author starts his article with a brief motivation why there is a need to consider a quantum theory of gravity. Next, he discusses several aspects of the different approaches presented in this volume. For instance, he contrasts background independent approaches with background dependent theories. The chapter closes with a brief summary of some of the main results obtained so far to achieve a quantum version of gravity.

In the second article **Claus Lämmerzahl** reports on the experimental status of quantum gravity effects. On the one hand, gravity is assumed to be "universal". On the other hand, quantum theory is regarded as being "fundamental". As a consequence, one should expect that a quantum theory of gravity will yield corrections to any physical process. Recent experiments, however, confirm with high precision the theory of relativity and quantum theory. Nonetheless, the author indicates how astrophysical as well as laboratory and satelite experiments may be improved in accuracy so that possible quantum gravity effects could be observed not too far in the future. Lämmerzahl describes at which scales and parameter ranges it could be more promising to push forward experimental efforts. The article closes with a discussion on recent proposals to increase experimental accuracy. A wealth of information is presented in a very readable style.

In their contribution the authors **Alfredo Macias** and **Hernando Quevedo** review in a very precise and compelling way the role of time in the process of (canonical) quantization. They discuss different approaches to solve the so-called "time paradox". The different conclusions drawn from their analysis imply that, after 70 years of attempts to quantize gravity, the fundamental "problem of time" is still an unresolved and fascinating issue.

In the search for quantum gravity the approach proposed by **Louis Kauffman** assumes non-commutative variables. In his contribution the author considers a non-commutative description of the world from an operational point of view. He introduces in a fascinating and straightforward approach a differential calculus that permits rephrasing some of the most basic notions known from classical differential geometry without the use of smooth manifolds. This includes, especially, the notion of Riemannian curvature, which is less simple to algebraically rephrase than the notion of Yang-Mills curvature of ordinary gauge theories. The discussion presented in this article should be contrasted with the contributions by Majid and Paschke as well as with the ideas presented by Grosse and Wulkenhaar concerning a non-commutative quantum field theory.

In contrast to the approach by Kauffman, the contribution to this volume by **Shahn Majid** uses the framework of Hopf algebras and (bi)covariant calculi on the former to address the problem of quantum gravity within an elaborated algebraic framework. The author starts by presenting the basic mathematical material in order to afterwards discuss a whole series of examples. These examples provide the reader with an introduction to a possible quantum theory of gravity on finite sets. The outlook of the article addresses further generalizations toward a purely functorial setting and a statement about the author's viewpoint on why this is needed to put quantum theory and gravity into a single framework. The author concludes his contribution with a number of remarks concerning some links to several other contributions of this volume.

Group field theory is an elaborated extension of Penrose's spin networks and spin foams. **Daniele Oriti** critically describes the challenges and achievements of group field theory. It seems possible that this way of generalized loop quantum gravity provides a richer framework that permits to handle problems like the Hamiltonian constraint. The author puts emphasis on the viewpoint that group field theory should be regarded as being a theoretical framework of its own.

Alain Connes' non-commutative geometry may provide an alternative approach to a quantum theory of gravity. In his contribution **Mario Paschke** gives an overview on the present status of this approach. The author focusses his attention on the role of the so-called "spectral action". In particular, he critically discusses the need to extend non-commutative geometry to Lorentzian signature and to study globally hyperbolic spectral triples. This viewpoint may provide a Lorentzian covariant and hence a more physically convincing approach to a non-commutative generalization of gravity. In this respect the article is closely related to the ideas concerning a covariant description of a perturbative quantum field theory as it is proposed by Brunetti and Fredenhagen in the next contribution.

Romeo Brunetti and **Klaus Fredenhagen** propose a certain background independent axiomatic formulation of perturbative quantum gravity. This formulation is based on a functorial mapping from the category of globally hyperbolic manifolds to the category of $*$-algebras. As explained in some details, the axioms for this functor are physically well motivated and permit to consider a quantum field as a fundamental local observable.

A major question of interest is the study of representations of the diffeomorphism group for any diffeomorphism invariant theory, like general relativity. In the case of globally hyperbolic space-times, these representations are related to the topology of spatially closed orientable 3-manifolds. One expects that a quantum theory of gravity would yield a super-selection structure that is induced by the topology of the classical (limiting) space. In his contribution to this volume, **Domenico Giulini** provides a well written introduction to this fascinating topic. He puts emphasis on a geometrical understanding of the mapping-class group of 3-manifolds and includes many illuminating pictures for illustration.

Next, **Christian Fleischhack** studies uniqueness theorems in loop quantum gravity analogous to the famous Stone – von Neumann theorem of ordinary quantum mechanics that guarantees the unitary equivalence of all the irreducible representations of the Heisenberg algebra. Due to the tremendously more complicated configuration space of loop quantum gravity, it is of utmost importance to know whether different quantization schemes may give rise to different physical predictions. Fleischhack's discussion of two uniqueness theorems proves that, under certain technical assumptions, an almost unique quantization procedure can be obtained for Ashtekar's formulation of a quantum gravity.

String theory is known to naturally include a spin-two field. When quantized, this field is commonly interpreted as graviton analogous to the photon in quantum electrodynamics. A basic object in any string theoretical formulation of a quantum theory of gravity plays the partition function defined in terms of appropriate functional integrals. A perturbative evaluation of the partition functions yields topological invariants of the background manifolds under consideration. **Kishore Marathe** discusses several aspects of the interplay between topological quantum field theory and quantum gravity. For instance, he discusses the Jones polynomial and other related knot invariants of low-dimensional smooth manifolds.

A rather different route to quantum gravity is proposed by **Felix Finster**. His "principle of the fermionic projector" summarizes the idea to start the formulation of a quantum theory of gravity from a set of points, a certain set of projectors related to these points and a discrete variational principle. The author summarizes the basic ideas how gravity and the gauge theory may be formulated within the framework presented in his contribution. Contrary to the common belief, the author considers locality and causality as fundamental notions only in the continuum limit of "quantum space-time".

Black holes are models of actual astrophysical effects involving strong gravitational fields. Hence, black holes are optimally suited as a theoretical laboratory for quantum gravity. In his article, **Thomas Mohaupt** deals with a string theoretical approach to black hole physics. The usual black hole quantum theory is heuristic and needs to be supported by a microscopic (statistical) theory. Formal arguments from string theory permit possible scenarios to construct densities of states that give rise to a statistical definition of entropy. Furthermore, Mohaupt's article demonstrates how effects even next to the leading order can be given a satisfactory explanation by the identification of different statistical ensembles.

Quantum mechanics originated in the attempt to understand experimental results which were in sharp contrast with Maxwell's electrodynamics. In this respect, one of the most crucial experimental effects was what is called today the "photon electric effect". Together with the black body radiation, the photon electric effect may be considered as the birth of the idea of the "quantum of (electromagnetic) energy". This idea, in turn, is known to have been fundamental for the development of the quantum theory of Maxwell's electrodynamics. The authors **Tekin Dereli** and **Robin W. Tucker** start out from the question of whether there is a similar effect related to the "quantum of gravitational energy". Analogous to electrodynamics one may look for a Hamiltonian that incorporates the energy of the classical gravitational field. In Einstein's theory of gravity this is known to be a non-trivial task. For this the authors introduce a different Lagrangian density which includes additional degrees of freedom and from which they derive an energy momentum tensor of the gravitational field. Moreover, the authors discuss specific gravitational (plane) wave like solutions of their generalized gravitational field equations which may be regarded as being similar to the electromagnetic plane waves in ordinary electromagnetism.

General relativity is known to be a perturbatively non-renormalizable theory. Such a theory needs the introduction of infinitely many free parameters ("counter terms"), which seems to spoil any predictive power of the corresponding quantum theory. Using the renormalization group **Oliver Lauscher** and **Martin Reuter** discuss the existence of non-Gaussian fixed points of the renormalization group flow such that the number of counter terms can be restricted to a finite number. Such a scenario can be obtained from numerical techniques called "asymptotic safety". Employing techniques from random walks one can show that a scale dependent effective theory which probes the nature of space-time on that particular scale can be obtained. The renormalization group trajectories permit discussing space-time properties on changing scales. While for large scales a smooth four dimensional manifold occurs, at small scales a fractal space-time of dimension two is obtained. Similar results are obtained using the idea of numerical dynamical triangulation which has been introduced by other groups.

The idea to change the structure of space-time at small distances in order to cure divergence problems in ordinary quantum field theory was introduced by Heisenberg and Schrödinger and was firstly published by Snyder. Recent developments concerning D-branes in string theory also support such a scenario. Yet another approach to a not point-like structure of space-time has been introduced by Dopplicher, Fredenhagen and Roberts in the 1990's. It is based on the idea to also obtain an uncertainty principle for the configuration space similar to the known phase space uncertainty of ordinary quantum mechanics. **Harald Grosse** and **Raimar Wulkenhaar** close this volume by a summary of their pioneering work on the construction of a specific model of a renormalizable quantum field theory on such a so-called "$\theta-$ deformed" space-time. They also describe the relation of their model to multi-scale matrix models.

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